

# Counterfactual quantum computation through quantum interrogation

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The logic underlying the coherent nature of quantum information processing often deviates from intuitive reasoning, leading to surprising effects. Counterfactual computation constitutes a striking example: the potential outcome of a quantum computation can be inferred, even if the computer is not run<sup>1</sup>. Relying on similar arguments to interaction-free measurements<sup>2</sup> (or quantum interrogation<sup>3</sup>), counterfactual computation is accomplished by putting the computer in a superposition of ‘running’ and ‘not running’ states, and then interfering the two histories. Conditional on the as-yet-unknown outcome of the computation, it is sometimes possible to counterfactually infer information about the solution. Here we demonstrate counterfactual computation, implementing Grover’s search algorithm with an all-optical approach<sup>4</sup>. It was believed that the overall probability of such counterfactual inference is intrinsically limited<sup>1,5</sup>, so that it could not perform better on average than random guesses. However, using a novel ‘chained’ version of the quantum Zeno effect<sup>6</sup>, we show how to boost the counterfactual inference probability to unity, thereby beating the random guessing limit. Our methods are general and apply to any physical system, as illustrated by a discussion of trapped-ion systems. Finally, we briefly show that, in certain circumstances, counterfactual computation can eliminate errors induced by decoherence.

The essential feature of Grover’s algorithm is that an amplitude-enhancement technique transfers the amplitude from a uniform database distribution to a particular element ‘marked’ by an ‘Oracle-type’ processor<sup>4,7</sup>. For instance, consider a database of four elements with input state  $|00\rangle$ . At the end of the algorithm, if the marked element (ME) is no. 1, no. 2, no. 3 or no. 4, the final state of the readout qubits is  $|00\rangle$ ,  $|01\rangle$ ,  $|10\rangle$  or  $|11\rangle$ , respectively.

Counterfactual computation (CFC) conceptually proceeds as follows: (0) the initial state of the computer can be written as  $|\psi_{in}\rangle = |\text{Off}\rangle|00\rangle$ , the  $|\text{Off}/\text{On}\rangle$  qubit being the ‘operating switch’; (1) apply a  $\frac{\pi}{2}$ -rotation ( $R(\frac{\pi}{2}) : |\text{Off}\rangle \rightarrow \frac{|\text{Off}\rangle + |\text{On}\rangle}{\sqrt{2}}$  and  $|\text{On}\rangle \rightarrow \frac{-|\text{Off}\rangle + |\text{On}\rangle}{\sqrt{2}}$ ) to the ‘switch’; (2) run the algorithm if the ‘switch’ is ‘On’; (3) apply  $R(\frac{\pi}{2})$  to the ‘switch’ only if the registers are in state  $|00\rangle$ . If the ME is no. 1, the effect is:

$$|\text{Off}\rangle|00\rangle \xrightarrow{R} \frac{|\text{Off}\rangle + |\text{On}\rangle}{\sqrt{2}}|00\rangle \xrightarrow{\text{Grover}} \frac{|\text{Off}\rangle|00\rangle + |\text{On}\rangle|00\rangle}{\sqrt{2}} \xrightarrow{R} |\text{On}\rangle|00\rangle \quad (1)$$

All the amplitude ends up in state  $|\text{On}\rangle|00\rangle$ , with equal amplitudes constructively interfering from histories with the computer ‘running’ and ‘not running’.

For other MEs (with  $xy = 01, 10$  or  $11$ ), we have:

$$|\text{Off}\rangle|00\rangle \xrightarrow{R} \frac{|\text{Off}\rangle + |\text{On}\rangle}{\sqrt{2}}|00\rangle \xrightarrow{\text{Grover}} \frac{|\text{Off}\rangle|00\rangle + |\text{On}\rangle|xy\rangle}{\sqrt{2}} \xrightarrow{R} \frac{|\text{Off}\rangle + |\text{On}\rangle}{2}|00\rangle + \frac{1}{\sqrt{2}}|\text{On}\rangle|xy\rangle \quad (2)$$

Now, there is a  $\frac{1}{4}$  probability to measure the final state  $|\text{Off}\rangle|00\rangle$ , from which we conclude that the ME is not no. 1 (as this term did not appear when ME = no. 1). There is no amplitude from a history with the computer ‘running’ in this outcome; therefore, we can conclude that the ME is not no. 1 without the computer ‘running’. (Later we describe a method to counterfactually determine the actual outcome.)

An optical realization of CFC is shown in Fig. 1 as an interferometer. We use the optical circuit in ref. 8 for Grover’s algorithm (shown as a black box in Fig. 1), with slight changes to improve performance (see Supplementary Information). This optical circuit takes in a single photon in path a with H (horizontal) polarization, that is,  $|aH\rangle$ . The output path a or b and polarization H or V (vertical) of the photon depend on the ME: no. 1  $\rightarrow |aH\rangle$ , no. 2  $\rightarrow |aV\rangle$ , no. 3  $\rightarrow |bH\rangle$  and no. 4  $\rightarrow |bV\rangle$ . Such single-photon encoding, though not scalable, suffices for our pedagogical purpose. Figure 2a shows that the algorithm as realized operates quite precisely, with an average error of less than 2.6%.

To realize CFC, the path lengths of the interferometer in Fig. 1 are adjusted to give destructive interference at detector  $D_1$  when the ME is no. 1, that is, if the photon traverses both paths indistinguishably, it is definitely detected at  $D_5$  (Fig. 1). If the ME is not no. 1, there is no amplitude exiting the search algorithm in the  $|aH\rangle$  mode, and no amplitude from the algorithm can reach the second beam-splitter (BS), thus eliminating the destructive interference at  $D_1$ . Therefore, this time there is a  $\frac{1}{4}$  probability of detecting the photon at  $D_1$ , indicating that the ME is not no. 1, even though the photon came from the algorithm-free path (that is, the computer did not run). Other possibilities are that with probability  $\frac{1}{2}$  we find out the answer but the photon comes from the algorithm ( $D_2, D_3$  or  $D_4$ ), and with probability  $\frac{1}{4}$  we detect the photon at  $D_5$  (after it has travelled the algorithm-free path) leading to no definite information (as a detection also occurs at  $D_5$  if ME = no. 1). The efficiency of CFC can be quantified as  $\eta = P_{D_1} / (P_{D_1} + P_{\text{Grover}}) = 1/3$ , where  $P_{D_1}$  is the probability of measuring the photon at  $D_1$ , a CFC; and  $P_{\text{Grover}}$  is the probability of the photon passing through the algorithm.

Experimentally, we used an equivalent but more stable interferometer configuration (see Supplementary Information); our CFC system performed as indicated in Fig. 2b, with an efficiency  $\langle \eta \rangle = 0.319 \pm 0.009$  (averaged over ME  $\neq$  no. 1). The interrogation interferometer had an intrinsic visibility of 97%, implying imperfect

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destructive interference at the  $D_1$  output when the ME was no. 1. A detection at the  $D_1$  output can thus no longer indicate a CFC with 100% certainty. We characterize this feature in terms of  $P_{\text{Succ}}$ , the probability of a successful CFC upon a detection at the interrogation detector  $D_1$ :  $P_{\text{Succ}|no.i} = P_{D_1|no.i} / (P_{D_1|no.1} + P_{D_1|no.i})$ . Here the probabilities are conditional on the ME ( $i = 2, 3, 4$ ). In the experiment we achieved  $\langle P_{\text{Succ}} \rangle = 0.943 \pm 0.009$ .

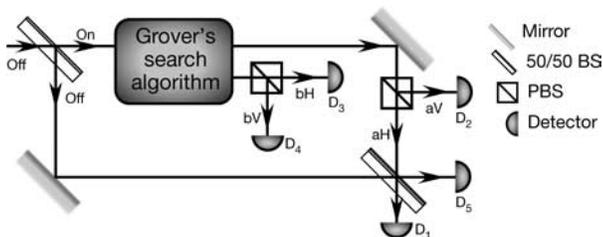
If one replaces the first (second) 50/50 beam splitter by a highly reflecting (transmitting) one, reducing the amplitude passing through the algorithm,  $\eta$  can be increased<sup>9</sup> to a maximum value of  $\frac{1}{2}$ . We tested this (here using attenuated coherent states) with a 5/95 BS, achieving  $\langle \eta \rangle = 0.472 \pm 0.007$  (0.487 theoretically), with a slight decrease in the CFC success:  $\langle P_{\text{Succ}} \rangle = 0.877 \pm 0.009$ . (See Supplementary Information for a modified version of CFC that interrogates all database elements simultaneously to determine the ME itself; however, this modified version is also limited to  $\eta = \frac{1}{2}$ ).

The efficiency can be increased from  $\frac{1}{2}$  to 1 using the quantum Zeno effect<sup>1</sup>, just as it was used for quantum interrogation<sup>3,9</sup>. (Before proceeding, we note that our approach should not be confused with the interesting proposal to combine two-photon absorption and the quantum Zeno effect to enable efficient optical quantum computation<sup>10</sup>, nor with the suggestion to use quantum interrogation inside Grover's search algorithm<sup>11</sup>.) Specifically, the switch is rotated successively in small steps ( $R(2\theta)$ :  $|Off\rangle \rightarrow \cos\theta|Off\rangle + \sin\theta|On\rangle$  and  $|On\rangle \rightarrow -\sin\theta|Off\rangle + \cos\theta|On\rangle$ ;  $\theta = \frac{\pi}{2N}$ , integer  $N \gg 1$ ) from state  $|Off\rangle$  to  $|On\rangle$ , and the output registers are monitored at each step. If the ME is no. 1, measurements on the output registers result in  $|00\rangle$  without affecting the evolution due to rotations, leaving the system in  $|On\rangle|00\rangle$  after  $N$  rotations. However, if the ME is not no. 1, then the system evolves as:

$$|Off\rangle|00\rangle \xrightarrow{R} (\cos\theta|Off\rangle + \sin\theta|On\rangle)|00\rangle \xrightarrow{\text{Grover}} \cos\theta|Off\rangle|00\rangle + \sin\theta|On\rangle|xy\rangle \xrightarrow{\text{Measure}} \approx \cos\theta|Off\rangle|00\rangle \quad (3)$$

The measurement on the output registers results in state  $|00\rangle$ , leaving the computer in the  $|Off\rangle$  state with probability  $\cos^2(\frac{\pi}{2N})$ . After a total of  $N$  cycles, the probability of finding the system in state  $|Off\rangle|00\rangle$  is  $\cos^{2N}(\frac{\pi}{2N})$ , which tends to 1 as  $N \rightarrow \infty$ . Thus, if the ME is not no. 1, the final state is  $|Off\rangle|00\rangle$  without the computer running; and if the ME is no. 1, the final state is  $|On\rangle|00\rangle$ , that is, this time the computer runs. Note that at best one can exclude a single value of the ME with these techniques.

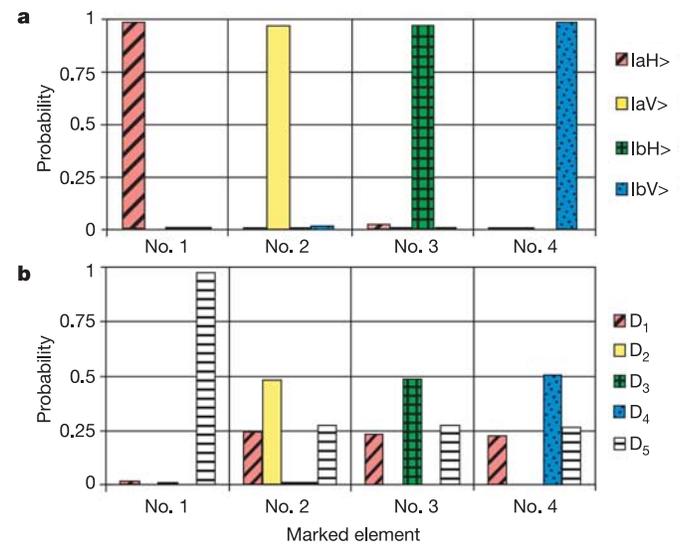
Now we describe a novel 'chained Zeno effect' that will permit us to counterfactually determine the actual ME with  $\eta \rightarrow 1$ . The strategy is to place the above 'Zeno scheme' inside another one, to avoid the computer running even if the ME is no. 1. For this purpose we use a third 'switch' state  $|Off'\rangle$ , in addition to  $|Off\rangle$  and  $|On\rangle$ , and



**Figure 1 | An optical realization of counterfactual computation.** By means of a 50/50 beam splitter (BS) (which serves as a  $\frac{\pi}{2}$ -rotation), an H-polarized single photon is put in a superposition of passing and not passing through the algorithm, encoding the 'operating switch' in different spatial modes, 'On' and 'Off'. Then on a second 50/50 BS, the two histories are interfered only if the photon after the algorithm is in the mode  $|aH\rangle$ . The modes  $|aH\rangle$  and  $|aV\rangle$  are distinguished via a polarizing beam splitter (PBS) which transmits H and reflects V.

define an additional rotation  $R'$  to couple  $|Off'\rangle$  to  $|Off\rangle$  ( $R'(2\theta')$ :  $|Off'\rangle \rightarrow \cos\theta'|Off'\rangle + \sin\theta'|Off\rangle$  and  $|Off\rangle \rightarrow -\sin\theta'|Off'\rangle + \cos\theta'|Off\rangle$ ;  $\theta' = \frac{\pi}{2N'}$ ). A possible optical implementation of the technique is shown in Fig. 3a. A single photon starts in cavity 'Off'. Using active optical elements (Pockels cells), a small amount of amplitude is exchanged between 'Off' and 'Off'' via BS<sub>1</sub> (see Methods for details). The small amplitude then performs the high-efficiency quantum Zeno cycles described above ( $N$  times between cavities 'Off' and 'On'). If the ME  $\neq$  no. 1, the small amplitude component effectively stays in cavity 'Off'. But if ME = no. 1, then first, all the small amplitude component transfers to cavity 'On', and then, via the Pockels cell in cavity 'On', is actively absorbed at A<sub>1</sub>. Now the entire procedure starting with the amplitude exchange between 'Off' and 'Off'' is repeated, a total of  $N'$  times. At the end of  $N' \times N$  total cycles, if ME = no. 1 ( $\neq$  no. 1) then the photon will be measured in cavity 'Off'' ('Off') with probability approaching one (Fig. 3e) as  $N' \rightarrow \infty$  ( $\frac{N'}{N} \rightarrow \infty$ ). In neither of these cases does the computer 'run'. One can then re-interrogate for the other elements one by one—thereby identifying the ME counterfactually—by changing the connections to the algorithm (Fig. 3b). Curves in Fig. 3e characterize a lossless system; for large  $N'$ ,  $N$ , even small losses become detrimental<sup>3</sup>, limiting the achievable performance in any real system.

The necessity to interrogate database elements one by one would negate the quantum speed-up advantage. However, it is possible to circumvent this by interrogating the logical value of each qubit one by one instead. To do this, we need to implement both the search algorithm and its adjoint—which undoes the search—and we need to perform the measurements (giving rise to the Zeno effect) between the algorithm and its adjoint. The first (second) optical circuit in Fig. 3c can be used in cavity 'On', to interrogate for the logical value of the first (second) qubit. If the value of the first (second) qubit is 'a' (H), then, at the end of the cycles, the photon will be measured in cavity 'Off''; likewise if the value is 'b' (V), then the photon will be



**Figure 2 | Experimentally determined probabilities for the output state of 670-nm single photons conditionally prepared through downconversion<sup>17</sup>.** **a**, The performance of the search algorithm (see Supplementary Information for details). The total probability of finding the photon in any incorrect output port is 2.6% (averaged over MEs); thus, the ME could be accurately determined with a single photon passing through the algorithm. **b**, Output state probabilities for the set-up in Fig. 1. Grid lines closest to data points represent theory. We attribute slight deviations from theory to imperfect beam splitting ratios, imperfect mode matching (apparently from wavefront distortions in various elements), and imperfect path-length balancing.

measured in cavity ‘Off’. In neither case does the computer (or its adjoint) run. (See Supplementary Information for more details, including the quantum circuit diagram of this method.)

The physical implementation of our proposals is not limited to optical systems. Consider qubits encoded using hyperfine ground states ( $|0\rangle$  and  $|1\rangle$ ) of two trapped ions. In the simplest scheme (the analogue of Fig. 1), the ions would start in trap ‘Off’ ( $|Off\rangle|00\rangle$ ). The atoms can be prepared in an equal superposition of both atoms being in trap ‘Off’ or being in another trap ‘On’ (see Methods). Then, a measurement on the atoms can be performed, causing at least one of them to fluoresce if the ME  $\neq$  no. 1 but leaving the system undisturbed if ME = no. 1. After interfering the two histories, if ME = no. 1 the atoms end up in trap ‘On’. If instead ME  $\neq$  no. 1, then there is a  $\frac{1}{4}$  probability of finding the atoms in trap ‘Off’ without observing any fluorescence during the measurement process above (which means that the algorithm did not run), from which we could counterfactually conclude that the ME  $\neq$  no. 1. In the ‘chained Zeno’ version, one would repeatedly look to see if the algorithm had run; but the atoms never appear in trap ‘On’, and the final location of atoms (trap ‘Off’ or ‘Off’) would counterfactually reveal the results.

To the best of our knowledge, we have achieved the most accurate (2.6% error) realization to date of Grover’s search algorithm, albeit with a non-scalable single-photon implementation. Using this set-up, we made the first proof-of-principle demonstration of CFC, inferring that a particular element was not the answer to the computation, even though the computer did not run (with efficiency  $\eta \approx 0.319$ ). Then, we showed in principle how to obtain complete information from the algorithm unconditionally, without the algorithm ever running, using a ‘chained’ Zeno effect. In fact, these CFC methods still work if the algorithm were to output non-orthogonal states as answers (for example, Grover’s algorithm in a larger

database), or if the ‘Oracle’ of the search algorithm were to be in a quantum superposition of marking different elements (see Supplementary Information).

Decoherence in quantum computing occurs owing to coupling to uncontrolled degrees of freedom—the environment—and results in incomplete interference between qubit states, leading to errors. In CFC, although the algorithm does not run, it still needs to be ready to run correctly, for all the critical interference effects to occur (for example, for the case of trapped ion computation, all the laser pulses for state manipulation still need to be sent into the trap, even if after the fact there was no ion there). For this reason, one might conclude that CFC would still necessarily be subject to decoherence-induced errors. In the Methods section, however, we show that by using a variant of the ‘chained’ Zeno CFC, in certain circumstances it is possible to protect computations against decoherence. When this scheme succeeds (that is, when the photon does not get absorbed; Fig. 3a and d), the total probability amplitude that has actually run through the algorithm is very small ( $\ll 1$ , though not zero). Consequently, coupling to the environment is small and decoherence-induced errors are suppressed. It remains to be seen whether the success probability can be made arbitrarily close to unity regardless of the amount of error. Nevertheless, by slightly modifying our approach, it may be possible to design error-suppressing counterfactual qubit gates, for use in scalable quantum computing architectures.

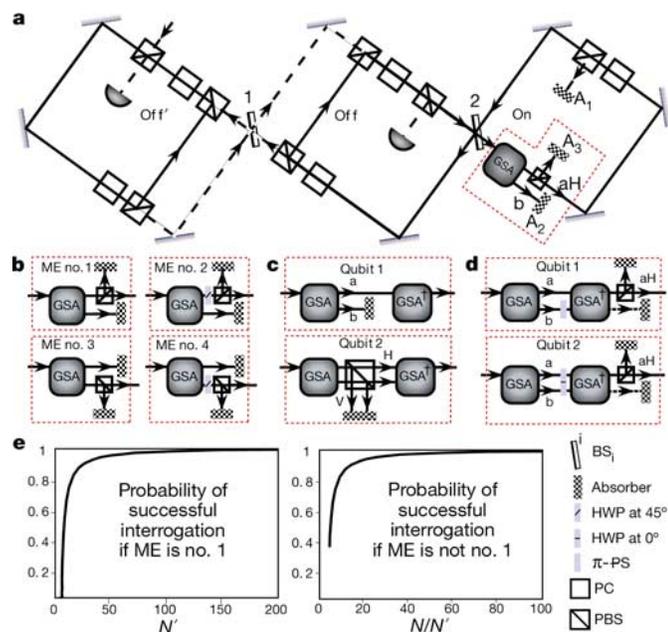
## METHODS

**CFC using the ‘chained Zeno effect’.** Using a polarizing BS and Pockels cells, a single photon is switched into cavity ‘Off’ (Fig. 3a). By firing the Pockels cells at appropriate times, cavities ‘Off’ and ‘Off’ are allowed a single coherent amplitude exchange via BS<sub>1</sub>, after which the cavities are isolated again. While the vast majority of the amplitude is cycling in cavity ‘Off’, the small amount of amplitude that has been allowed to leak into cavity ‘Off’ performs the high-efficiency quantum Zeno cycles described in the text ( $N$  times between cavities ‘Off’ and ‘On’). At each step, the amplitude exiting the algorithm is directed to an absorber (A<sub>2</sub> or A<sub>3</sub>) if the ME is different from no. 1, effectively projecting the leaked amplitude into cavity ‘Off’, assuming  $N \gg 1$ . In contrast, if the ME is no. 1, at the end of  $N$  cycles all of the small amplitude component has coherently moved into cavity ‘On’; when the Pockels cell in this cavity is fired to terminate the amplitude at A<sub>1</sub>, no amplitude is left in either cavity ‘Off’ or ‘On’. In fact, this effectively projects the state into cavity ‘Off’, assuming  $N' \gg 1$ .

Now we once again temporarily couple cavities ‘Off’ and ‘Off’, and repeat the ‘Off’–‘On’ (inner) cycling another  $N$  times. The procedure ends after repeating this entire procedure (outer cycle)  $N'$  times, for a total of  $N' \times N$  inner cycles. Upon successful operation, if the ME is no. 1 (not no. 1), the photon ends up in cavity ‘Off’ (‘Off’).

**Error suppression.** In our theoretical model, decoherence causes depolarization. Assume that, with probability  $\epsilon$ , the search algorithm becomes entangled with the environment, and outputs  $(1 - \frac{3\epsilon}{4})|00\rangle\langle 00| + \frac{\epsilon}{4}|01\rangle\langle 01| + \frac{\epsilon}{4}|10\rangle\langle 10| + \frac{\epsilon}{4}|11\rangle\langle 11|$  instead of  $|00\rangle\langle 00|$ , outputs  $\frac{\epsilon}{4}|00\rangle\langle 00| + (1 - \frac{3\epsilon}{4})|01\rangle\langle 01| + \frac{\epsilon}{4}|10\rangle\langle 10| + \frac{\epsilon}{4}|11\rangle\langle 11|$  instead of  $|01\rangle\langle 01|$ , and so on. In an attempt to suppress these errors, we use a variant of the ‘qubit-by-qubit’ ‘chained’ Zeno CFC (note that the algorithm adjoint will also introduce errors, making the net probability of entanglement with the environment  $2\epsilon - \epsilon^2$ ). Instead of measurements between the algorithm and its adjoint (Fig. 3c), we apply an extra  $\pi$ -phase shift to the state if the ‘operating switch’ is  $|0\rangle$  and the state of the qubit being interrogated is  $|1\rangle$  (Fig. 3d). After the algorithm adjoint we perform a measurement on the output registers to ensure that the state is  $|00\rangle$  (that is, absorb the photon if it is not in mode ‘aH’), as the search algorithm and its adjoint together should leave the output registers unchanged, that is,  $|00\rangle$ , in the case of no errors. If the state of the qubit being interrogated is  $|1\rangle$ , then because of the extra  $\pi$ -phase shift, any amplitude entering the ‘On’ state is interferometrically directed back to ‘Off’ (of course degraded by decoherence). But, if the state is  $|0\rangle$ , then the amplitude coherently builds up in ‘On’ (also degraded by decoherence), depleting the amplitude in ‘Off’, in effect inhibiting the coherent flow from ‘Off’ to ‘Off’. At the end, if the qubit being interrogated is  $|0\rangle$  ( $|1\rangle$ ), we expect the system to stay in (move to) ‘Off’ (‘Off’).

Applying the extra  $\pi$ -phase shifts, instead of measurements, is analogous to the Super-Zeno effect<sup>12</sup> (a bang-bang type control<sup>13</sup>) instead of the Zeno effect, and preserves the quantum state of a system much more efficiently. Optimum calculations ( $N' = 70$ ,  $N = 40$ ;  $\frac{N'}{N} \gg 1$  not being necessary for the Super-Zeno



**Figure 3 | Proposed set-up for the ‘chained Zeno effect’.** Three cavities correspond to three states of the switch ( $|Off\rangle$ ,  $|Off\rangle$  and  $|On\rangle$ ), separated by BSs—the rotation operators. Pockels cells (PCs) rotate the polarization by 90° on demand; half-wave plates (HWP) at 45° rotate the polarization by 90°. GSA, Grover’s search algorithm. **a**, Interrogation for element no. 1. A<sub>1</sub>–A<sub>3</sub> are absorbers. **b**, Settings for the interrogation of different elements. **c**, Set-up configurations for qubit-by-qubit interrogation. GSA<sup>†</sup> undoes the action of GSA. **d**, Configurations for error suppression.  $\pi$ -PS induces a  $\pi$ -phase shift on path b; HWP at 0° induces a  $\pi$ -phase shift on polarization V. **e**, Probability of successful interrogation for the set-up in **a**, as a function of the cycling parameters  $N'$  and  $N$  (numerically evaluated).

version) show that, for single-pass error probability  $\epsilon = 0.05$  ( $2\epsilon - \epsilon^2 = 0.0975$ ), error-suppressing CFC yields the value of the qubit being interrogated 0.944 of the time with an error probability of only  $\sim 2 \times 10^{-4}$ . The remaining 0.056 ( $< 2\epsilon - \epsilon^2$ ) of the time the system suffers decoherence and fails, and the photon gets absorbed at one of the absorbers in Fig. 3d. Even for a much larger  $\epsilon = 0.5$  ( $2\epsilon - \epsilon^2 = 0.75$ ), we still learn the value of the qubit 0.508 of the time, with an error of only  $\sim 0.035$ .

**CFC with trapped ions.** Consider ions starting in state  $|\text{Off}\rangle|00\rangle$ . In order to put the computer in a superposition, the atoms can first be prepared in the entangled state  $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$  (by applying the rotation  $R(\frac{\pi}{2})$  (see main text) to the first qubit, and then applying a controlled-not gate on the qubits); with the aid of appropriate laser beams, a state-dependent spatial force can then be applied to the atoms, resonantly driving and separating the wavepackets of different internal states<sup>14</sup>. At this point a new external potential can be introduced, which traps  $|0\rangle$  states in trap 'Off', and  $|1\rangle$  states in trap 'On':  $\frac{|\text{Off}\rangle|00\rangle+|\text{On}\rangle|11\rangle}{\sqrt{2}}$ . After flipping the states of the amplitudes in trap 'On', we can apply the algorithm<sup>15</sup> (again using suitable laser pulses) in this trap:  $\frac{|\text{Off}\rangle|00\rangle+|\text{On}\rangle|xy\rangle}{\sqrt{2}}$ . The states of the atoms can be measured efficiently using a state-selective fluorescence technique<sup>16</sup>, which scatters light only from atoms in state  $|1\rangle$ . But this measurement gives no signal if the output of the algorithm is  $|xy\rangle = |00\rangle$  (that is, if the ME = no. 1), and thus cannot distinguish which trap the ions are in, leaving the state  $\frac{|\text{Off}\rangle|00\rangle+|\text{On}\rangle|00\rangle}{\sqrt{2}}$ . The amplitudes in the two traps can then be coherently interfered by reversing the amplitude-splitting procedure described above. The extension to high efficiency interrogation can be accomplished by preparing the entangled state  $\cos\theta|00\rangle + \sin\theta|11\rangle$  (which only differs in applying the rotation  $R(2\theta)$  instead of  $R(\frac{\pi}{2})$  to the first qubit).

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**Supplementary Information** is linked to the online version of the paper at [www.nature.com/nature](http://www.nature.com/nature).

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