Experimental Investigation of a Two-Qubit Decoherence-Free Subspace

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We thoroughly explore the phenomenon of a decoherence-free subspace (DFS) for two-qubit systems. Specifically, we both collectively and noncollectively decohere entangled polarization-encoded two-qubit states using thick birefringent crystals. These results characterize the basis-dependent effect of decoherence on the four Bell states, the robustness of the DFS state against perturbations in the assumption of collective decoherence, and the existence of a DFS for each type of stable noncollective decoherence. Finally, we investigate the effects of collective and noncollective dissipation.

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Quantum computation enables the use of certain algorithms — factoring [1], simulation of quantum systems [2], database searching [3] — superior to any available on a classical system. To realize a working quantum computer, however, the problem of decoherence must be overcome. Decoherence occurs when quantum bits, internal to the quantum computer, couple to external degrees of freedom that are unmeasured. A pure quantum superposition of qubits is thereby transformed into a mixed state. While error-correcting codes [4] or dynamical decoupling [5] act to minimize these effects, it is possible to perform quantum operations in a space fundamentally immune to certain types of decoherence. Qubits can be imbedded in a "decoherence-free subspace" (DFS) [6] in such a way as to be unaffected by collective decoherence or dissipation (energy loss), and robust against noncollective perturbations. While the existence of a DFS was first experimentally demonstrated in 2000 [7] and subsequently verified in several qubit systems [8,9], and even recently used to implement actual quantum algorithms [10], until now no experiment has systematically explored how noncollective, multiple-basis environments affect input states. Here we experimentally investigate how a complete basis of polarization-encoded Bell states, including a DFS, evolve under collective and noncollective decohering and dissipative conditions. All theoretical curves found in this Letter were produced using both a full quantum optical treatment of polarization states in birefringent media (see [11]) and a simple model of spin states in randomly varying magnetic fields. This not only emphasizes the generality of these results but allows the use of the simpler spin model for illustration of the fundamentals of decoherence-free subspaces.

To this end, consider decohering a spin-$\frac{1}{2}$ particle initially in the pure state $|\psi_i\rangle = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2}$. Let a random magnetic field—not controllable by the experimenter — have an equal chance to apply either an $e^{i\pi}$ (field A) or an $e^{i2\pi}$ (field B) phase factor to $|\downarrow\rangle$. This produces the entangled state $|\psi\rangle_{\rm total} = \frac{1}{2}[(|\uparrow\rangle - |\downarrow\rangle) \otimes |{\rm field}_A\rangle + (|\uparrow\rangle + |\downarrow\rangle) \otimes |{\rm field}_B\rangle]$. Tracing over the state of the field, we find the system in the totally mixed ensemble $\rho_{\rm mixed} = \frac{1}{2}(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)$. The presence of a degree of freedom which is, in principle, measurable has made the $|\uparrow\rangle + |\downarrow\rangle$ and $|\uparrow\rangle - |\downarrow\rangle$ states distinguishable [12] and has destroyed their coherence. This model of decoherence can vary in either basis (with the random phases applied to a state other than $|\downarrow\rangle$) or strength [13].

It is impossible to avoid decoherence on a single qubit without eliminating all external couplings. At the two-qubit level, however, we find that a single state is immune to many effects of this type. Specifically, when the singlet state $|\psi^-\rangle \equiv (1/\sqrt{2})(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$ is subjected to a random magnetic field which induces arbitrary phases $e^{i\chi_A}$ or $e^{i\chi_B}$ to the $|\downarrow\rangle$ state of *both* qubits 1 and 2, the resulting state is

$$\begin{aligned}|\psi\rangle_{\rm total} =& \frac{1}{2}[(|\uparrow\rangle_1 e^{i\chi_A}|\downarrow\rangle_2 - e^{i\chi_A}|\downarrow\rangle_1|\uparrow\rangle_2) \otimes |{\rm field}_A\rangle \\ &+ (|\uparrow\rangle_1 e^{i\chi_B}|\downarrow\rangle_2 - e^{i\chi_B}|\downarrow\rangle_1|\uparrow\rangle_2) \otimes |{\rm field}_B\rangle] \\ =& |\psi^-\rangle \otimes \frac{1}{\sqrt{2}}(e^{i\chi_A}|{\rm field}_A\rangle + e^{i\chi_B}|{\rm field}_B\rangle). \qquad (1)\end{aligned}$$

Under the assumption of collective decoherence, that the field acts in the same basis for both qubits, the field has no effect on the singlet state, regardless of the specific phases χ_A and χ_B. Note that the state $|\psi^+\rangle = (1/\sqrt{2}) \times (|\uparrow\rangle_1|\downarrow\rangle_2 + |\downarrow\rangle_1|\uparrow\rangle_2)$ is also immune to decoherence in the $\uparrow - \downarrow$ basis; together $|\psi^-\rangle$ and $|\psi^+\rangle$ form the simplest DFS, a 1-qubit basis immune to collective decoherence restricted to the $|0\rangle - |1\rangle$ basis ($|\uparrow\rangle - |\downarrow\rangle$ for spin, $|H\rangle - |V\rangle$ for polarization) [8]. While $|\psi^+\rangle$ will degrade when subjected to decoherence in any basis other than $\uparrow - \downarrow$, $|\psi^-\rangle$ is decoherence-free in *every* basis.

If the decoherence affecting qubit 1 differs in either strength or basis from that affecting qubit 2, then it is "noncollective." Although the singlet state is largely unaffected even by somewhat noncollective decoherence (see below), there is always a completely decoherence-free state — not necessarilya Bell state — for each type of stable noncollective decoherence.

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We have investigated collective and noncollective decoherence using a source of entangled photon pairs [14]. Our logical basis states are horizontal (H) and vertical (V) polarization. The photon pairs are produced via spontaneous parametric down-conversion in thin nonlinear crystals (beta-barium-borate, BBO) crystals cut for type-I phase matching. An 80-mW 45°-polarized pump beam (at 351 nm) is directed through two orthogonally oriented thin BBO crystals such that the pump can either down-convert into two H photons in the first crystal or two V photons in the second crystal (at 702 nm). This yields the Bell state $|\psi^+\rangle = (1/\sqrt{2}) \times (|H\rangle|H\rangle + |V\rangle|V\rangle)$. The other three Bell states can be generated by applying a simple unitary operation in one arm. The Bell states are then used as the input to various decohering or dissipating apparatuses (see Fig. 1).

Quantum state tomography [15] allows analysis and characterization of these Bell states both with and without decoherence. State tomography uses a series of correlation measurements (e.g., HH, HV, and $V45°$) to reconstruct the density matrix of the incident state. Each correlation measurement is performed using a polarization analyzer in each arm, consisting of a half wave plate, quarter wave plate, and polarizing beam splitter, which together allow projection into any polarization basis.

Decoherence in our experiment is controllably introduced using a thick (11 mm) piece of birefringent quartz in each arm; the quartz separates the ordinarily and extraordinarily polarized wave packets by approximately 140 wavelengths along the propagation axis, approximately the coherence length of the down-converted light after a 5-nm (FWHM) bandwidth interference filter. Because the H and V components of the light are separated by the coherence length, these become, in principle, temporally distinguishable with respect to one another. This acts as a label (the relative timing and the polarization become entangled) and therefore induces decoherence in the ordinary-extraordinary basis of the quartz crystal. Half wave plates before and after the decoherers (see Fig. 1) induce rotations which allow each decoherer to effectively operate in any linear polarization basis. For a complete treatment of this type of photon state decoherence, see reference [11].

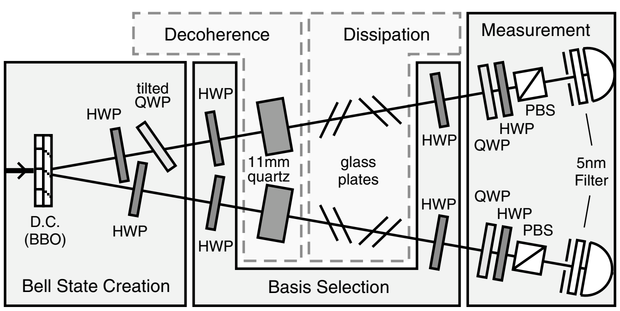

FIG. 1. A schematic of our experimental setup. Photons in the maximally entangled state $|\psi^+\rangle = (|HH\rangle + |VV\rangle)/\sqrt{2}$ are produced when 45° polarized pump light is directed through two adjacent nonlinear crystals [14]. Two half wave plates immediately after the crystals are used to interchange between the four Bell states within a phase factor. This phase factor is adjusted by tilting a quarter wave plate. Depending on the experiment, either decohering elements or dissipative elements are inserted into both paths. The final state of the light is determined by making a series of polarization correlation measurements in various bases, and from these deducing what the density matrix of the output light is, which may then be compared with the input density matrix.

For collective decoherence, these wave plates rotate together [16], ensuring that the two environments always operate in the same basis. Figure 2 illustrates the effect of collective decoherence by plotting the fidelity [17] between input Bell states and the same states after they pass through the decoherer; as predicted, the singlet state is decoherence-free in every collective linear basis.

Another predicted [18] (but until now unverified) benefit of a DFS is that it should be robust against perturbations; i.e., when the decoherence has a small noncollective component, the DFS basis states will still be largely decoherence-free. One way to investigate the dependence of a DFS's effectiveness on noncollective effects is to apply the decoherence in each arm in a different basis, using the basis selection half wave plates (see Fig. 1). Figure 3(a) shows the fidelity of the output state with the input Bell state when the decohering basis in arm 1 is fixed at 15° while the decohering basis in arm 2 is varied from 0° to 30°. Notice that the fidelity of the DFS state $|\psi^-\rangle$ falls off only quadratically (rather than linearly) with angle, showing that it is robust to perturbations in the normal assumption of collective decoherence.

Figure 3(b) illustrates a second type of noncollective decoherence. By varying the thickness of quartz in one arm, the *strength* of the decoherence can be changed, ranging from no decoherence (no crystal present) to total decoherence (asymptotically approached for an infinitely long crystal). The crystal-induced separation between the o and e waves in arm 1 is fixed at 140λ, while the thickness of the crystal in arm 2 is varied. Again, the DFS state is robust (scaling quadratically) to perturbations in the assumption of collective decoherence.

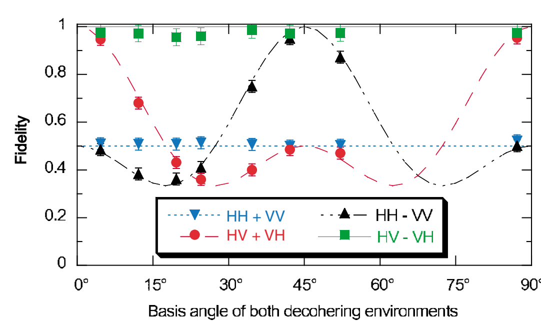

FIG. 2 (color). Plot showing the effect of collective decoherence on the four Bell states, when the decoherence is applied in a number of different (linear polarization) bases. Solid lines are the theoretical predictions.

The normal DFS state, the singlet state, *does* decohere somewhat under conditions of noncollective decoherence, but a different DFS state exists to compensate for these conditions. In fact, for every pair of orientations for two equal strength decohering environments there exist two special DFS states which are completely decoherence-free under these conditions. For example, the DFS for the conditions 15° basis in arm 1 and 45° basis in arm 2 is spanned by $|\psi\rangle_{\text{special}} = (1/\sqrt{2})(|15°\rangle|45°\rangle \pm |105°\rangle|135°\rangle)$. Both $|\psi^-\rangle$ and one of these states were subjected to this environment. Figure 4 shows their density matrices before and after the noncollective environment and, as expected, the singlet state decoheres while $|\psi\rangle_{\text{special}}$ does not. These results, coupled with the ability to exactly characterize any source of decoherence (via quantum process tomography [19]), allow the construction of a DFS optimized for any (static) environment.

A problem separate from decoherence is dissipation, whereby entire qubits have some probability of being lost, dissipated into an unmeasured mode. Consider using a state subject to dissipation for quantum cryptography [20]. In addition to requiring additional qubits for the same size key, if the dissipation is basis dependent (e.g., dissipating the $|0\rangle$ state more frequently than the $|1\rangle$ state), Bob has a chance to incorrectly measure the $(|0\rangle \pm |1\rangle)/\sqrt{2}$ states sent by Alice. Dissipation of a single qubit may be characterized by a basis (e.g., $|0\rangle$ and $|1\rangle$) and a ratio (e.g., dissipates twice as much $|0\rangle$ as $|1\rangle$). As for the case of decoherence, dissipation can be collective (affecting each qubit of a multiple qubit system identically) or noncollective (differing from qubit to qubit in either basis or ratio). A DFS state subject to collective dissipative conditions will sometimes be destroyed, but never measured incorrectly.

Our dissipative environments were experimentally realized using tilted glass plates so that H polarization had different transmission than V ($T_H = 0.86, T_V = 0.21$). As before, we subjected the Bell states to both collective and noncollective environments. In Fig. 5(a) the collective dissipation results show that $|\psi^+\rangle$ and $|\psi^-\rangle$ form a dissipation-free subspace (subjecting these states to unbalanced dissipation in the H-V basis causes a net loss, but never results in the states being incorrectly measured). This is to be expected, as a dissipative environment in the H-V basis causes $|H\rangle|V\rangle \pm |V\rangle|H\rangle$ to be measured as $\sqrt{T_H}|H\rangle\sqrt{T_V}|V\rangle \pm \sqrt{T_V}|V\rangle\sqrt{T_H}|H\rangle = \sqrt{T_H} \times \sqrt{T_V}(|H\rangle|V\rangle \pm |V\rangle|H\rangle)$, i.e., the same state after renormalization. Because the singlet state has the same

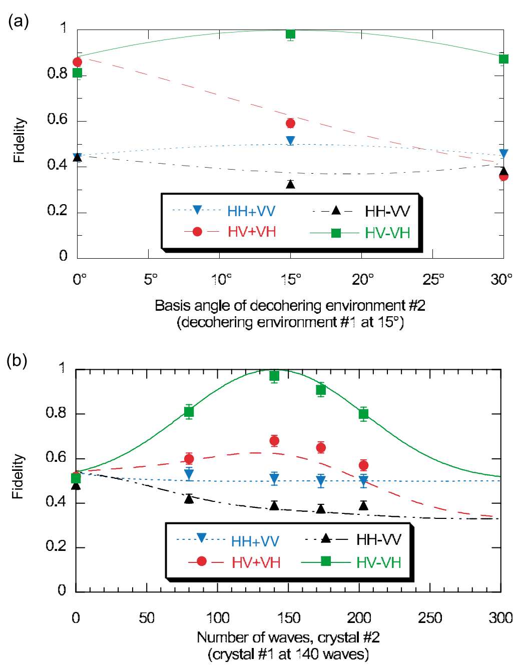

FIG. 3 (color). Results showing the effect of noncollective decoherence. Solid lines are theoretical predictions. (a) The strength of the decoherence affecting each qubit is the same (corresponding to the thickness of the decohering piece of quartz, measured in terms of the induced longitudinal separation between the *o* and *e* wave packets), but the relative bases in which this decoherence occurs is changed: the decoherence basis is fixed at 15° for photon number 1, while the basis is varied from 0° to 30° for photon number 2. (b) The orientation of the bases are now kept fixed (15° in both arms), while the amount of decoherence is varied for photon 2 relative to photon 1. In all cases we see that the singlet state $|\psi^-\rangle$ is robust against perturbations to the assumptions of collective decoherence, falling off quadratically rather than linearly.

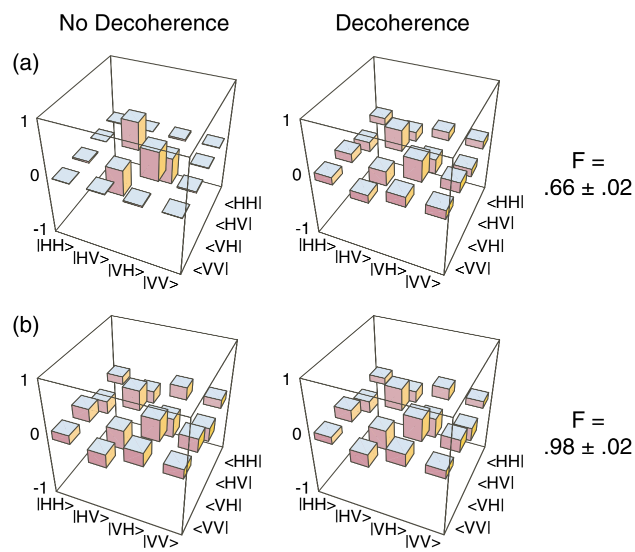

FIG. 4 (color). Measured density matrix elements demonstrating that DFSs exist even when the environments acting on the two qubits are very different. We show the effect of different decohering elements (15° basis in arm 1 and 45° basis in arm 2, but both the same strength) on both the singlet state and a special state specifically calculated as the DFS for these environments (see text). (a) The singlet state is heavily decohered; the fidelity between the initial and final state is (66 ± 2)%. (b) For these environments, a true DFS is shown. The fidelity between the initial and final state is (98 ± 2)%.

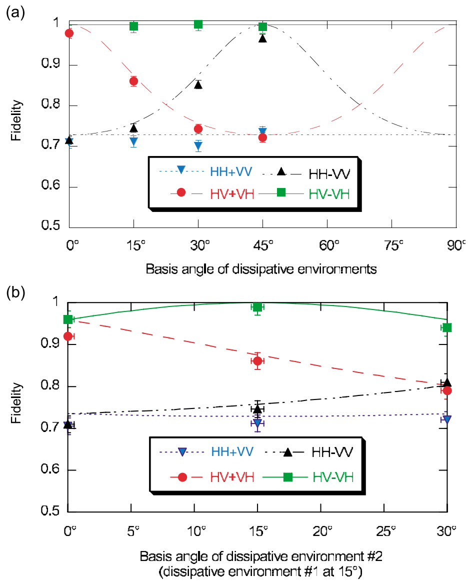

FIG. 5 (color). Plots showing the effect of dissipation on the four Bell states. (a) The effects of collective dissipation. (b) The effects of noncollective dissipation. The dissipating environment in arm 1 is applied at 15° while the environment in arm 2 is rotated between 0° and 30°. Solid lines are theoretical predictions. The singlet state $|\psi^-\rangle$ is robust against both collective and noncollective dissipation.

representation in every basis, it is never affected by collective dissipation. Figure 5(b) shows noncollective dissipation with the environment in arm 1 fixed at 15° and the environment in arm 2 varied from 0° to 30°. As with decoherence, $|\psi^-\rangle$ is robust (scaling quadratically) against perturbations to the assumption of collective dissipation. Though not shown here, a special state can also be constructed that is completely unperturbed by this type of stable noncollective dissipation.

Our investigations of decoherence-free states have been performed with polarization-encoded qubits. However, the results are valid for *any* physical qubit implementation, e.g., ions [8] or nuclear spins [21], as long as the qubits used to encode the decoherence-free subspace experience collective noise effects. In fact, we have verified that the theoretical curves presented in this Letter can be arrived at either by a direct calculation of the optical propagation of the down-conversion photons [11] or equivalently by the simple model of spin-1/2 particles in a random magnetic field discussed earlier. The fact that the DFS state is insensitive even to slightly noncollective noise effects has important implications for fault-tolerant quantum computing: the error introduced into the singlet state fidelity by a 0.4° basis difference in the two decohering environments is still less than 10^{-4}.

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[12] In most cases the extra degree of freedom is not accessible to the experimenter; for example, if the particle is in contact with a heat bath, the extra information may be unrecoverable. Nevertheless, the coupling of a quantum system to a single unobserved degree of freedom is the fundamental mechanism for decoherence (whether or not that degree of freedom can actually be observed).
[13] For example, if there were only a 10% chance of the decohering magnetic fields affecting the system, it would create the partially mixed state $\rho = 0.9|\psi_i\rangle\langle\psi_i| + 0.1\rho_{\text{mixed}}$ — the result of relatively weak decoherence.
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[16] To compensate for frequency anticorrelations in the two daughter photons, the fast axes of the decohering crystals in arms 1 and 2 were oriented at 90° to each other [11].
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