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Hyperentangled Bell-state analysis

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It is known that it is impossible to unambiguously distinguish the four Bell states encoded in pairs of photon polarizations using only linear optics. However, hyperentanglement, the simultaneous entanglement in more than one degree of freedom, has been shown to assist the complete Bell analysis of the four Bell states (given a fixed state of the other degrees of freedom). Yet introducing other degrees of freedom also enlarges the total number of Bell-like states. We investigate the limits for unambiguously distinguishing these Bell-like states. In particular, when the additional degree of freedom is qubitlike, we find that the optimal one-shot discrimination schemes are to group the 16 states into seven distinguishable classes, and that an unambiguous discrimination is possible with two identical copies.

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Just as the controlled-NOT gate [1] is one of the most important two-qubit gates in quantum computation, Bell measurement is one of the most important two-qubit measurements, as it enables many applications in quantum-information processing, such as superdense coding [2,3], teleportation [4–6], quantum fingerprinting [7,8], and direct characterization of quantum dynamics [9]. However, it was shown that complete Bell-state analysis (BSA) using linear optics is not possible [10,11], and that the optimal probability of success is only 50% [11–13], for which the optimal BSA schemes have been realized experimentally [3,14,15]. But Kwiat and Weinfurter (KW) [16] showed that with additional degrees of freedom, such as timing or momentum, it is indeed possible to achieve complete BSA for four Bell states, given that the additional degrees are in a fixed entangled state. Other similar BSA schemes have also been proposed [17–19] and implemented [20,21]. In all of these schemes, such states are called "hyperentangled" [22], and such measurements are termed "embedded BSA" [16]. Hyperentangled states with polarization and orbital angular momentum of two photons have recently been created and characterized [23]. Furthermore, the KW scheme for BSA has recently been implemented by Schuck *et al.* [24]. Nevertheless, adding additional degrees of freedom also enlarges the Hilbert space, and hence the number of Bell-like states (e.g., see Table I); all previous investigations on embedded BSA have focused on a subset of these states (e.g., states with fixed $|\phi^+\rangle$). It is, therefore, important to set theoretical limits on optimal BSA in the enlarged Hilbert space.

In this Rapid Communication, we investigate the optimality of hyperentanglement-assisted BSA, with both degrees of freedom being qubitlike, such as polarization (H and V), plus either two momenta (spatial directions) or two orbital angular momenta or two time bins. The resulting Bell-like states for two photons thus total 16. We show that an unambiguous state discrimination is impossible, but that the optimal scheme divides the 16 Bell states into seven distinct groups. We also show by construction that an unambiguous discrimination of any of the 16 states requires two copies of the same states. Finally, we discuss the implications for superdense coding, teleportation, and quantum fingerprinting.

KW showed that when the momentum degrees of freedom are in a fixed entangled state, the four polarization Bell states can be unambiguously distinguished [16]. Let us introduce the 16 Bell-like states, constructed from two photons with polarization and momentum (or spatial mode) or timing degrees of freedom: (1) $\{H,V\}\otimes\{a,c\}$ and (2) $\{H,V\}\otimes\{b,d\}$ [25]. These states result from the different combinations of the four polarization Bell states,

$$|\Phi^\pm\rangle \equiv (|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2)/\sqrt{2}, \tag{1a}$$

$$|\Psi^\pm\rangle \equiv (|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2)/\sqrt{2}, \tag{1b}$$

and the four momentum Bell states,

$$|\phi^\pm\rangle \equiv (|a\rangle_1|b\rangle_2 \pm |c\rangle_1|d\rangle_2)/\sqrt{2}, \tag{1c}$$

$$|\psi^\pm\rangle \equiv (|a\rangle_1|d\rangle_2 \pm |c\rangle_1|b\rangle_2)/\sqrt{2}. \tag{1d}$$

The detection patterns for the KW scheme (Fig. 1) are shown in Table I. The 16 states are divided into seven distinct classes according to the measurement outcome [26]. Except that one class contains four states, all others each have two states. Thus, no single state can be unambiguously distin-

TABLE I. Detection signature table. $\Phi^\pm \equiv (H_1H_2 \pm V_1V_2)$, $\Psi^\pm \equiv (H_1V_2 \pm V_1H_2)$, $\phi^\pm \equiv (a_1b_2 \pm c_1d_2)$, and $\psi^\pm \equiv (a_1d_2 \pm c_1b_2)$. The subscript 45 indicates the port associated with transmission through the polarizing beam splitter and $\overline{45}$ that with reflection. The final row lists a unique detection signature, corresponding, however, to states outside the Hilbert space spanned by the 16 hyperentangled Bell states [26].

Class	State	Detector signature
1	$\Phi^+\otimes\phi^+$, $\Phi^-\otimes\phi^-$	$\alpha_{45}\alpha_{45}$, $\alpha_{\overline{45}}\alpha_{\overline{45}}$, $\beta_{45}\beta_{45}$, $\beta_{\overline{45}}\beta_{\overline{45}}$
	$\Psi^+\otimes\psi^-$, $\Psi^-\otimes\psi^+$	$\delta_{45}\delta_{45}$, $\delta_{\overline{45}}\delta_{\overline{45}}$, $\gamma_{45}\gamma_{45}$, $\gamma_{\overline{45}}\gamma_{\overline{45}}$
2	$\Phi^-\otimes\phi^+$, $\Phi^+\otimes\phi^-$	$\alpha_{45}\alpha_{\overline{45}}$, $\beta_{45}\beta_{\overline{45}}$, $\delta_{45}\delta_{\overline{45}}$, $\gamma_{45}\gamma_{\overline{45}}$
3	$\Psi^-\otimes\psi^-$, $\Psi^+\otimes\psi^+$	$\alpha_{45}\beta_{45}$, $\alpha_{\overline{45}}\beta_{\overline{45}}$, $\delta_{45}\gamma_{45}$, $\delta_{\overline{45}}\gamma_{\overline{45}}$
4	$\Psi^+\otimes\phi^+$, $\Phi^+\otimes\psi^-$	$\alpha_{45}\delta_{45}$, $\alpha_{\overline{45}}\delta_{\overline{45}}$, $\beta_{45}\gamma_{45}$, $\beta_{\overline{45}}\gamma_{\overline{45}}$
5	$\Psi^+\otimes\phi^-$,$\Phi^-\otimes\psi^-$	$\alpha_{45}\delta_{\overline{45}}$, $\alpha_{\overline{45}}\delta_{45}$, $\beta_{45}\gamma_{\overline{45}}$, $\beta_{\overline{45}}\gamma_{45}$
6	$\Psi^-\otimes\phi^+$, $\Phi^+\otimes\psi^+$	$\alpha_{45}\gamma_{45}$, $\alpha_{\overline{45}}\gamma_{\overline{45}}$, $\beta_{45}\delta_{45}$, $\beta_{\overline{45}}\delta_{\overline{45}}$
7	$\Psi^-\otimes\phi^-$, $\Phi^-\otimes\psi^+$	$\alpha_{45}\gamma_{\overline{45}}$, $\alpha_{\overline{45}}\gamma_{45}$, $\beta_{45}\delta_{\overline{45}}$, $\beta_{\overline{45}}\delta_{45}$
*	$\Psi^\pm\otimes(a_1c_2-b_1d_2)$	$\alpha_{45}\beta_{\overline{45}}$, $\alpha_{\overline{45}}\beta_{45}$, $\delta_{45}\gamma_{\overline{45}}$, $\delta_{\overline{45}}\gamma_{45}$

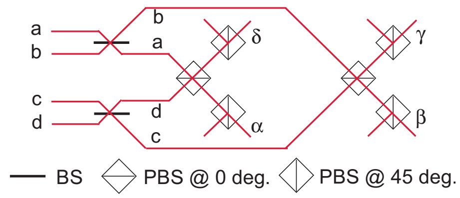

FIG. 1. (Color online) Kwiat-Weinfurter scheme for the embedded Bell-state analysis.

guished using this scheme. If the momentum state is ϕ^+, the four states with distinct polarization Bell states belong to four distinct classes, and hence can be distinguished. Similarly, if the polarization state is Φ^+, the states with four distinct momentum Bell states can be distinguished. Therefore, the same setup can perform BSA for either degree of freedom.

One may wonder what the optimal Bell-state analysis is. Calsamiglia [13] showed that any element $|u_i\rangle\langle u_i|$ in a generalized measurement (i.e., positive operator-valued measure $\Sigma_i\lambda_i|u_i\rangle\langle u_i|=\mathbb{1}$, with $\Sigma_i\lambda_i=1$) on two i-qudits (qudits composed of identical particles) of linear optics can have a Schmidt number at most of 2. As our hyperentangled Bell states have Schmidt number 4, this means that no single state can be distinguished from any other, and so unambiguous and complete BSA for the 16 states is not possible. Thus, the optimal scheme groups the states into classes; in our case, at most eight distinguishable classes. However, our analysis of the KW scheme (Table I) identifies only seven classes. Now we shall prove that seven is in fact the upper limit.

We utilize the method of van Loock and Lütkenhaus to test whether eight classes can be discriminated. They showed that a necessary condition for the distinguishability of the states ψ_i and ψ_j $(i\neq j)$ is [27]

$$\langle\psi_i|c_s^\dagger c_s|\psi_j\rangle=0 \quad \text{with } c_s=\sum_{i=1}^{N}\nu_i c_i, \tag{2}$$

where c_s is the annihilation operator, linearly composed of N modes (both input and auxiliary) via some unitary transformation, and thus the ν_i's cannot all be zero. The rationale behind Eq. (2) is that in order for ψ_i and ψ_j to be distinguishable, the remaining states should maintain orthogonality after a single-photon detection at mode s. In addition, ancillary photons do not assist state discrimination if either input or auxiliary states have a fixed number of photons. This means that in Eq. (2), N can be set as the number of input modes.

For the setup shown in Fig. 1, we relabel the input modes as $|1\rangle\equiv|H\rangle\otimes|a\rangle$, $|2\rangle\equiv|H\rangle\otimes|c\rangle$, $|3\rangle\equiv|V\rangle\otimes|a\rangle$, $|4\rangle\equiv|V\rangle\otimes|c\rangle$, $|5\rangle\equiv|H\rangle\otimes|b\rangle$, $|6\rangle\equiv|H\rangle\otimes|d\rangle$, $|7\rangle\equiv|V\rangle\otimes|b\rangle$, and $|8\rangle\equiv|V\rangle\otimes|d\rangle$, where H and V denote the polarization degree of freedom and a,b, c, and d denote the momentum or direction (or angular-momentum) degree of freedom. Thus, the Bell states can be written as

$$|\Psi^{(\mu)}\rangle=\sum_{i,j=1,\ldots,8}W_{ij}^{(\mu)}c_i^\dagger c_j^\dagger|0\rangle, \tag{3}$$

where the symmetric matrices $W^{(\mu)}$ are 8×8 invertible (i.e., with nonzero determinant) and characterize the 16

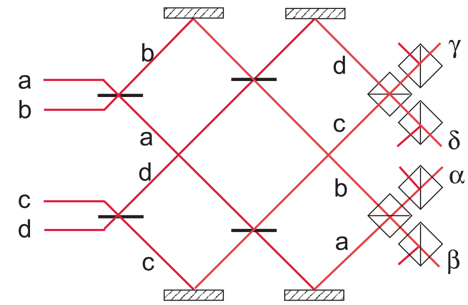

FIG. 2. (Color online) Modified KW scheme.

$(\mu=1,\ldots,16)$ Bell states. If the optimal BSA groups the 16 Bell states into eight classes, there must exist sets of eight states for which the conditions set by Eq. (2) are satisfied. On the other hand, if seven is the optimal number of classes, no set of eight states satisfy Eq. (2). To see whether the former or the latter is true, we have to check whether Eq. (2) can be satisfied for all possible combinations of eight out of the 16 Bell states ($C_8^{16}=12\,870$, though this number can be reduced by considering the group structure of operations that transform the 16 states onto themselves.)

First, as an example, take two states from class 1 and one from each of the other six classes: $\Phi^+\otimes\phi^+$, $\Phi^-\otimes\phi^-$, $\Phi^-\otimes\phi^+$, $\Psi^-\otimes\psi^-$, $\Psi^+\otimes\phi^+$, $\Psi^+\otimes\phi^-$, $\Psi^-\otimes\phi^+$, and $\Psi^-\otimes\phi^-$. Applying Eq. (2) to these states, we have, after simplifying the equations,

$$|\nu_1|=|\nu_3|,\quad |\nu_2|=|\nu_4|,\quad |\nu_5|=|\nu_7|,\quad |\nu_6|=|\nu_8|, \tag{4a}$$

$$|\nu_1|^2+|\nu_5|^2=|\nu_2|^2+|\nu_6|^2, \tag{4b}$$

$$\nu_7^*\nu_5=\nu_2^*\nu_4=\nu_6^*\nu_8=\nu_3^*\nu_1=0. \tag{4c}$$

These lead to the only solution $\nu_i=0$, which is a contradiction. This shows that one cannot discriminate any state from the above eight states.

We checked all 12 870 cases by programming MATHEMATICA to examine the conditions derived from Eq. (2), supplemented by the normalization condition $\Sigma_i|\nu_i|^2=1$. This is achieved by first enumerating and simplifying the equations generated from Eq. (2), as well as the normalization condition, and then by using the function FindInstance[ ] to find an instance of solutions. One feature of FindInstance[ ] is that it will *always* find a solution if there is one. For all the 12 870 cases, FindInstance[ ] returns an empty set, showing no solution. Therefore, we conclude that it is impossible to reliably distinguish among any set of eight Bell-like hyperentangled states, and that seven is the optimal, as is realized in the KW scheme.

Having seen that a one-shot measurement is unable to perfectly discriminate any Bell state, it seems natural to ask how many copies are necessary to enable such discrimination. We show here by construction that two copies are sufficient. First, we introduce a slightly modified measurement scheme from that of KW, shown in Fig. 2. The corresponding detection patterns are shown in Table II. From Tables I and II, we see that no two states share the same class of detector signature. Therefore, we imagine letting one copy go through

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TABLE II. Detection signature for the scheme in Fig. 2.

Class	State	Detector signature
1′	$\Phi^+\otimes\phi^-$, $\Psi^-\otimes\phi^-$ $\Phi^+\otimes\psi^+$, $\Psi^-\otimes\psi^+$	$\alpha_{45}\alpha_{45}$, $\alpha_{\overline{45}}\alpha_{\overline{45}}$, $\beta_{45}\beta_{45}$, $\beta_{\overline{45}}\beta_{\overline{45}}$, $\delta_{45}\delta_{45}$, $\delta_{\overline{45}}\delta_{\overline{45}}$, $\gamma_{45}\gamma_{45}$, $\gamma_{\overline{45}}\gamma_{\overline{45}}$
2′	$\Phi^-\otimes\phi^-$, $\Phi^-\otimes\psi^+$	$\alpha_{45}\alpha_{\overline{45}}$, $\beta_{45}\beta_{\overline{45}}$, $\delta_{45}\delta_{\overline{45}}$, $\gamma_{45}\gamma_{\overline{45}}$
3′	$\Psi^+\otimes\phi^-$,$\Psi^+\otimes\psi^+$	$\alpha_{45}\beta_{45}$, $\alpha_{\overline{45}}\beta_{\overline{45}}$, $\delta_{45}\gamma_{45}$, $\delta_{\overline{45}}\gamma_{\overline{45}}$
4′	$\Psi^+\otimes\phi^+$, $\Phi^-\otimes\psi^-$	$\alpha_{45}\delta_{45}$, $\alpha_{\overline{45}}\delta_{\overline{45}}$, $\beta_{45}\gamma_{45}$, $\beta_{\overline{45}}\gamma_{\overline{45}}$
5′	$\Phi^+\otimes\psi^-$, $\Psi^-\otimes\psi^-$	$\alpha_{45}\delta_{\overline{45}}$, $\alpha_{\overline{45}}\delta_{45}$, $\beta_{45}\gamma_{\overline{45}}$, $\beta_{\overline{45}}\gamma_{45}$
6′	$\Phi^+\otimes\phi^+$, $\Psi^-\otimes\phi^+$	$\alpha_{45}\gamma_{45}$, $\alpha_{\overline{45}}\gamma_{\overline{45}}$, $\beta_{45}\delta_{45}$, $\beta_{\overline{45}}\delta_{\overline{45}}$
7′	$\Phi^-\otimes\phi^+$, $\Psi^+\otimes\psi^-$	$\alpha_{45}\gamma_{\overline{45}}$, $\alpha_{\overline{45}}\gamma_{45}$, $\beta_{45}\delta_{\overline{45}}$, $\beta_{\overline{45}}\delta_{45}$

the KW scheme and the other through the scheme in Fig. 2. Suppose we obtain signatures in 1 and 2′. Combining both outcomes enables us to uniquely determine which of the 16 states was analyzed, e.g., $\Phi^-\otimes\phi^-$ in the example given [28].

We have shown that, with one additional qubitlike degree of freedom for each photon, there exist seven states (out of 16) that can be distinguished from one another. Next, we consider for each photon n qubitlike degrees of freedom in total. In this case there are 4^n Bell-like states. What is the maximum number of distinguishable subsets of these states?

Let us begin by noting that we can express the 4^n Bell-like states in the form of Eq. (3), where the upper limit in the sum is now the number of input modes, 2^{n+1}. The matrices $W^{(\mu)}$ are now $(2^{n+1})\times(2^{n+1})$. If one makes a unitary transformation of the modes (using the fact that one can take the number of modes equal to the number of input modes, ignoring any auxiliary mode), $a_i^\dagger=\Sigma_j U_{ij}c_j^\dagger$, the necessary condition for discrimination between states $\Psi^{(\mu)}$ and $\Psi^{(\nu)}$ $(\mu\neq\nu)$ is

$$\langle\Psi^{(\mu)}|a_i^\dagger a_i|\Psi^{(\nu)}\rangle=0 \Leftrightarrow \langle\psi_i^{(\mu)}|\psi_i^{(\nu)}\rangle=0, \tag{5}$$

where we have defined $|\psi_i^{(\mu)}\rangle\equiv a_i|\Psi^{(\nu)}\rangle$. Because of the unitarity of W and U, $|\psi_i^{(\mu)}\rangle$ has nonzero norm and is equivalent to a 2^{n+1}-component vector. The above orthogonality condition then implies that there can be at most 2^{n+1} linearly independent vectors of $\psi_i^{(\mu)}$ for fixed i. Thus, we see that the maximum number of Bell states that can be distinguished is bounded above by 2^{n+1}. This means that the ratio of the maximal number of mutually distinguishable sets of Bell states to the total number of Bell states decreases exponentially with n: $2^{n+1}/4^n=2^{-n+1}$.

We conjecture that $2^{n+1}-1$ is a good upper bound, as it is true for $n=1$ (e.g., polarization only) and $n=2$ (e.g., polarization plus two spatial modes). Generalizing to different dimensions of the degrees of freedom, the absolute upper bound on distinguishable Bell states can be shown to be $2d_1d_2d_3\cdots d_n$.

(*a*) *Superdense coding*. Given that we can choose seven Bell states such that they can be distinguished from one another, we can then take one of them as a shared entanglement and use seven operations, taking the state to itself or six others, to encode seven messages. For example, Alice and Bob share $\Psi^-\otimes\psi^-$. She can locally transform the state into six other states, $\Phi^+\otimes\phi^+,\Phi^-\otimes\phi^+,\Psi^+\otimes\phi^+,\Psi^-\otimes\phi^+,\Phi^-\otimes\psi^-$, and $\Phi^-\otimes\psi^+$. As these seven states can be distinguished using the KW scheme, Bob can uniquely determine the message encoded by Alice, giving a superdense coding of $\log_2 7\approx 2.8$ bits. For two photons entangled only in polarization, a superdense coding encodes only $\log_2 3\approx 1.58$ bits [3]. Even though its extension to two pairs encodes $2\log_2 3\approx 3.17$ bits, the four-photon detection efficiency η^4 is typically much smaller than the two-photon efficiency η^2, where η is the single-photon detection efficiency (usually much smaller than 70%). In fact, as long as the efficiency is less than $\sqrt{7/9}\approx 88\%$, the single-pair hyperentangled scheme is superior. Thus, hyperentanglement for superdense coding seems more practical than multipair entanglement.

(*b*) *Quantum fingerprinting*. Fingerprinting is a communication protocol in which two parties, Alice and Bob, want to test whether they receive the same message from a supplier, but they cannot have direct communication with each other. Therefore, they have to communicate through a third party to test whether the two messages are the same. Instead of sending the whole messages, they send the corresponding "fingerprint" (a much shorter message) of their messages to the third party. A quantum protocol is superior to its classical counterpart because the former allows 100% fingerprinting success. It was shown that shared two-qubit Bell states enable perfect fingerprinting of binary-encoded $\{0,1\}$ messages [7,8]. Here, we propose using hyperentanglement of a pair of photons to achieve perfect fingerprinting of $\{0,1,\ldots,6\}$ encoded messages. Analogously to dense coding with hyperentanglement, Alice and Bob share the state $\Psi^-\otimes\psi^-$, and both parties can locally transform the shared state into the seven states $\Psi^-\otimes\psi^-$, $\Phi^+\otimes\phi^+,\Phi^-\otimes\phi^+,\Psi^+\otimes\phi^+,\Psi^-\otimes\phi^+,\Phi^-\otimes\psi^-$, and $\Phi^-\otimes\psi^+$. Thus, they encode their fingerprints locally by applying the required operations, and a referee can perform the BSA on the resulting two-photon state to determine whether the fingerprints are the same.

(*c*) *Quantum teleportation*. A shared Bell-like state enables the teleportation of an unknown state. However, as complete BSA of a two-photon polarization state alone is not possible, schemes employing additional degrees of freedom have been proposed [16,17]. The embedded Bell-analysis schemes proposed in Refs. [17,19,20], however, cannot be used for teleportation, as their measurements do not require two photons to interfere, and can be performed locally. If these schemes could enable teleportation, it would imply that entanglement can be created locally by distant parties; but it is well known that local operations and classical communication cannot generate entanglement. Our analysis shows that the KW scheme enables the teleportation of an arbitrary state encoded in either polarization or momentum (not both) with a 50% probability of success, the same probability as the two-photon polarization BSA. Suppose a photon in Alice's laboratory is in a state with known momentum but arbitrary polarization $|\psi\rangle=(\alpha|H\rangle_1+\beta|V\rangle_1)\otimes|h\rangle_1$, where $\{h,v\}$ is used to indicate its momentum degree of freedom. Alice and Bob share the Bell state $(\Phi^+\otimes\phi^+)_{23}$ of photons 2 and 3. If Alice performs the KW BSA on photons 1 and 2, there is a 50% probability (and she knows whether it succeeds) that Bob can transform his photon into the state $(\alpha|H\rangle_1+\beta|V\rangle_1)$, by performing the corresponding local operation according to Alice's measurement outcome, and postselecting the photon from his momentum modes b or d in $\phi^{+}=(a_1b_2+c_1d_2)$. Similarly, an arbitrary momentum state $|H\rangle\otimes(\alpha|h\rangle+\beta|v\rangle)$ can be teleported. The use of hyperentanglement of photons, unfortunately, does not offer advantages for teleportation over the conventional polarization-only teleportation [5,6], both having only 50% probability of success.

We have investigated the optimal Bell-state analysis using projective measurements in linear optics for hyperentangled Bell states. The results are relevant, as there has been recent experimental progress in realizing BSA of hyperentangled states [20,21,24]. In particular, we have shown that when the additional degrees of freedom are also qubitlike, the resulting 16 Bell-like states can be, at best, divided into seven distinct classes. Moreover, we have provided a method to unambiguously discriminate any of the 16 Bell states, given two copies of the state. We have also discussed the implications for superdense coding, fingerprinting, and teleportation. We conclude with two open issues for future study: (1) how generalized measurements might be used to help Bell analysis in general; and (2) whether other methods such as that of Eisert [29] may provide alternative approaches to understand the results presented here.

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[1] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, U.K., 2000).

[2] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).

[3] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996).

[4] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[5] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, Nature (London) **390**, 575 (1997).

[6] D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998).

[7] H. Buhrman, R. Cleve, J. Watrous, and R. de Wolf, Phys. Rev. Lett. **87**, 167902 (2001).

[8] R. T. Horn, S. A. Babichev, K.-P. Marzlin, A. I. Lvovsky, and B. C. Sanders, Phys. Rev. Lett. **95**, 150502 (2005).

[9] M. Mohseni and D. A. Lidar, Phys. Rev. Lett. **97**, 170501 (2006).

[10] L. Vaidman and N. Yoran, Phys. Rev. A **59**, 116 (1999).

[11] N. Lütkenhaus, J. Calsamiglia, and K.-A. Suominen, Phys. Rev. A **59**, 3295 (1999).

[12] J. Calsamiglia and N. Lükenhaus, Appl. Phys. B: Lasers Opt. **72**, 67 (2000).

[13] J. Calsamiglia, Phys. Rev. A **65**, 030301(R) (2002).

[14] J. A. W. van Houwelingen, N. Brunner, A. Beveratos, H. Zbinden, and N. Gisin, Phys. Rev. Lett. **96**, 130502 (2006).

[15] R. Ursin, T. Jennewein, M. Aspelmeyer, R. Kaltenbaek, M. Lindenthal, P. Walther, and A. Zeilinger, Nature (London) **430**, 849 (2004). T. Jennewein, R. Ursin, M. Aspelmeyer, and A. Zeilinger, e-print arXiv:quant-ph/0409008.

[16] P. G. Kwiat and H. Weinfurter, Phys. Rev. A **58**, R2623 (1998).

[17] S. P. Walborn, S. Pádua, and C. H. Monken, Phys. Rev. A **68**, 042313 (2003).

[18] S. P. Walborn, W. A. T. Nogueira, S. Pádua, and C. H. Monken, Europhys. Lett. **62**, 161 (2003).

[19] X.-F. Ren, G.-P. Guo, and G.-C. Guo, Phys. Lett. A **343**, 8 (2005).

[20] M. Barbieri, G. Vallone, P. Mataloni, and F. De Martini, Phys. Rev. A **75**, 042317 (2007).

[21] J. T. Barreiro, Tzu-Chieh Wei, and P. G. Kwait (unpublished).

[22] P. G. Kwiat, J. Mod. Opt. **44**, 2173 (1997).

[23] J. T. Barreiro, N. K. Langford, N. A. Peters, and P. G. Kwiat, Phys. Rev. Lett. **95**, 260501 (2005).

[24] C. Schuck, G. Huber, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. Lett. **96**, 190501 (2006).

[25] The basis states for photon 1 are therefore composed of $\{|Ha\rangle_1,|Hc\rangle_1,|Va\rangle_1,|Vc\rangle_1\}$ and those for photon 2 $\{|Hb\rangle_2,|Hd\rangle_2,|Vb\rangle_2,|Vd\rangle_2\}$; see Fig. 1.

[26] There is one class of four detector outcomes missing. However, this class, indicated in the last line of Table I, could be realized, e.g., by the following two states: $(H_1V_2-V_1H_2)\times(a_1c_2-b_1d_2)$ and $(H_1V_2+V_1H_2)(a_1c_2-b_1d_2)$, which reside outside the Hilbert space spanned by the 16 Bell states and are composed of photon 1 having spatial modes a and b and photon 2 having c and d.

[27] P. van Loock and N. Lütkenhaus, Phys. Rev. A **69**, 012302 (2004).

[28] We found that a modification of the scheme in Ref. [17] also results in complete discrimination with two copies, consistent with the findings of S. Ghosh, G. Kar, A. Roy, and D. Sarkar, Phys. Rev. A **70**, 022304 (2004).

[29] J. Eisert, Phys. Rev. Lett. **95**, 040502 (2005).