Optical Implementation of Quantum Orienteering

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We present results from an optical implementation of quantum orienteering, a protocol for communicating directions in space using quantum bits. We show how different types of measurements and encodings can be used to increase the communication efficiency. In particular, if Alice and Bob use two spin-1/2 particles for communication and employ joint measurements, they do better than is possible with local operations and classical communication. Furthermore, by using oppositely oriented spins, the achievable communication efficiency is further increased. Finally, we discuss the limitations of an optical approach: our results highlight the usually overlooked nonequivalence of different physical encodings of quantum bits.

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Quantum orienteering is a quantum communication protocol addressing the problem "how can Alice most efficiently communicate a direction in space to Bob"? This problem can be solved using classical or quantum communication. If Alice and Bob share a reference frame (such as that provided by well-known distant stars, or Earth's magnetic field), they can transmit a classical signal, e.g., a binary encoding of the polar and azimuthal angles, allowing straightforward communication of a direction to a precision limited only by the number of bits they use. However, if they do not share a frame, Alice and Bob must transmit a physical object, such as a spinning gyroscope whose angular momentum points in the desired direction. Classical objects may, in principle, be measured to arbitrary precision, yielding a perfect transmission fidelity. However, for small systems, such as a single spin-1/2 particle, quantum mechanical uncertainty limits the accuracy with which Bob can estimate the direction. Here we examine how Alice and Bob can cooperate to maximize the information Bob can extract from such a system.

For quantum orienteering using two spin-1/2 particles, it has been proven that a straightforward measurement of the two particles independently does not yield the optimum fidelity of communication, but can be beaten by a joint measurement on the two particles [1,2]. Furthermore, if Alice changes her encoding by flipping the direction of the second spin, Bob can make even more precise measurements [3]. We have designed and implemented a sample protocol using photon polarization as our effective spin-1/2 system. Below we discuss the advantages and limitations of such an implementation. Others have investigated optimal strategies for more than two spin-1/2 particles [4,5] but here we consider only one- and two-spin cases. Quantum orienteering is similar to optimal detection [6] and unambiguous state discrimination [7,8], protocols for distinguishing between a fixed set of nonorthogonal states. Recently, Pryde *et al.* [9] showed the advantage of using collective measurement to distinguish whether a pair of spins were parallel or antiparallel. In contrast to these, orienteering involves providing the best estimate for an arbitrary state or direction.

In order to compare different protocols for orienteering, we must quantify the accuracy of transmission. We define the average transmission fidelity as

$$F \equiv \int d\Omega \frac{1 + \cos(\theta)}{2}, \tag{1}$$

where θ is the angle between Bob's guess and the direction Alice tried to send, and the integral is an average over all possible directions Alice might choose. For a random guess, then, the average fidelity will be 1/2, while perfect transmission gives a fidelity of 1. When Alice is only allowed a single spin-1/2 particle to transmit a direction, Bob's measurement is simple: he picks any direction $\hat{r}$ and measures the spin in that direction, e.g., with a Stern-Gerlach magnet. Bob's guess then is $\pm\hat{r}$, depending on whether the outcome of his measurement is $\pm 1/2$. This achieves a transmission fidelity of 2/3, the optimal value using only a single spin [2].

When Alice sends Bob two spins polarized in the same direction, Bob has several options. The simplest extension uses two independent measurements, one on each particle. The best way to do this is to measure in bases at 90° to each other, e.g., along $\hat{X}$ and $\hat{Y}$. This gives a fidelity of 73.3%. However, if Bob instead makes a single collective measurement on the two spins, he can make a slight improvement to $F = 75\%$ [2] by projecting into the basis

$$|\psi_k\rangle = \frac{\sqrt{3}}{2}|\hat{n}_k, \hat{n}_k\rangle + \frac{1}{2}e^{i\phi_k}|\psi^-\rangle. \tag{2}$$

The $|\hat{n}_k\rangle$ are the tetrahedral states shown in Fig. 1, while $|\psi^-\rangle$ is the antisymmetric Bell state $\frac{|HV\rangle - |VH\rangle}{\sqrt{2}}$. Adjusting the phase term $e^{i\phi_k}$ allows the set of states to be made orthonormal (see Table I).

Remarkably, even this is not the optimal protocol, though it is the best Bob can do if Alice polarizes both spins in the same direction. If instead Alice polarizes the

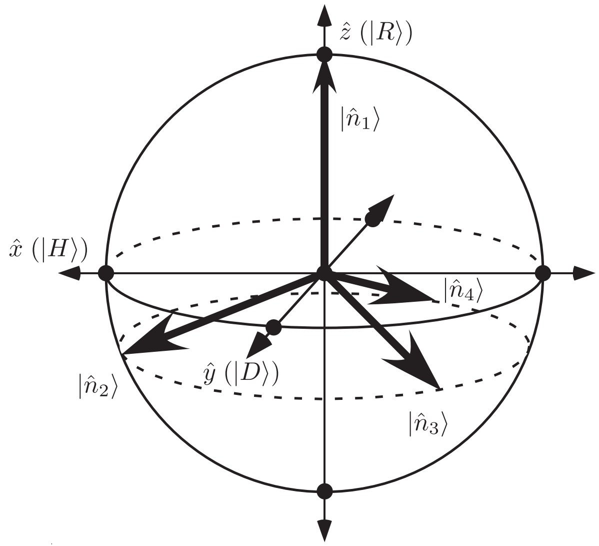

FIG. 1. The four $\hat{n}_j$ equally spaced directions in space corresponding to the corners of a regular tetrahedron. In our implementation we associate directions in real space with directions on the Poincaré sphere of polarization states: right-circular polarization ($|R\rangle$) is mapped to the $\hat{z}$ direction, horizontal polarization ($|H\rangle$) is mapped to $\hat{x}$, and 45° polarization ($|D\rangle$) to $\hat{y}$.

second spin in the opposite direction (and informs Bob she is doing so) Bob can make a different set of joint measurements:

$$|\psi'_k\rangle = \frac{\sqrt{3}}{2}\frac{|\hat{n}_k, -\hat{n}_k\rangle - |-\hat{n}_k, \hat{n}_k\rangle}{\sqrt{2}} + \frac{1}{2}e^{i\phi'_k}|\psi^-\rangle. \quad (3)$$

Again, the ϕ'_k's are chosen to make this an orthonormal basis. Using these measurements, Bob can increase the fidelity of his guess to 78.9%, the maximum achievable using two spin-1/2 particles [3]. The advantage of two antiparallel spins over two parallel spins has frequently been qualitatively explained as follows: the former states span the full Hilbert space, while the latter do not (they have no overlap with the singlet entangled state). Thus, the parallel spins have a smaller effective ''alphabet'' for encoding the direction. However, this intuitive argument has not been made rigorous, and recent work has in fact shown that the argument is incomplete and actually fails in some cases [10].

TABLE I. The coordinates of the four tetrahedral directions, along with a set of phases sufficient to make the measurement sets $|\psi_k\rangle$ and $|\psi'_k\rangle$ orthonormal.

	(X, Y, Z)	ϕ_k	ϕ'_k
$\hat{n}_1$	$(0, 0, 1)$	0	0
$\hat{n}_2$	$\frac{1}{3}(\sqrt{8}, 0, -1)$	π	0
$\hat{n}_3$	$\frac{1}{3}(-\sqrt{2}, \sqrt{6}, -1)$	1.897	0
$\hat{n}_4$	$\frac{1}{3}(-\sqrt{2}, -\sqrt{6}, -1)$	-1.897	0

While it is common to regard all two-level systems as equivalent, for this application they are not. Here we are not only interested in the geometry of state space, but its embedding into a physical object. It is thus important to note that while the spin of electrons and other massive fermions can be imagined to ''point'' in a specific direction (within the limits of the uncertainty principle), that is not the case for photons [11]. Consequently, photons cannot directly implement orienteering as described above. Neither our implementation, nor any other using photon polarization as the communication medium, can transmit a direction in space without some previously agreed upon reference [12]. Nevertheless, we are still able to demonstrate the operating principles of orienteering using photons, and, in particular, show explicitly the advantages of collective measurements and antiparallel encodings of the direction.

A number of factors make a direct photon- implementation difficult. The most important of these is that, as mentioned above, the polarization of a photon does not necessarily point in a particular direction. However, Alice and Bob can use the representation of polarization states on the Poincaré sphere to construct a mapping, such as that shown in Fig. 1, to convert photon polarization estimates into directions. Obviously, there are an infinite number of equivalent mappings. Bob and Alice's agreement on a particular one is tantamount to possessing a shared reference frame.

The second difficulty is that making the required joint measurements is challenging, equivalent to full Bell-state analysis. While one or two Bell states may be measured at a time, it is not, in general, possible to perform an efficient, arbitrary, four-outcome measurement of the two-photon polarization state with current technology [13]. However, if Alice and Bob do share a reference frame (required for the polarization-to-direction mapping already), they can combine part of the measurement into state creation. Specifically, since an arbitrary joint measurement without ancilla can be thought of as a two-qubit unitary operator $\hat{U}$ followed by a separable measurement $\hat{M}$ [18], we can directly apply the unitary to Alice's source state $|\psi\rangle$, allowing Bob to make a simple separable measurement: $(\hat{M}\cdot\hat{U})|\psi\rangle = \hat{M}(\hat{U}|\psi\rangle)$. This requires Alice to precisely create specific partially entangled states such as

$$\begin{aligned} |\psi_{111}\rangle \approx\, & 0.683|HH\rangle + (0.463 + 0.358i)|HV\rangle \\ & + (-0.208 + 0.383i)|VH\rangle \\ & + (-0.011 - 0.025i)|VV\rangle, \end{aligned} \quad (4)$$

equivalent to a parallel encoding of the direction $\frac{1}{\sqrt{3}} \times (1, 1, 1)$.

Figure 2 shows the system used to create these entangled states and perform the separable measurements on them.

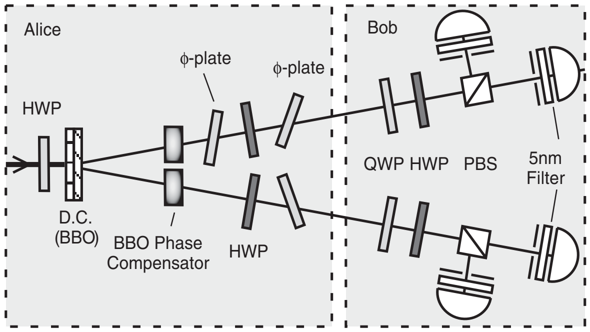

FIG. 2. Alice creates the necessary entangled states using parametric down-conversion in two BBO crystals. By changing the first half wave plate (HWP) she can create a nonmaximally entangled state of the form $\cos\theta|HH\rangle + e^{i\phi}\sin\theta|VV\rangle$. The BBO phase compensators correct for a spatial dependence of ϕ in the initial entangled state, allowing greater state purity [21]. Following that are several wave plates which allow Alice to create an arbitrary pure two-qubit state. The plates marked "ϕ plate" are wave plates (with their optic axes at 0°) which can be tipped to provide an arbitrary phase (ϕ) between $|H\rangle$ and $|V\rangle$, while the HWPs perform rotation by π about any linear axis. Bob uses a QWP, HWP, and polarizing beam splitter (PBS) in each arm, enabling an arbitrary projection on each qubit. This allows him to make the separable measurement for orienteering, and also to perform full state tomography [24].

Alice uses a tunable source of polarization entanglement based on spontaneous parametric down-conversion (SPDC) in a pair of nonlinear crystals (BBO) [19]. These crystals have their optic axes oriented such that when pumped by a UV laser (in our case, an Ar^+ laser at 351.1 nm), the first crystal generates horizontally polarized down-conversion pairs ($|HH\rangle$) while the second crystal generates vertically polarized pairs ($|VV\rangle$). If the coherence length of the pump is long, those processes are indistinguishable, resulting in the entangled state $\cos\theta|HH\rangle + \sin\theta|VV\rangle$. The weight of the two terms—and thus the degree of entanglement—is controlled by the pump polarization; the other 5 parameters in a general two-qubit pure state can be set by wave plates after the crystals [20]. In order to achieve the high state quality needed to be able to resolve the optimal fidelities of the different encodings, we also implemented a phase-compensation technique, described in detail elsewhere [21,22]. In each case, the required states for our experiment were created with a fidelity greater than 98%. [In fact, although transferring the unitary operator onto the state preparation step leads to particular relative phases as in Eq. (4)—Bob's separable measurement is actually insensitive to these, depending only on the magnitudes.]

We measured the transmission fidelity for three different two-photon protocols (Table II; for comparison we also list the measured fidelities if Alice sends only a single photon). First, we implemented the naïve approach where Bob makes separable measurements. The second protocol used the optimal joint measurements for a parallel encoding, while the final case used the optimal measurements for antiparallel encoding. In each case, we determined the fidelity for a large number of possible directions (see Fig. 3). The average orienteering fidelities (Table II) are near the theoretical values for all trials, thereby verifying the principle of using quantum encoding for orienteering. In particular, the experimental results for the antiparallel protocol exceed the theoretical bound for parallel spin protocols, and experimental results for joint measurements exceed the upper bound for separable measurements.

In addition, we examined cases where Alice is restricted to only send a subset of possible directions. For example, if Alice is constrained to sending states on the equatorial plane (equivalent to transmitting a phase), the optimal fidelity of 85.3% [4] is realized using orthogonal separable measurements (also in the plane). The greatest advantage of the antiparallel encoding is observed when Alice chooses to send one of the four tetrahedral directions, yielding a fidelity of 95.5% [3], much higher than the 83.3% using joint measurements on a parallel encoding.

Our results confirm the possibility of superior orienteering using quantum resources, i.e., that joint measurements on antiparallel spins outperform separable measurements. It is important to realize, however, that none of these algorithms is actually optimal when Alice and Bob share a reference frame. In that case, they actually do the best if Alice simply sends a 2-bit classical encoding of the direction, which amounts to telling Bob which of the tetrahedral directions from Fig. 1 is closest to her desired direction. This gives an average transmission fidelity of 85.3%, significantly higher than the optimal measurement on antiparallel spins (78.9%).

Quantum orienteering in the absence of a shared reference frame is interesting theoretically since it is a recent addition to a relatively small set of quantum communication protocols. Nevertheless, realizing a practical imple-

TABLE II. The average fidelities for each of the four protocols—the single-spin case and three variations on the two-spin protocol. We also show the average when Alice is confined to transmitting a direction on the equatorial plane, or picking one of the four tetrahedral directions. The (statistical) error on each value is ±0.1%, the theoretical limits are shown in [].

class	single-spin	separable	joint parallel	antiparallel
sphere	66.5 [66.7]	73.2 [73.6]	74.0 [75.0]	78.2 [78.9]
equator	74.8 [75.0]	84.1 [85.3]	74.0 [75.0]	78.4 [78.9]
tetrahedron	69.3 [69.4]	73.6 [73.6]	82.5 [83.3]	94.9 [95.5]

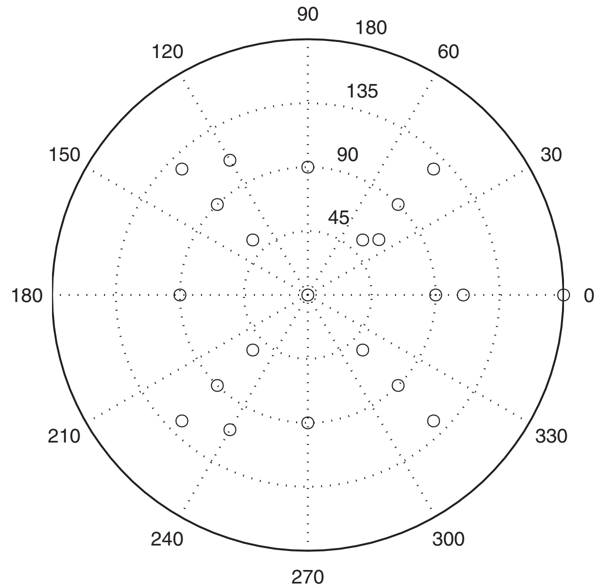

FIG. 3. For all four protocols, each of the directions indicated on this polar plot of the surface of Poincaré sphere were encoded. These include the four tetrahedral directions, the cardinal directions: $\pm\{\hat{X}, \hat{Y}, \hat{Z}\}$, four additional states on the equator at $\pm 45°$ and $\pm 135°$, and all eight points at the corner of an inscribed cube: $(\pm 1, \pm 1 \pm 1)/\sqrt{3}$. The center of the plot corresponds to the state $|R\rangle$ while the outer rim represents $|L\rangle$, encoding the states $\pm\hat{z}$, respectively.

mentation is likely to be problematic. While a system using electron spins, nuclear spins, or Rydberg atoms [which can also be used for a related protocol, transmitting a reference frame [23]] could avoid the photon-implementation issues discussed above, it introduces others, most notably sensitivity to stray magnetic fields, which would cause precession of the spins. For example, it seems unlikely that two parties could be in a position where it was technologically difficult or impossible to share a reference frame, yet they could control the magnetic fields well enough to communicate using quantum orienteering.

However, this opens up a new avenue for exploration: instead of agreeing on a global idea of a direction, Alice and Bob may wish to measure the rotation applied by the background fields. This, for instance, would allow partners with quantum computers to align their computational basis in the same direction, defined so that when Alice sends a "1," that is what Bob receives, and similarly for "0." Now, Alice is no longer trying to transmit a physical direction, and so this application may be performed not only with massive spins, but, e.g., with photon polarization. In this instance, two parties connected by a fiber optic cable, which in general performs an arbitrary rotation of the polarization, may identify a basis in which they can communicate most efficiently.

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