Little–Parks oscillations at low temperatures: Gigahertz resonator method

Andrey Belkin,[a)] Matthew Brenner, Thomas Aref, Jaseung Ku, and Alexey Bezryadin
Department of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA

(Received 16 March 2011; accepted 27 April 2011; published online 14 June 2011)

A thin-film Fabry–Perot superconducting resonator is used to reveal the Little and Parks (LP) effect [Phys. Rev. Lett. **9**, 9 (1962)], even at temperatures much lower than the critical temperature. A pair of parallel nanowires is incorporated into the resonator at the point of the supercurrent antinode. As the magnetic field is ramped, Meissner currents develop, changing the resonance frequency of the resonator. The LP oscillation is revealed as a periodic set of distorted parabolas observed in the transmission of the resonator and corresponds to the states of the wire loop having different vorticities. We also report a direct observation of single and double phase slip events. © *2011 American Institute of Physics*. [doi:10.1063/1.3593482]

The experimental demonstration of the fluxoid quantization was first reported by Little and Parks[1] (LP) who showed that the critical temperature of a thin-walled superconducting cylinder is a periodic function of the magnetic flux. The phase diagram of the thin-walled hollow cylindrical superconductor, obtained by LP from multiple resistance versus temperature measurements, revealed the presence of a clearly defined series of parabolic variations in the critical temperature with magnetic field.[2,3] More recently, Vakaryuk[4] predicted that at low temperatures the magnetic moment of a superconducting loop should oscillate with the applied field either with the LP period or with a doubled period.

So far measurements of the LP effect were mostly done near the critical temperature where the resistance is still high. At low temperatures the LP effect is hard to detect because the resistance is immeasurably low. Here we measure the effect of changing of *kinetic inductance* of a pair of parallel wires, which, together with superconducting electrodes, form a closed loop. These electrodes constitute a superconducting coplanar waveguide (CPW) resonator. The kinetic inductance variations,[5] caused by the Meissner currents, lead to a change in the resonance frequency, which produces changes in the transmission of the resonator. We show that the transmission coefficient S_{21} of the resonator-double-wire device has multiple, periodically spaced branches as a function of the magnetic field. Here, $S_{21}=V_{out}/V_{in}$, $S_{21}[\mathrm{dB}]=20\log|S_{21}|$, where V_{in} and V_{out} are, correspondingly, the voltage amplitudes of the applied and transmitted electromagnetic waves. The transition from one branch to the next one corresponds to a single Little's phase slip[6] (LPS) event, taking place in one of the wires. The periodic structure of the $S_{21}(H)$ dependence has the same origin as the LP critical temperature $T_c(H)$ periodic oscillations, which occur with a change in the applied magnetic field H. These oscillations are due to the oscillation of the supercurrent magnitude and, correspondingly, the free energy.

Our device consists of a pair of parallel superconducting nanowires incorporated in the center of a superconducting CPW resonator [Fig. 2 inset]. It is the same as in Fig. 1 of Ref. 7. The nanowires with thickness ~25 nm and length ~100 nm are produced by the molecular templating technique as follows:[8,9] a carbon nanotube is placed over a 100 nm wide trench on a SiO_2 substrate and sputter-coated with a superconducting alloy of $Mo_{79}Ge_{21}$. The superconducting critical temperature of the 25 nm film is expected to be close[10] to the critical temperature of the bulk $Mo_{79}Ge_{21}$, which is 7.36 K, while the transition temperature of nanowires is expected to be higher than 4.8 K, based on the comparison to previously measured thinner samples from Ref. 11. The resonators were patterned by means of optical lithography, followed by wet chemical etching in H_2O_2. The width of the center conductor is $w=10$ μm and the gap between the center conductor and ground plane is 6 μm. The length of the center conductor between the input and the output "mirrors" is 10 mm. The gap between the center conductor and the input/output electrode is 3 μm with corresponding capacitances of about 45 fF each. Thus, the resonators are over-coupled and their quality factors are about 500.

Two samples have been measured with different nanowire spacings and nanowire widths. In sample A the spacing is 6.63 μm, and in sample B it is 4.26 μm. The widths of the nanowires in sample A and sample B are ~21 nm and ~25 nm, correspondingly, based on SEM images. Each measurement is performed in a ^{3}He cryostat equipped with two semirigid coaxial cables with thermalized attenuators and a low temperature gigahertz amplifier LNF-LNC4_8A. The transmission measurements are performed using a network analyzer Agilent PNA5230A.

The measurements, shown in Fig. 1(a) for sample A and in the inset of Fig. 1(b) for sample B, reveal that $S_{21}(H)$ is a periodic multivalued function. Each branch of the $S_{21}(H)$ curve corresponds to a state with a particular number of phase vortices (or vorticity) trapped in the nanowire loop. A single branch of the function $S_{21}(H)$ (for sample A) measured at a temperature of 0.36 K is extracted and plotted in Fig. 1(b). As one can see, the transmission function is not truly parabolic as it has a flat top. This profile is preserved in a wide temperature interval [Fig. 1(b), inset].

To examine the periodicity of the transmission coefficient in the external magnetic field we compare the mean distance in magnetic field between the intersections of branches with the following vorticities: $(n,n\pm1)$ (not shown), $(n,n\pm2)$ [shown as circles in Fig. 1(a)], and $(n,n\pm3)$ [shown as triangles Fig. 1(a)]. This analysis yields the mean value of $\langle\Delta H(1.80\ \mathrm{K})\rangle=0.426$ Oe. The period is independent of the temperature, as was previously observed

[a)]Electronic mail: abelkin@illinois.edu.

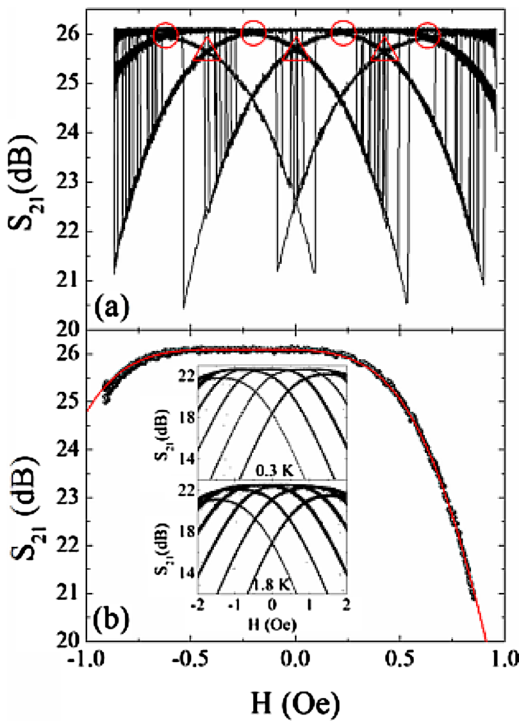

FIG. 1. (Color online) (a) The transmission coefficient of sample A as a function of the external magnetic field, measured at the zero-field resonance frequency and 360 mK. (b) A single branch of the experimental $S_{21}(H)$ dependence (black dots). The red curve shows the theoretical fit. The inset shows $S_{21}(H)$ for sample B measured at 0.3 K (top graph) and 1.8 K (bottom graph).

with dc measurements of double-wire devices.[12] The value of the period in a magnetic field allows one to establish the relationship between the applied magnetic field and the phase change in the superconducting order parameter of the studied double-wire system. Indeed, the period of the LP oscillations corresponds to the phase change of 2π on the closed loop; $\Delta H \times \beta = 2\pi$ from where we find the field-phase geometrical parameter $\beta \approx 14.8\ \text{Oe}^{-1}$. We have also tested that the observed multivalued response function is independent of the input power. The powers tested were -60, -70, and -80 dBm, referring to the output of the network analyzer. This is four, five, and six orders of magnitude lower than the critical power at which the current amplitude in the nanowires reaches the critical value.[7] The observed independence of the measured $S_{21}(H)$ on the input power proves that the measurement current is negligible compared to the Meissner current in the loop.

If the magnetic field is exactly perpendicular to the sample's surface, the period is given by[12] $\Delta B = (\pi^2/8G) \times (\Phi_0/aw)$, where a is the distance between the wires and $G = 0.916$ is the Catalan number[13] (the formula is exactly correct only if $w \gg a$). In the experiment the angle between the magnetic field and the sample's surface was $\theta \approx 35° - 40°$. The corresponding expression for the period is $\Delta B = (\pi^2/8G) \times (\Phi_0/aw \sin \theta)$. With $a = 6.63\ \mu$m and $w = 10\ \mu$m, we get $\Delta B = 0.59$ to 0.66 Oe. This is close to the measured value. The difference is due to the fact that the condition $w \gg a$ is not satisfied.

The transmission coefficient of the resonator depends on the resonance frequency f_0, the probe frequency f, and the resonator's quality factor Q, and is described by a Lorentzian $S_{21} = 10 \log\{A_0/[(f_0/Q)^2 + 4(f - f_0)^2]\}$, where A_0 is some constant. The resonance frequency $f_0(H)$ is a function of the magnetic field H. The field-dependent transmission coefficient is $S_{21}(H) = S_{21}(0) - 10 \log\{1 + 4Q^2[1 - f_0(H)/f_0(0)]^2\}$. This is derived under the assumption that the probe signal is fixed at $f_0(0)$. It is also assumed that H is so weak that Q does not depend on it. The above expression is used to generate the theoretical fit in Fig. 1(b) (red curve). The total inductance (L) of the sample can be estimated as the sum of the inductance of the resonator itself (L_{res}) and the kinetic inductance of the nanowires (L_{nw}), due to the inertia of the moving condensate, thus $L = L_{res} + L_{nw}$. The kinetic inductance of the wire $L_{nw} = (dI/d\phi)^{-1}(\hbar/2e)$ depends on the current-phase relationship (CPR) $I(\phi)$ of the wire. Here e is the electronic charge, $\hbar$ is the reduced Planck's constant, ϕ is the phase difference between the ends of the wire, and I is the supercurrent in a wire. We have performed numerical simulations using the Likharev[14] CPR expression for a single long wire ($l \gg \xi$, where l is the length of the nanowire and ξ is the coherence length); $I = (3\sqrt{3}I_C/2)[\phi\xi/l - (\phi\xi/l)^3]$. Here I_C is the critical current of the wire. From the two previous formulas, the inductance of the nanowires is $L_{nw}(H) = \hbar/(3\sqrt{3}e)(I_{C1} + I_{C2})^{-1}[\xi/l - 3(\xi/l)^3(\beta H/2)^2]^{-1}$. Here I_{C1} and I_{C2} are the critical currents of the wires. Using the expression for the resonance frequency $f_0 = (4\pi^2 LC)^{-1/2}$, we obtain the shift in the resonance: $f_0(0) - f_0(H) = (4\pi^2 C_{res})^{-1/2}\{[L_{res} + L_{nw}(0)]^{-1/2} - [L_{res} + L_{nw}(H)]^{-1/2}\}$. Here C_{res} is the capacitance of the resonator and $\phi = \beta H/2$, where the factor of 2 originates from the fact that the total phase difference generated on the wire loop is shared, presumably equally, between the two wires. Using these expressions we are able to fit the experimental data with the fitting parameters $I_{C1} = I_{C2} = 33\ \mu$A, $l/\xi = 20$ (thus $\xi = 5$ nm, which is consistent with previous results[15,16]), $L = 6$ nH and $\beta = 12$. The result is illustrated in Fig. 1(b) (red curve).

The observed jumps between the parabolas show exactly how many vortices have entered or escaped the loop. The vertical lines in Fig. 1(a) correspond to single or double or triple phase slip events. These lines appear because in the figure the data points are connected by lines and the vorticity changes almost instantaneously. Thus they show from which parabola to which other parabola the jump takes place. If the line connects two neighboring parabolas then just one LPS has happened. If it connects the next to the near neighbor parabolas then there have been two LPS, etc.

It was theoretically predicted[17] that transitions between states with different vorticities have unequal probabilities. Under certain conditions the entrance of two vortices at once into a superconducting loop is expected to be more probable. Analysis of our data reveals that at $T < 1.5$ K the rate of double-LPS is significantly higher than single-LPS (Fig. 2). Around 2 K the two rates become equal and $n \to n \pm 3$ transitions become very rare. A further rise of temperature leaves only single-LPS.

We are able to provide an upper bound on the rate of quantum phase slips (QPSs) in nanowires in the zero-bias limit.[18] Considering all the data we have obtained on both samples, we conclude that no QPS events occur in the "linear regime," i.e., within the flat top of the S_{21} parabolas. This corresponds to the QPS rate being less than $2 \cdot 10^{-4}\ \text{s}^{-1}$. Experimentally we find that the jumps from one parabola to another can only occur if the field is such that the screening current is strong enough to significantly change L_{nw}. This

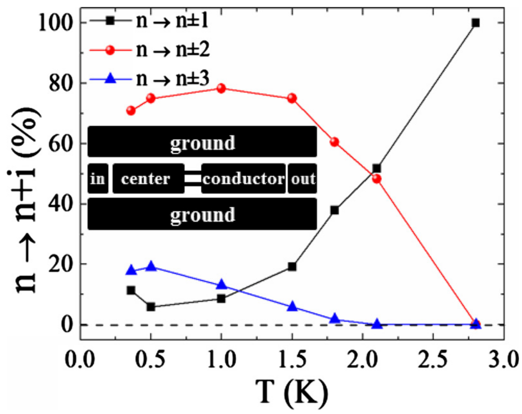

FIG. 2. (Color online) Relative frequencies for transitions of the type $n \rightarrow n\pm1$ (black squares), $n\rightarrow n\pm2$ (red circles), and $n\rightarrow n\pm3$ (blue triangles), calculated for ~90 transitions. Here n is the number of phase vortices in the loop. The inset shows the schematics of the studied resonator-nanowire-loop device with two nanowires in the center. The black color corresponds to the MoGe film. The center conductor is capacitively coupled to the input and output electrodes.

result is in agreement with the predictions of Golubev and Zaikin,[18] and Tinkham and Lau,[19] for wires with rather large cross sections (~500 nm^2 in our case).

In conclusion, we have shown that a CPW resonator can be used to study the LP periodicity at low temperatures and at low bias, and to detect single and multiple LPSs in thin superconducting wires. No QPSs are observed *at low currents* in MoGe wires with diameters ~21–25 nm, and the wire stays coherent on a scale of ~1 h at least, unless a strong current is applied.

The work was supported by the DOE Grant No. DE-FG02-07ER46453 and by the NSF Grant No. DMR 10-05645. A. Belkin and M. Brenner contributed to the work equally.

[1] W. A. Little and R. D. Parks, Phys. Rev. Lett. **9**, 9 (1962).
[2] R. D. Parks and W. A. Little, Phys. Rev. **133**, A97 (1964).
[3] R. P. Groff and R. D. Parks, Phys. Rev. **176**, 567 (1968).
[4] V. Vakaryuk, Phys. Rev. Lett. **101**, 167002 (2008).
[5] J. Ku, V. Manucharyan, and A. Bezryadin, Phys. Rev. B **82**, 134518 (2010).
[6] W. A. Little, Phys. Rev. **156**, 396 (1967).
[7] M. W. Brenner, S. Gopalakrishnan, J. Ku, T. J. McArdle, J. N. Eckstein, N. Shah, P. M. Goldbart, A. Bezryadin, Phys. Rev. B **83**, 184503 (2011).
[8] A. Bezryadin, C. N. Lau, and M. Tinkham, Nature (London) **404**, 971 (2000).
[9] A. Bezryadin, J. Phys.: Condens. Matter **20**, 043202 (2008).
[10] A. T. Bollinger, A. Rogachev, M. Remeika, and A. Bezryadin, Phys. Rev. B **69**, 180503(R) (2004).
[11] A. T. Bollinger, A. Rogachev, and A. Bezryadin, Europhys. Lett. **76**, 505 (2006).
[12] D. S. Hopkins, D. Pekker, P. M. Goldbart, and A. Bezryadin, Science **308**, 1762 (2005).
[13] I. S. Gradshtein and I. M. Ryzhik, *Table of Integrals, Series, and Products* (AP, New York, 1965), Vol. 1036.
[14] K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).
[15] J. M. Graybeal and M. R. Beasley, Phys. Rev. B **29**, 4167 (1984).
[16] A. Belkin, V. Novosad, M. Iavarone, J. Fedor, J. Pearson, A. Petrean-Troncalli, and G. Karapetrov, Appl. Phys. Lett. **93**, 072510 (2008).
[17] S. E. Korshunov, JETP Lett. **45**, 434 (1987).
[18] D. S. Golubev and A. D. Zaikin, Phys. Rev. B **64**, 014504 (2001).
[19] M. Tinkham and C. N. Lau, Appl. Phys. Lett. **80**, 2946 (2002).