PATH INFORMATION IN QUANTUM INTERFEROMETRY

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1 Introduction

Ever since the famous Bohr-Einstein dialogue,[1] it has been known that it is not possible in an interference experiment to have a maximum visibility interference pattern *and* path information at the same time. This feature of quantum mechanics, necessary for its consistency, has been elevated by Feynman[2] to a principle: whenever it is not possible, not even in principle, to obtain path information, then one has to superpose probability amplitudes instead of adding probabilities for making experimental predictions.

Bohr introduced the notion of complementarity to describe situations where two observables cannot be known exactly at the same time, of which the Heisenberg uncertainty principle is a special case. For Bohr, complementarity was a consequence of the fact that the very design of an apparatus for measuring one quantity, say, the position, excludes measurement of the complementary quantity, here, momentum.

In the present paper we discuss three explicit cases of complementarity between interference and path information and we present some interesting consequences. In section 2, the experimental realization of a two-photon quantum eraser is given, in section 3 we discuss the realization of a new intense source for entangled photons based on these ideas, and in section 4 a nontrivial relation between Aharonov-Bohm and Einstein-Podolsky-Rosen nonlocalities, which is intimately related to path information considerations, is presented.

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2 Two-Photon Quantum Erasers

An experiment was explicitly proposed by Scully and Drühl[3] where interference can be recovered if one destroys (erases) path information. Recently, we have performed a series of experiments[4] where we demonstrate quantum eraser features in a very explicit way.

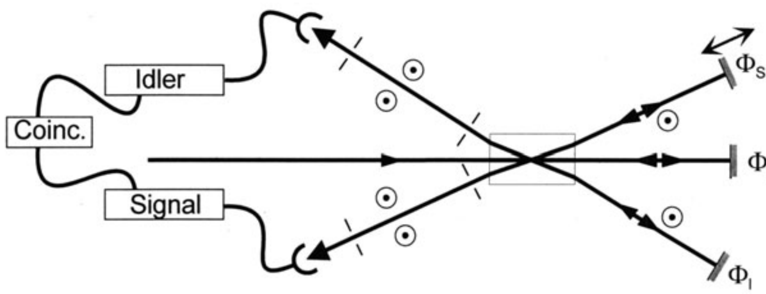

Figure 1: A photon pair, consisting of the signal photon and the idler photon, can be created in two possible ways: Either by the pump on the passage from left to right or by the pump on passage from right to left.[5] Mirrors and diaphragms are then used in such a way that it becomes indistinguishable for the photon pair arriving at the detectors whether it was created in the first or the second process. Thus, interference arises for each photon alone. Coincidence registration then demonstrates that this is a two-photon, i.e. quantum, process.

Consider first the setup of Fig. (1). There, we have obtained quantum interference by having a pump beam pass twice through a suitable nonlinear crystal.[5] In that way, a photon pair can be created by the pump either on a passage from left to right or during the return passage from right to left. External mirrors and suitable diaphragms are arranged such that the modes into which the photons are emitted during forward passage overlap completely with the modes into which the photons are emitted during backward passage; furthermore, since type-I down-conversion is used in these experiments, the photons produced carry the same polarization ($\odot$). Thus, if certain coherence conditions on the distances of the mirrors from the crystal are met, no possible measurement on the two outgoing beams can decide on which passage the photons were created, and therefore complete coherence results. Since the photons, historically called signal and idler, are always created in pairs, this implies that the two singles intensities (I_s, I_i) and the coincidence rate (I_c) are identical:

$$I_s = I_i = I_c = 2I_o(1 + cos\phi), \tag{1}$$

where $\phi = \phi_s + \phi_i - \phi_p$ with the individual phases depending on the positions of the retroflection mirrors.

We now consider Fig. (2, top) where path information (a quantum marker) is introduced for both photons. This is done by inserting quarter-wave plates of proper orientation. Each photon passes twice through its quarter-wave plate and as a result finds its polarization rotated by 90 degrees. Thus measuring the polarization of either photon in the $\odot$-$\updownarrow$ basis we could determine whether the pair was created during forward passage or during backward passage; therefore no interference is observable (Fig. (2, left)). Each photon carries path information for both photons because of the two-photon character of our experiment. The state of the two photons emerging from the

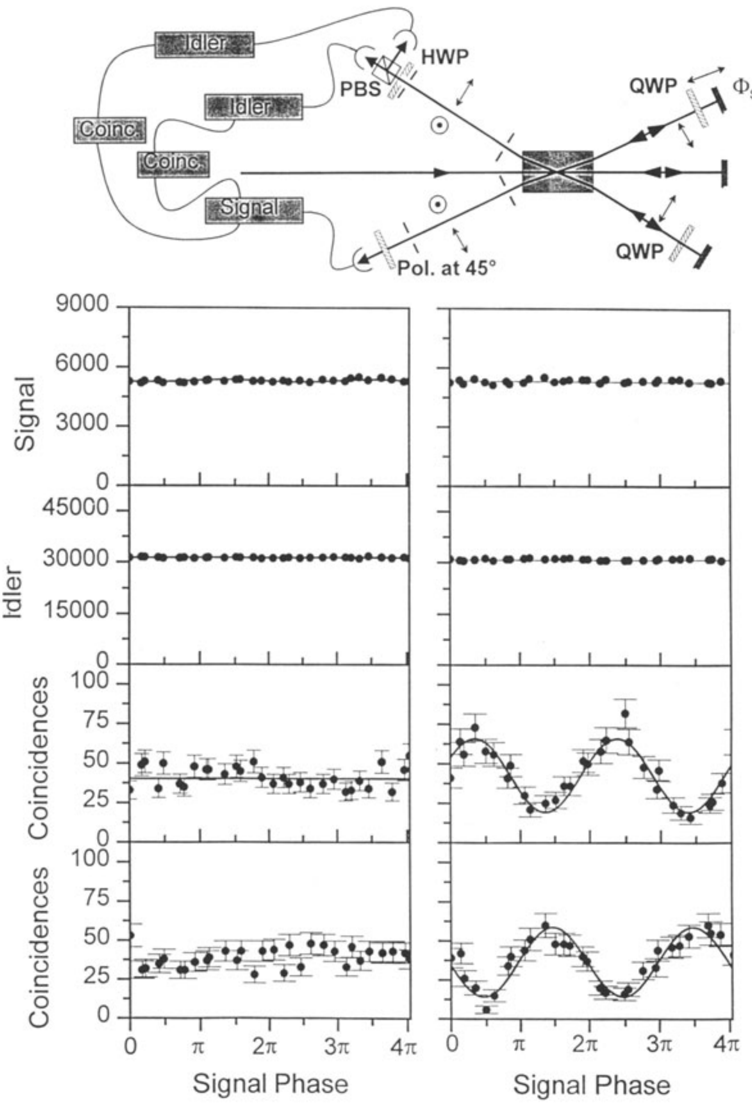

Figure 2: Principle of the two photon quantum eraser experiment (top). Quarter-wave plates (QWP) are inserted both into the signal beam and into the idler beam, rotating the polarization from ⊙ to ↕. Thus, measurement of the polarization of the outgoing beam implies determination of the path. A polarizer oriented at 45° in the signal beam erases its path information. A polarizing beam splitter (PBS), together with the half-wave plate (HWP), is inserted into the outgoing idler beam. Intensities and coincidence rates are monitored as functions of the signal phase (Φ_S). With the half-wave plate at 0°, polarization measurement via PBS implies path information, and no interferences result (left). If the half-wave plate is oriented at 22.5°, the outgoing beams represent polarizations along the +45° and −45° directions respectively, and therefore they carry no path information. In that case, the individual count rates (right) still don't show interference, because each photon seperately does not "know" whether the other one is interrogated for path information or not. Yet interference fringes arise in coincidence registration (right, bottom), thus demonstrating complete erasure of path information.

crystal is

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\odot\rangle_i|\odot\rangle_s + e^{i\phi}|\updownarrow\rangle_i|\updownarrow\rangle_s), \tag{2}$$

which is a maximally entangled state. However, it is now possible to recover interference by erasing the path information in both photons through insertion of a polarizer oriented at 45° into each output beam. The interesting observation is that still none of the individual intensities after the polarizer shows interference fringes (Fig. (2, right)). This is again a consequence of the fact that both photons initially carry path information and, colloquially speaking, neither photon "knows" whether we decide to read out or to erase the path information for the other one. Interference fringes therefore can only arise in the coincidence count rates, as shown in Fig. (2, right). Registration of both photons is actually necessary to destroy the path information.

In related experiments,[4] we studied the influence of a quantum marker / quantum eraser inserted in the path of only one of the photons. Alternatively, we also used photon flight time for path information.

3 An Intense Polarization-Entangled Photon Source

In type I down-conversion, as used in the experiments mentioned above, the two photons created have the same polarization. Thus, in order to utilize such a source for tests of Bell's inequalities, one either has to change the polarization of some of the modes and subsequently postselect polarization-entangled states[6] or one has to select different spatiotemporal modes and superimpose them.[7] In their experimental realization, all these experiments therefore had some disadvantages as to either the inherent non-entangled nature of the initial photon state or impurities of the actual correlations.

Recently,[8] in collaboration with A.V. Sergienko and Y.H. Shih, we have realized a type-II down-conversion source for polarization entangled photons. In type-II down-conversion, one of the resulting photons is extraordinary- and the other one ordinary-polarized. For certain orientations of the crystal optic axis and the pump, the two photons emerge on cones, as indicated in Fig. (3). In general the two photons can be distinguished by their polarization, and knowledge of the path into which a photon is emitted implies knowledge of the polarization. Yet, along the two directions where the two cones intersect, the polarization of the two photons is undefined, and this path information does not yield polarization information. Thus we obtain the polarization entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\odot\rangle_1|\updownarrow\rangle_2) + e^{i\phi}|\updownarrow\rangle_1|\odot\rangle_2, \tag{3}$$

where the phase ϕ is a property of experimental details.

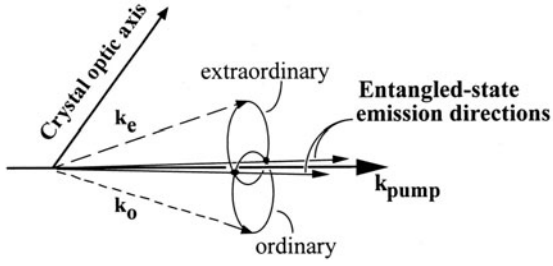

Figure 3: Spontaneous down-conversion cones present with type-II phase matching. Correlated photons lie symmetrically on opposite sides of the pump beam.

We should notice[8] that the state directly emerging from the crystal is more complicated, because birefringence results in - partial - longitudinal and transverse separation of the two polarization states. Yet, this can be sufficiently well compensated in our experiment (see Fig. (4)) for the state of Eq. (3) to emerge finally.

The primary advantage of our new source is that it is polarization entangled and that the emission directions of the photons are well defined. One direct application of this feature is that it is easy to convert the resulting state into any one of the four Bell basis states using just half-wave plates and quarter-wave plates, as indicated in Fig. (4). These four states are

$$
\begin{aligned}
|\psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|\odot\rangle_1|\updownarrow\rangle_2 \pm |\updownarrow\rangle_1|\odot\rangle_2) \\
|\phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}(|\odot\rangle_1|\odot\rangle_2 \pm |\updownarrow\rangle_1|\updownarrow\rangle_2).
\end{aligned}
\tag{4}
$$

They form a maximally entangled basis of the two-particle Hilbert space, and are of key importance for many quantum communication and quantum information schemes.

The high coincidence fringe visibility for the Bell states is an indication of the purity of the entangled states. A further indication is the observed violation of a Bell-type inequality[9] which for any realistic theory implies $|S| \leq 2$, where S is a certain combination of coincidence rates. Experimentally, we obtained for the $|\psi^+\rangle$ state $S = -2.6489 \pm 0.0064$ and for the $|\psi^-\rangle$ state $S = -2.6900 \pm 0.0066$: a violation of Bell's inequality by more than 100 standard deviations in a measurement time of less than 5 minutes!

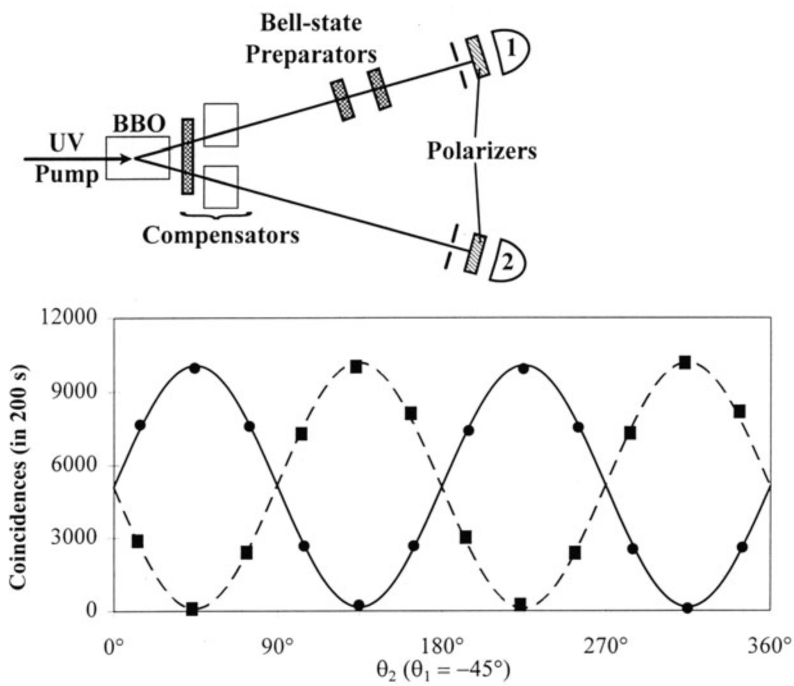

Figure 4: Principle of our method to directly produce polarization entangled states from the down-conversion crystal (top). Coincidence fringes for the $|\psi^+\rangle$ state (□) and for the $|\psi^-\rangle$ state (•) as a function of the angular settings of the polarizers (bottom).

4 Entanglement and Gauge Invariance

In quantum mechanics, both Einstein-Podolsky-Rosen correlations[10] and the Aharonov-Bohm effect[11] are usually understood to be indicative of nonlocality. It has been suggested by A. Shimony[12] that it would "indeed be surprising if these two nonlocalities were not related". Here we show that gauge invariance in an Aharonov-Bohm type experiment with entangled charged particles[13] demands that in such an experiment no first order fringes can exist. Only second order fringes as implied by entangled states are gauge invariant. This also represents an interesting special case of the complementarity between first- and second-order interference.[14]

Consider the gedanken experiment of Fig. (5), left picture. A particle with charge q_0 emerging from S propagates to the two slits S_1 and S_2, where it decays into two new particles with charges q and q' respectively. The particle q will be observed on the observation screen while the particle q' is allowed to escape. We then look for Aharonov-Bohm fringes for particle q upon variation of the magnetic field $\vec{B}$ in the solenoid enclosed by the beam path. The intensity at the observation point P is determined by the phase difference between the two paths leading to P and this may be written as

$$\Delta\phi_P = \Delta\phi_P(\vec{k}, \vec{K}) + \Delta\phi_P(\vec{A}), \tag{5}$$

where $\Delta\phi_P(\vec{k}, \vec{K})$ is the spatial part of the phase difference, and $\Delta\phi_P(\vec{A})$ is its gauge part:

$$\Delta\phi_P(\vec{A}) = \frac{q_0}{\hbar c} \int\limits_{S_1-S-S_2} \vec{A}\cdot d\vec{s} - \frac{q}{\hbar c} \int\limits_{S_1-P-S_2} \vec{A}\cdot d\vec{s}. \tag{6}$$

where, e.g., $S_1 - S - S_2$ implies the path integral from S_1 to S_2 via S. Thus, we immediately find that the resulting one-particle phase difference is not gauge invariant, because it cannot be rewritten as one or more closed loop integrals. We are therefore forced to conclude that no single particle interference pattern arises in this situation. This is indicative of the fact that the state after decay of the mother particle is really an entangled two-particle state. The nonexistence of an interference pattern for each particle separately may be traced to Welcher-Weg detection considerations. Since both particles are created together, it is evident that observation of one particle may provide

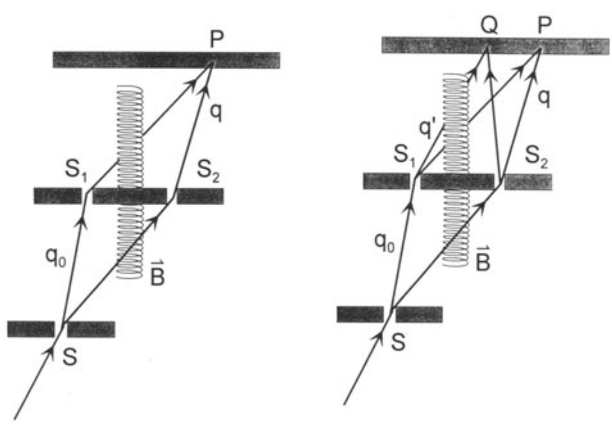

Figure 5: A particle with charge q_0 is subject to a double-slit experiment such that it decays into two particles with charge q and q' in the region of the two slits S_1 and S_2. An infinitely long solenoid is used to create an Aharonov-Bohm type phase shift, either for the single particle fringes (left) or for the two-particle fringes (right).

path information about the other particle. It is only through registration of both particles that the path information is irrecoverably destroyed. Thus, interference fringes can only arise in the two-particle coincidences.

Now, we analyze the phase conditions for two-particle interference fringes. Calculating the probability to find particle q at P and particle q' at Q, we have to superpose the probability amplitude that particle q arrives at P via S_1 and particle q' arrives at Q via S_2, with the probability amplitude that particle q arrives at P via S_2 and particle q' arrives at Q via S_1. The resulting overall phase difference for the particle pair arriving at the observation points P and Q may be written as

$$\Delta\phi_{PQ} = \Delta\phi_{PQ}(\vec{k}, \vec{K}) + \Delta\phi_{PQ}(\vec{A}), \tag{7}$$

where $\Delta\phi_{PQ}(\vec{k}, \vec{K})$ is the spatial part of the phase difference, and $\Delta\phi_{PQ}(\vec{A})$ is its gauge part which is

$$\Delta\phi_{PQ}(\vec{A}) = \frac{q_0}{\hbar c} \int\limits_{S_1-S-S_2} \vec{A} \cdot d\vec{s} - \frac{q}{\hbar c} \int\limits_{S_1-P-S_2} \vec{A} \cdot d\vec{s} - \frac{q'}{\hbar c} \int\limits_{S_1-Q-S_2} \vec{A} \cdot d\vec{s}. \tag{8}$$

Finally we obtain for this two-particle phase difference

$$\Delta\phi_{PQ}(\vec{A}) = \frac{q}{\hbar c} \oint_I \vec{A} \cdot d\vec{s} + \frac{q'}{\hbar c} \oint_{II} \vec{A} \cdot d\vec{s}, \tag{9}$$

where the path integral I is taken around the loop $S - S_1 - P - S_2 - S$ and the path integral II around the loop $S - S_1 - Q - S_2 - S$. It is evident that $\Delta\phi_{PQ}(\vec{A})$ is now gauge invariant, as necessary.

A most important consequence of our analysis is that, if one only calculates the phase difference for singles fringes, that is, for one particle alone, one obtains a phase which is not gauge invariant. This implies that in such an experiment, whenever one particle can be used to determine the path of the other, the requirement of gauge invariance in itself can only be fulfilled by two-particle fringes. This, we submit, implies a very deep connection between gauge invariance and entanglement. Also, it is interesting that Eq. (9) only follows from Eq. (8) - and hence the phase $\Delta\phi_{PQ}$ is only gauge invariant - if exact charge conservation holds.

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