

Remote state preparation: arbitrary remote control of photon polarization

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(Dated: October 22, 2004)

We experimentally demonstrate the first remote state preparation of arbitrary single-qubit states, encoded in the polarization of photons generated by spontaneous parametric downconversion. Utilizing degenerate and nondegenerate wavelength entangled sources, we remotely prepare arbitrary states at two wavelengths. Further, we derive theoretical bounds on the states that may be remotely prepared for given two-qubit resources.

PACS numbers: 03.67.Hk, 42.65.Lm, 03.67.Mn

Quantum communication is concerned with the transmission, manipulation, and detection of quantum information. If a sender (Alice) wants to transmit an *unknown* quantum state to a receiver (Bob), they may use teleportation [1]. However, it has been shown that the classical communication costs for sending a *known* state using the remote state preparation protocol (RSP) are less than those of teleportation [2–4]. RSP is a quantum communication protocol that relies on correlations between two entangled qubits to similarly prepare Bob's qubit in a particular state determined by Alice, conditional on the outcome of a measurement on her qubit. However, unlike teleportation, RSP does not require the sender to perform full Bell-state analysis, currently an experimental challenge for optical implementations.

Thus far, several RSP demonstrations with varying degrees of control over remotely prepared qubits have been reported: pseudo-pure states using liquid-state NMR [5], pure-state superpositions of vacuum and single-photon states [6], and some mixed states of a polarization qubit [7, 8]. However, until now, no RSP implementation has achieved control over the three parameters required to prepare *arbitrary* single qubit states, which we report here. Specifically, we achieve arbitrary mixed state RSP by using arbitrary polarization measurement on one photon of a polarization-entangled pair. In addition, we derive bounds on the states that may be remotely prepared using arbitrary two-qubit entangled resources and discuss two specific cases in detail.

First, we describe the general idea of RSP and give several examples. Although we will make reference to photon polarization qubits, the methods described here can be generalized to any physical qubit implementation. Consider the two-photon maximally entangled state: $|\phi^+\rangle \equiv (|H_t H_{rp}\rangle + |V_t V_{rp}\rangle)/\sqrt{2} \equiv (|D_t D_{rp}\rangle + |A_t A_{rp}\rangle)/\sqrt{2}$, where the subscripts label the trigger and remotely prepared photons, $|H\rangle$ and $|V\rangle$ label horizontal and vertical polarization states and $|D\rangle \equiv (|H\rangle + |V\rangle)/\sqrt{2}$ and $|A\rangle \equiv (|H\rangle - |V\rangle)/\sqrt{2}$ label diagonal and anti-diagonal polarizations states, respectively. Measurement of the trigger photon in the state $|D_t\rangle$ (i.e., detecting the trigger photon after a diagonal polarizer) prepares

the other photon in the state $|D_{rp}\rangle$. To remotely prepare an arbitrary *pure* state $|\psi_{rp}(\theta, \phi)\rangle \equiv \cos\theta|D\rangle + \sin\theta e^{i\phi}|A\rangle$, Alice can act on the target photon with a quarter-wave plate (QWP) and a half-wave plate (HWP), such that the two-photon state $|\phi^+\rangle \rightarrow (|\zeta_t(\theta, \phi)D_{rp}\rangle + |\zeta_t^\perp(\theta, \phi)A_{rp}\rangle)/\sqrt{2} \equiv (|D_t\psi_{rp}(\theta, \phi)\rangle + |A_t\psi_{rp}^\perp(\theta, \phi)\rangle)/\sqrt{2}$, where $|\zeta_t(\theta, \phi)\rangle \equiv \cos\theta|D\rangle - e^{-i\phi}\sin\theta|A\rangle$ [9]. Thus when the target qubit is projected into $\langle D|$ ($\langle A|$), the remotely prepared qubit is in the state $|\psi(\theta, \phi)\rangle$ ($|\psi^\perp(\theta, \phi)\rangle$) [10].

If instead, the trigger polarizer is removed, the trigger photon is measured in a polarization-*insensitive* way, tracing over its polarization state. This prepares the remaining photon in the completely mixed state (i.e., unpolarized), according to

$$\rho_{rp} = \langle D_t|\phi^+\rangle\langle\phi^+|D_t\rangle + \langle A_t|\phi^+\rangle\langle\phi^+|A_t\rangle = \frac{1}{2}(|D_{rp}\rangle\langle D_{rp}| + |A_{rp}\rangle\langle A_{rp}|) = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (1)$$

By using a *partial* polarizer to tune between the two limiting cases discussed above, we can control the *strength* of the polarization measurement on the trigger, and thus the resulting mixedness of the remotely prepared qubit (RPQ). Combined with the wave plates that allow us to prepare arbitrary pure states, the partial polarizer allows us to prepare completely arbitrary mixed states:

$$\rho_{rp}((\theta, \phi, \lambda)) = (1 - \lambda)|\psi(\theta, \phi)\rangle\langle\psi(\theta, \phi)| + \frac{\lambda}{2}\mathbf{1}, \quad (2)$$

where the value λ is determined by the partial polarizer.

The experiment divides into three logical sections: entangled resource state creation, detection of the trigger to remotely prepare a qubit, and tomography of the RPQ. Photons are created via spontaneous parametric downconversion by pumping two type-I phasematched BBO nonlinear crystals with a cw Ar-ion 351-nm pump laser. The two crystals have their optic axes oriented in perpendicular planes to give an entanglement superposition $(|HH\rangle + |VV\rangle)/\sqrt{2}$ [11]. An initial tomography of the entangled resource state is taken by measuring 36 polarization correlations (such as $|HH\rangle$, $|HV\rangle$, $|DH\rangle$, ..., etc.) from which a density matrix is calculated using a maximum likelihood technique [12].

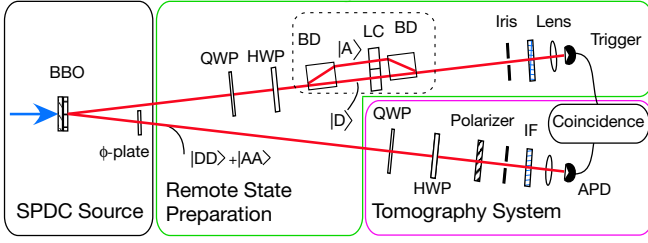


FIG. 1: Experimental arrangement for remote state preparation. The entangled state $(|DD\rangle + |AA\rangle)/\sqrt{2}$ is generated by equally pumping two BBO crystals whose optic axes are in perpendicular planes [the relative phase is adjusted by tipping a HWP (ϕ -plate) about its vertical optic axis]. The trigger photon is then *partially* projected into an arbitrary polarization state with a quarter-wave plate (QWP) and a half-wave plate (HWP) located before a partial polarizer [13] shown in the dashed box. Conditional on detection of this photon, the sister photons is prepared into the desired state.

Next, the trigger photon of the entangled pair is projected into an arbitrary polarization state with an adjustable strength polarizer [13] to remotely prepare a qubit of the form (2). For perfect wave plates (i.e., the birefringent retardance is 180° for a HWP and 90° for a QWP), the precise wave plate angles can be readily calculated [14, 15].

The strength of the partial polarizer is determined by the transmission of two orthogonal polarization components, T_D and T_A , each normalized with respect to each other by $N \equiv 1/(T_D + T_A)$. Unit transmission of one component, coupled with zero transmission of the other, is equivalent to an ideal polarizer for the transmitted component. In contrast, if both transmitted components have the same amplitude, then the partial polarizer behaves as if no polarizer is present (though the overall amplitude may be reduced). The action of the partial polarizer alone remotely prepares qubits according to [7]

$$\begin{aligned} \rho_{rp}(T_D, T_A) &= \frac{N(T_D \langle D_t | \phi^+ \rangle \langle \phi^+ | D_t \rangle + T_A \langle A_t | \phi^+ \rangle \langle \phi^+ | A_t \rangle)}{N(T_D + T_A)} \\ &= \frac{N}{2} (T_D |D_{rp}\rangle \langle D_{rp}| + T_A |A_{rp}\rangle \langle A_{rp}|) \\ &= \frac{1}{2} \begin{pmatrix} 1 & N(T_D - T_A) \\ N(T_D - T_A) & 1 \end{pmatrix}, \end{aligned} \quad (3)$$

where the final density matrix is in the $|H\rangle$, $|V\rangle$ basis. After the partial polarizer, the trigger photons pass via a 2.2-mm iris, an interference filter (discussed below), and a collection lens, which focuses them onto a photon-counting avalanche photodiode (APD; Perkin-Elmer SPCM-AQR-13). The classical communication from Alice to Bob is implemented by counting the RPQs in coincidence (within a 4.5-ns window) with their triggering photons. The requirement for coincidence counting (which necessarily requires the (sub)luminal transfer of the APD signals) precludes all possibility of superluminal communication.

The final verification step is the tomographic measure-

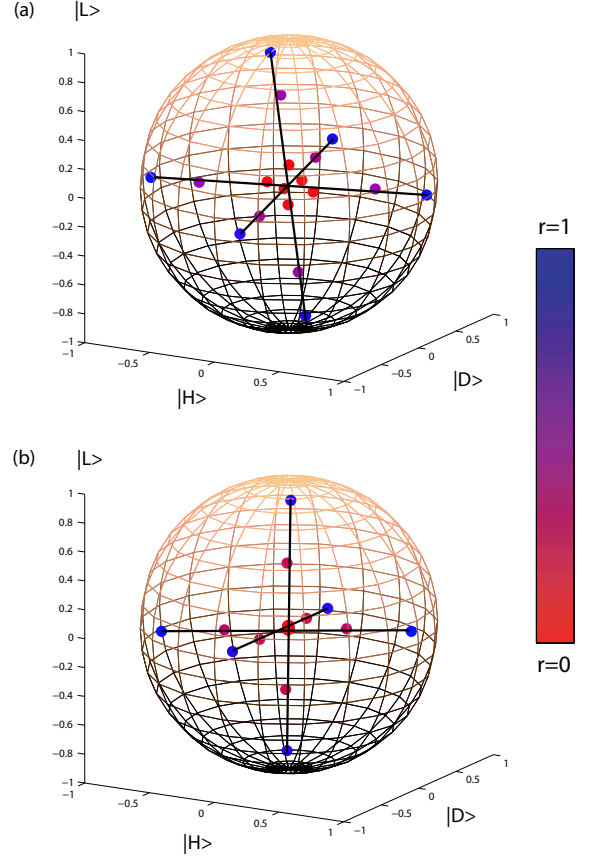


FIG. 2: Remotely prepared states shown in the Poincaré sphere. (a) States remotely prepared at 702 nm using frequency *degenerate* entanglement. (b) States remotely prepared at 670 nm (using a 737-nm trigger). In either case, the distance of the remotely prepared state from the origin is indicated by its color: red \rightarrow mixed, blue \rightarrow pure. Lines are drawn along the data to guide the eye.

ment of the RPQ. Using the wave plates and polarizer of the tomography system shown in Fig. 1, the remotely prepared ensemble is projected into $\langle H|$, $\langle V|$, $\langle D|$ and $\langle A|$ states, as well as the left and right circular polarization states, $\langle L| \equiv \langle H| - i\langle V|$ and $\langle R| \equiv \langle H| + i\langle V|$, respectively. The results of this complete polarization analysis are converted to the closest physically valid density matrix using a maximum likelihood technique [12].

A summary of states remotely prepared in this way is shown in Fig. 2(a), along with a color bar indicating the distance of each RPQ from the center of the Poincaré sphere; the color corresponds to the state purity, from blue (pure) to red (mixed). We tested our ability to precisely remotely prepare arbitrary states by creating six states along each of three (nearly orthogonal) axes in the Poincaré sphere. To calculate experimental agreement between the state we prepared (ρ_p) and the remote state we expected (ρ_e) given the parameters of our system, we use the fidelity $F(\rho_e, \rho_p) \equiv |\text{Tr}(\sqrt{\sqrt{\rho_e}\rho_p\sqrt{\rho_e}})|^2$ [16];

$F = 1$ (0) for identical (orthogonal) states. The average fidelity for our data is 0.996, with all 18 states above 0.99.

The previous results were all taken using *degenerate* qubits, i.e., both trigger and RPQ were at 702 nm, as defined by the cut of the BBO crystals, the position of the collection irises (corresponding to a 3°-opening angle with respect to the pump beam), and 2-nm bandpass filters in front of each APD [17]. To demonstrate the ability to remotely prepare qubits at other wavelengths, we additionally performed a similar set of measurements using *non-degenerate* entangled pairs: Detection of a trigger photon after a 5-nm bandpass filter at 737 nm corresponded to a RPQ at 670 nm. Note that all of the same physical resources, e.g., the crystals, the wave plates and the partial polarizer, were used at the different wavelengths [15]. Results are shown in Fig. 2(b). The average fidelity was 0.996, with 17 of the 18 measured states above 0.99. The flexibility to remotely prepare qubits at various wavelengths could be useful, e.g., for optimizing detector sensitivity, fiber or atmospheric transmission, or coupling to other quantum systems. One could even envision a sort of nonlocal wavelength division multiplexing scheme: using an adjustable filter before the trigger detector, arbitrary states could be remotely prepared at one of several detectors, each receiving a slightly different wavelength band.

While a maximally entangled state resource enables the remote preparation of *any* state, it is important to consider the limits on the remotely preparable states when the two-qubit resource is mixed or only partially entangled, as in practice all realizable states are of this type. We consider the scenario that Bob simply keeps or discards his photon, based on transmission of a single classical bit from Alice. Furthermore, we restrict Alice to single-qubit operations, i.e., no collective manipulation of her qubits. This consideration is realistic, as efficient optical CNOT gates do not yet exist.

The most general operations Alice can perform on her qubit can be described by at most four local filters [18]:

$$\rho_A \rightarrow \sum_{i=1}^4 p_i \mathcal{M}_i \rho_A \mathcal{M}_i^\dagger, \quad (4)$$

where $\sum_i p_i \mathcal{M}_i^\dagger \mathcal{M}_i \leq \mathbb{1}$, and each local filter \mathcal{M}_i can be expressed in the singular-value decomposition

$$\mathcal{M} = V^\dagger \mathcal{D} U. \quad (5)$$

Here \mathcal{D} is a non-negative, no-greater-than-unity diagonal matrix, and U and V are unitary matrices, not necessarily adjoint to each other.

Under the general operation (which can be non-trace preserving) performed by Alice, the initial joint two-qubit state ρ_{AB} becomes

$$\rho'_{AB} = \sum_{i=1}^4 p_i \mathcal{M}_i \otimes \mathbb{1} \rho_{AB} \mathcal{M}_i^\dagger \otimes \mathbb{1}, \quad (6)$$

neglecting normalization, and Bob's qubit becomes $\rho'_B = \text{Tr}_A \rho'_{AB}$. Thus, the most general states Alice can remotely prepare are mixtures of states which she can prepare from a single local filter. The states preparable from a single filter form a surface inside the Poincaré sphere, and all the states she can remotely prepare lie on or inside the convex hull of this surface. We now analyze the capability of a general local filter applied to RSP.

The decomposition of a general local filter (5) can be interpreted as a three-step procedure: (i) first, apply a local unitary transformation U , followed by (ii) a ‘‘Procrustean’’ operation [19–22]

$$\mathcal{D} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}, \quad (7)$$

with $0 \leq (a, b) \leq 1$, and lastly by (iii) another unitary transformation V^\dagger . The last step V^\dagger has no effect on Bob's state, so it can be ignored in the analysis of RSP. With a suitable parameterization of U , e.g., $U = \cos \theta \mathbb{1} + i \hat{n} \cdot \vec{\sigma} \sin \theta$, where \hat{n} is a unit vector and $\vec{\sigma}$ are the Pauli spin matrices, it is straightforward to analyze the states that can be remotely prepared by a single filter:

$$\rho_B = \text{Tr}_A \left[(\mathcal{D} U) \otimes \mathbb{1} \rho_{AB} (\mathcal{D} U)^\dagger \otimes \mathbb{1} \right], \quad (8)$$

where ρ_{AB} is the initial shared two-qubit state (unnormalized).

To illustrate the results, we first consider the case where ρ_{AB} is a pure (but non-maximally) entangled state: $\sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle$, assuming $p > 1/2$ without loss of generality. In fact, analysis of Eq. (8) reveals that with this state Alice can prepare *arbitrary* single-qubit states for Bob. She first uses Procrustean distillation \mathcal{D} [19–22], with $a = \sqrt{(1-p)/p}$ and $b = 1$, to obtain the perfect Bell state $|00\rangle + |11\rangle$ (though with probability < 1), with which she may remotely prepare arbitrary states as we have demonstrated experimentally.

As a rather different example, consider the starting state to be of the form [23]

$$\rho_{AB} = \frac{1}{4} (\mathbb{1} \otimes \mathbb{1} + t_1 \sigma_x \otimes \sigma_x + t_2 \sigma_y \otimes \sigma_y + t_3 \sigma_z \otimes \sigma_z), \quad (9)$$

which has eigenvalues $\lambda_1 = (1 - t_1 + t_2 + t_3)/4$, $\lambda_2 = (1 + t_1 - t_2 + t_3)/4$, $\lambda_3 = (1 + t_1 + t_2 - t_3)/4$, and $\lambda_4 = (1 - t_1 - t_2 - t_3)/4$. This state, when described by (t_1, t_2, t_3) , lies on the surface of or inside a tetrahedron, with the four vertices being $(-1, -1, -1)$, $(-1, 1, 1)$, $(1, -1, 1)$, and $(1, 1, -1)$. The state is entangled if any of the λ_i are greater than $1/2$. Eq. (9) describes a wide range of interesting resource states, as judicious choice of t_i changes the state from a maximally entangled pure state to a Werner state [24] to states with varying classical correlations. Again analyzing Eq. (8) we obtain that the states (in the Poincaré sphere) that Alice can remotely prepare lie on or inside the ellipsoid

centered at the origin, with three axes of length $|t_1|$, $|t_2|$, and $|t_3|$, respectively. To remotely prepare states on the surface of the ellipsoid, Alice simply rotates her qubit via $i\hat{n} \cdot \vec{\sigma}$, followed by projection onto $|0\rangle$. As she varies the rotation axis $\hat{n} = (\sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha)$, Bob's states will then follow the corresponding trace $(t_1 \sin 2\alpha \cos \beta, t_2 \sin 2\alpha \sin \beta, t_3 \cos 2\alpha)$ on the ellipsoid. To obtain states inside the ellipsoid, the projection onto $|0\rangle$ is replaced by the more general partial projection (Eq. (7)).

We have seen that pure-state entanglement allows remote preparation of arbitrary states. However, pure-state entanglement may not be *required* to remotely prepare some states. Consider that the tetrahedron state (9) has purity

$$P_{AB} = \text{Tr} \rho_{AB}^2 = \frac{1}{4}(1 + t_1^2 + t_2^2 + t_3^2), \quad (10)$$

and is unentangled if (t_1, t_2, t_3) lies inside the octahedron embedded in the tetrahedron [23]. The maximum purity of the states Alice can remotely prepare via the tetrahedron state is

$$\max P_B = \frac{1}{2}(1 + \max(t_1^2, t_2^2, t_3^2)). \quad (11)$$

Interestingly, there appears to be no general requirement for entanglement in the two-qubit resource to be able to remotely prepare a one-qubit state of arbitrary purity. Consider the classically correlated state $\rho_{cc} = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$ (i.e., $t_1 = t_2 = 0$ and $t_3 = 1$). This classically correlated two-qubit state can be used to remotely prepare any state of the form $\cos^2 \theta |0\rangle\langle 0| + \sin^2 \theta |1\rangle\langle 1|$, possessing any purity. For unentangled resources where the classical correlations are less than in ρ_{cc} , Alice can only remotely prepare states near the origin of the Poincaré sphere.

We have demonstrated the first arbitrary remote state preparation of qubits, preparing a broad range of states spanning the Poincaré sphere. Moreover, by varying the acceptance wavelength of the trigger photon (using a nondegenerate entangled source) we can also control the wavelength of the remotely prepared qubit. Such a capability may assist in the preparation of states at wavelengths more optimal for other quantum communication protocols, e.g., quantum cryptography. Finally, we have derived bounds on the single-qubit states that may be remotely prepared using certain two-qubit resource states.

This work was supported by the U. S. Army Research Office (Grant No. DAAD19-03-1-0282).

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