Edge Excitations of Non-Abelian Quantum Hall States and Quasiparticle Tunneling

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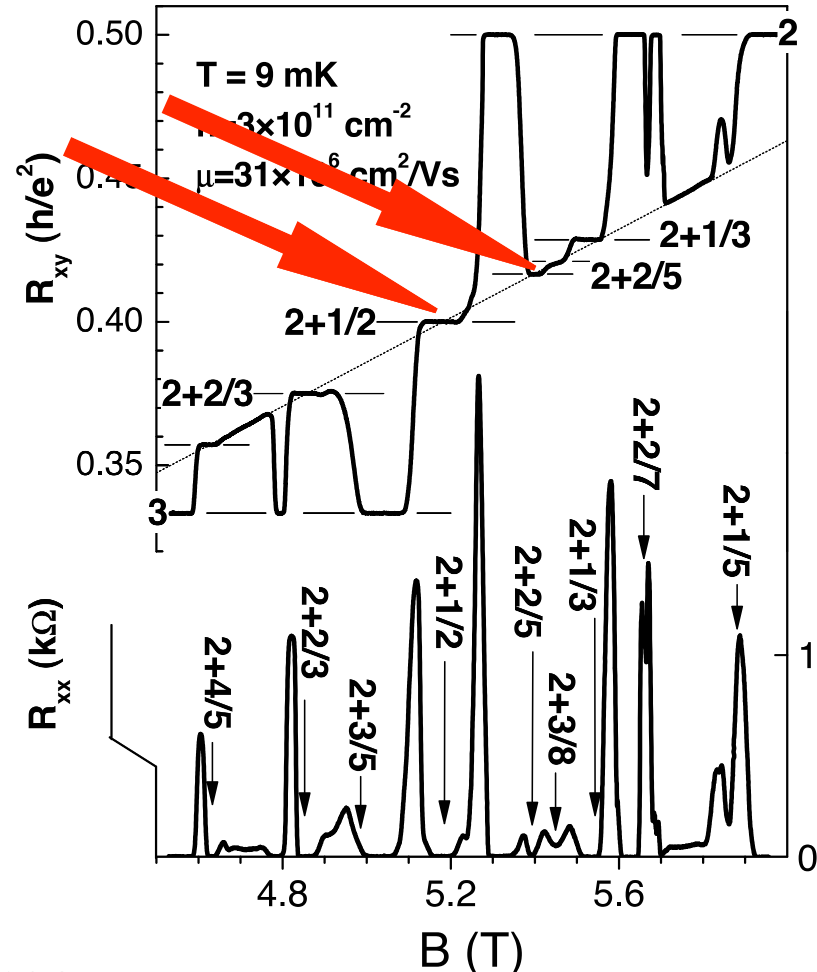
- Topological quantum computation depends on the existence in nature of non Abelian topological phases of matter.

- The leading candidate is $\nu = 5/2$.

  Perhaps $\nu = 12/5$ as well ....



J.S. Xia *et al.*, '04;
J.P. Eisenstein *et al.*, '02.

How will we know if they are?

- The belief that the 5/2 state is non-Abelian comes primarily from numerical solutions of small systems.
  Morf ’98; Rezayi and Haldane ’00; Feiguin *et al.* ’07; Peterson et al. ’08

- Ultimately, experimental proof will have to come from experiments in which quasiparticles are braided, e.g. two point contact interferometers.
  Chamon *et al.* ’97; Fradkin *et al.* ’98;
  Das Sarma, Freedman, and Nayak ’05;
  Bonderson et al. ’06; Stern and Halperin ’06

- However, some simpler experiments can give important corroborating evidence about these states. And are a partial step towards interferometry anyway.

Edge Excitations

One important resource at our disposal in a chiral topological state: gapless edge excitations.

The edge of the system provides us with a cheap supply of 'test' quasiparticles with which we can probe quasiparticles in the bulk by, e.g. the non-Abelian version of the Aharonov-Bohm effect.

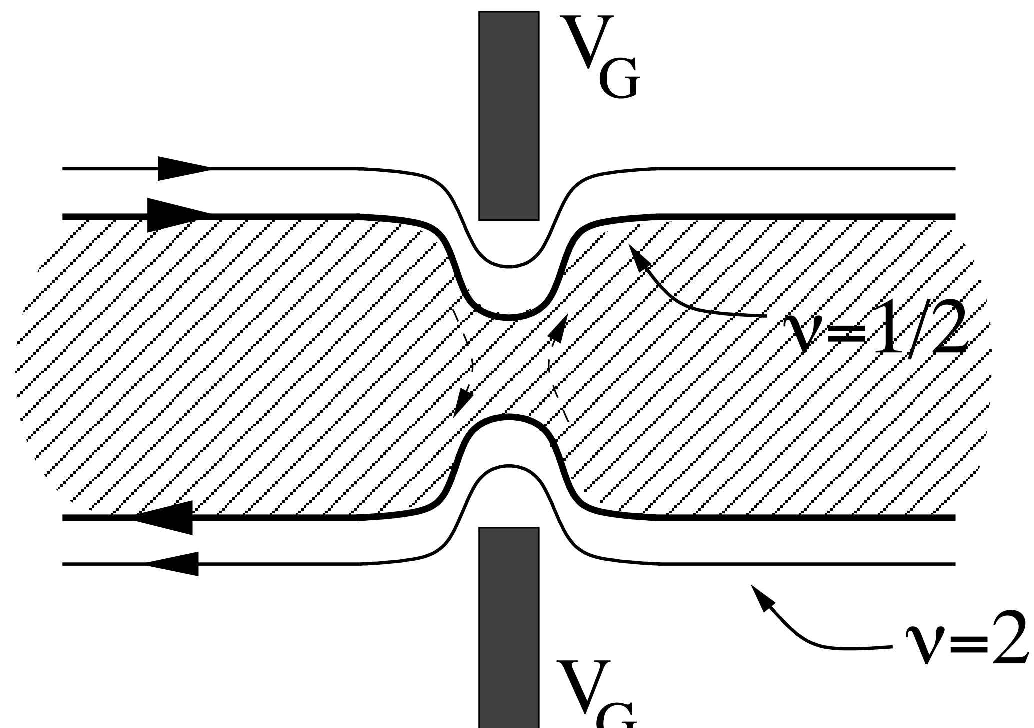
On the other hand, the edge does not seem like a good way to probe the topological structure of a state. It is gapless, so there may not be well-defined quasiparticles.

Point Contacts

To have a useful probe of topological properties, we need some kind of interplay between edge and bulk: **quasiparticle tunneling from one edge to another**.

The edge need not have well-defined quasiparticles, but the bulk does.

Tunneling through the bulk selects these.



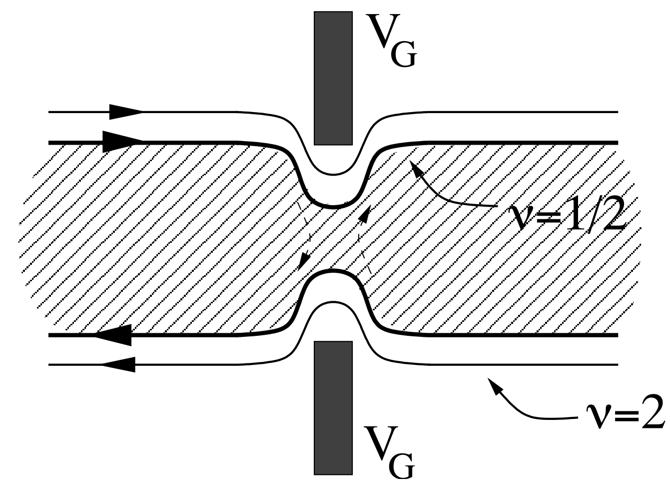
Effective Theory for the MR Pfaffian Edge

$$\mathcal{L}^{\rm edge} = \frac{1}{4\pi}\partial_x\phi_c(\partial_t + v_c\partial_x)\phi_c + \frac{1}{2\pi}\,\psi(\partial_t + v_n\partial_x)\psi$$

Milovanovic and Read '95

The 'quasiparticles' of this edge theory are just free bosons and fermions.

However, tunneling favors the edge counterparts of bulk qps.



$$\begin{aligned}\mathcal{L}^{\rm tun} \quad &= \quad \lambda_{1/4}\,\sigma_\alpha\sigma_\beta\, e^{i(\phi_{c\alpha}-\phi_{c\beta})/2\sqrt{2}} \;+\; \text{h.c.} \\ &\quad + \lambda_{1/2}\, e^{i(\phi_{c\alpha}-\phi_{c\beta})/\sqrt{2}} + \text{h.c.} \;+\; i\lambda_1\psi_\alpha\psi_\beta\end{aligned}$$

$$\mathcal{L}^{\rm tun} = \lambda_{1/4}\,\sigma_\alpha\sigma_\beta\, e^{i(\phi_{c\alpha}-\phi_{c\beta})/2\sqrt{2}} + \text{h.c.}$$
$$+\ \lambda_{1/2}\, e^{i(\phi_{c\alpha}-\phi_{c\beta})/\sqrt{2}} + \text{h.c.} + i\lambda_1\psi_\alpha\psi_\beta$$

$$\text{expect } \lambda_{1/2} \ll \lambda_{1/4} \text{ because } \lambda_{1/2} \sim \lambda_{1/4}^2$$

Tunneling of charge e/4 qps. is the most relevant perturbation.

$$\begin{aligned}\frac{d}{d\ell}\lambda_{1/2} &= \frac{1}{2}\,\lambda_{1/2}\\ \frac{d}{d\ell}\lambda_{\psi,0} &= 0\\ \frac{d}{d\ell}\lambda_{1/4} &= \frac{3}{4}\,\lambda_{1/4}\end{aligned}$$

Therefore, we expect e/4 qps to dominate tunneling transport. If they do ... good news for interferometry.

Effective Theory for anti-Pfaffian Edge

- If there is no Landau-level mixing, the Hamiltonian is particle-hole symmetric. $c_m^\dagger \longrightarrow c_m, c_m \longrightarrow c_m^\dagger$

$$
\begin{aligned}
H_2 &= \sum_{klmn} V_{klmn} c_k^\dagger c_m c_l^\dagger c_n - \mu \sum_m c_m^\dagger c_m \,, \\
\tilde{H}_2 &= \sum_{klmn} V_{klmn} c_k^\dagger c_m c_l^\dagger c_n + \left(\mu - 2\mu_{1/2}\right) \sum_m c_m^\dagger c_m .
\end{aligned}
$$

Particle-hole transf: $\sigma_{xy} \longrightarrow 1 - \sigma_{xy}, \quad \kappa_{xy} \longrightarrow 1 - \kappa_{xy}$

in units of $\frac{e^2}{h}, \frac{\pi^2 k_B^2 T}{3h}$

$$\mathcal{L}^{\text{edge}} = \frac{1}{4\pi} \partial_x \phi_c (\partial_t + v_c \partial_x) \phi_c + \frac{1}{2\pi} \psi (\partial_t + v_n \partial_x) \psi \quad \Longrightarrow \quad \kappa_{xy} = \frac{3}{2}$$

The Pfaffian state is not particle-hole symmetric.

Analogy to Lattice p-wave SC

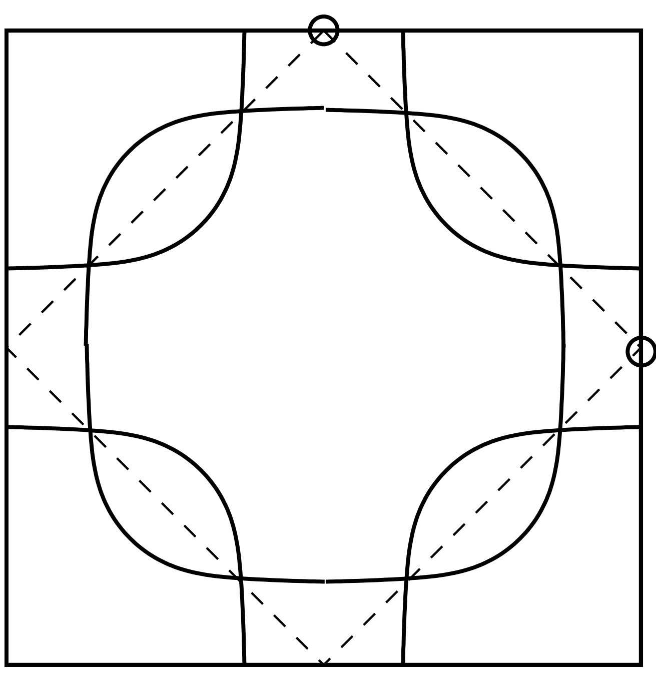
- Spinless electrons, square lattice, p-wave SC order.

$$H = \sum_{\langle i,j\rangle}\left(-tc_i^\dagger c_j + \Delta_{ij} c_i^\dagger c_j^\dagger + \text{h.c.}\right) - \mu \sum_i c_i^\dagger c_i - \sum_{\langle\langle i,j\rangle\rangle} \varphi\, c_i^\dagger c_j + \text{h.c.}$$

- Particle-hole symmetric if $\mu = 0, \varphi = 0$

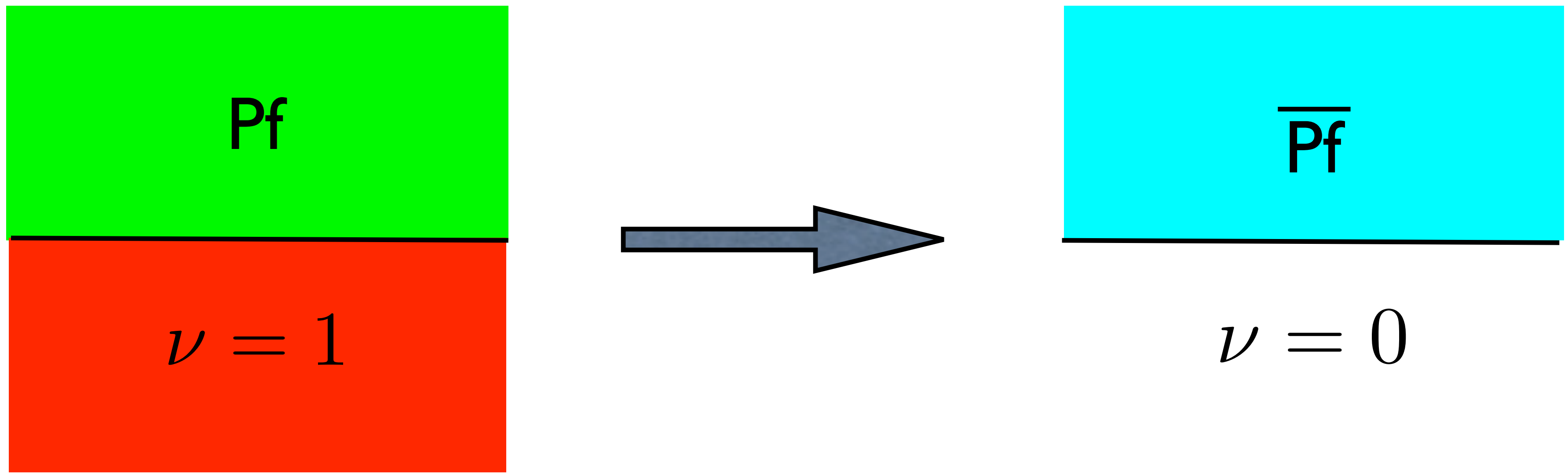
spontaneous 2nd neighbor hopping

Gapless Majorana fermion at $(\pi, 0),\ (0\pi)$



- If p/h symm is broken, $\varphi \neq 0$ then $\kappa_{xy} = \pm\frac{1}{2}$

Edge Theory of the Anti-Pfaffian



$$\mathcal{L} = \frac{1}{4\pi}\partial_x\phi_1(-i\partial_t + v_1\partial_x)\phi_1 + \mathcal{L}_{\rm Pf}(\psi_1, \phi_2)$$
$$+\frac{1}{4\pi}2v_{12}\partial_x\phi_1\partial_x\phi_2 + \xi(x)\,\psi_1\,e^{i(\phi_1-2\phi_2)} + \text{h.c.}.$$

U(1) SU(2)$_2$

$$\mathcal{L} = \frac{2}{4\pi}\partial_x\phi_\rho(-i\partial_t + v_\rho\partial_x)\phi_\rho + \psi_a(-\partial_t + iv_\sigma\partial_x)\psi_a$$
$$+2i\psi_1(\xi_1\psi_3 + \xi_2\psi_2) + \delta v_1\psi_1 i\partial_x\psi_1 + iv\psi_2\psi_3\,\partial_x\phi_\rho$$

Anti-Pfaffian Point Contact

- There are two charge e/4 qp ops. at the anti-Pfaffian edge:

$$\Phi^{(1)} = \sigma_3\, e^{i(\phi_\rho - \phi_\sigma)/2}$$
$$\Phi^{(2)} = \mu_3\, e^{i(\phi_\rho + \phi_\sigma)/2}$$

where: $e^{i\phi_\sigma} = \psi_1 + i\psi_2$

- 4 qp tunneling ops. at a point contact: $T_{ij} = \Phi_a^{(i)\dagger} \Phi_b^{(j)}$

- All four have scaling dim. =1/2, (different from the Pfaffian, dim=1/4). Charge e/2 tunneling has same scaling dim.

Fendley, Fisher, Nayak, in prep.

(3,3,1) Point Contact

- Pf: $S_z = 1$ triplet

$$\Psi_{\mathrm{Pf}} = \prod_{j<k}(z_j - z_k)^m \prod_j e^{-|z_j|^2/4}\, \mathrm{Pf}\left(\frac{|\uparrow\rangle_j |\uparrow\rangle_k}{z_j - z_k}\right)$$

$$\vec{d} = \hat{x} - i\hat{y}$$

- (3,3,1): $S_z = 0$

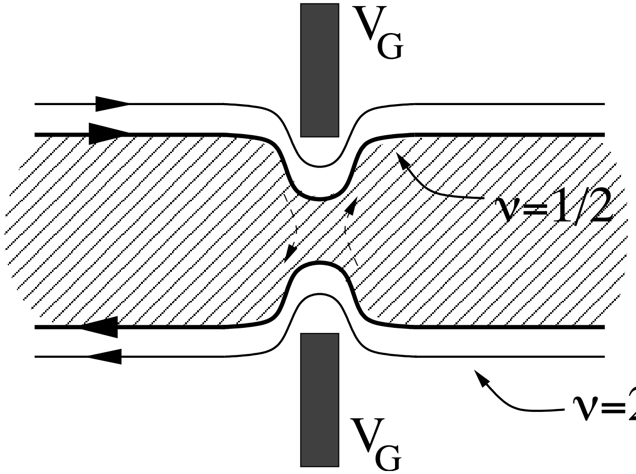
$$\Psi_{(3,3,1)} = \prod_{j<k}(z_j - z_k)^m \prod_j e^{-|z_j|^2/4}\, \mathrm{Pf}\left(\frac{|\uparrow\rangle_j|\downarrow\rangle_k + |\downarrow\rangle_j|\uparrow\rangle_k}{z_j - z_k}\right)$$

$$\vec{d} = \hat{z}$$

- $$\mathcal{L}_{331} = \frac{1}{4\pi} K^{IJ} \partial\phi_I^a \partial\phi_J^a + (a \to b) + t\sum_I \cos\left(\phi_I^a - \phi_I^b\right)$$

- e/4 qp tunneling op. has scaling dim=3/8.

Weak Backscattering Limit



MR Pfaffian: $$R_{xx} \sim \lambda_{1/4}^2 T^{-3/2}$$

Fendley, Fisher, Nayak PRL '06

(3,3,1): $$R_{xx} \sim \lambda_{1/4}^2 T^{-5/4}$$

anti-Pfaffian: $$R_{xx} \sim \lambda_{1/4}^2 T^{-1}$$

3 Majorana fermions

Lee, Ryu, Nayak, Fisher '07; Levin, Halperin, Rosenow '07

T and V Dependence in Weak-BS. Limit

- At finite T and V,

$$I = |\Gamma|^2 \, |V|^{2g-1} \, A\left(\frac{eV/4}{k_B T}\right)$$

$$g = \frac{1}{4}, \; \frac{3}{8}, \; \frac{1}{2}$$

Pf — (3,3,1) — $\overline{\text{Pf}}$

$$A(x) = \frac{1}{\sqrt{2}} \, \frac{e}{4} \, \frac{\pi x^{1-2g}}{\Gamma(2g)} \left| \Gamma\left(g + i\frac{x}{2\pi}\right)\right|^2 \sinh(x/2)$$

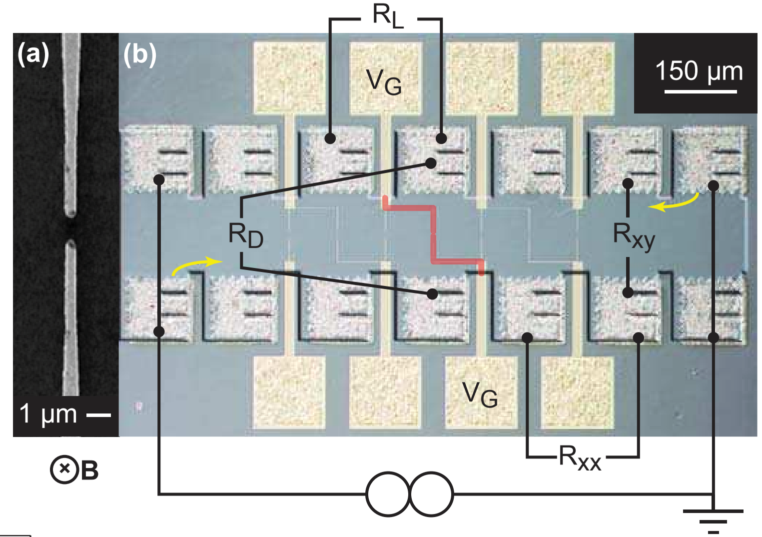
Bishara and Nayak ’08

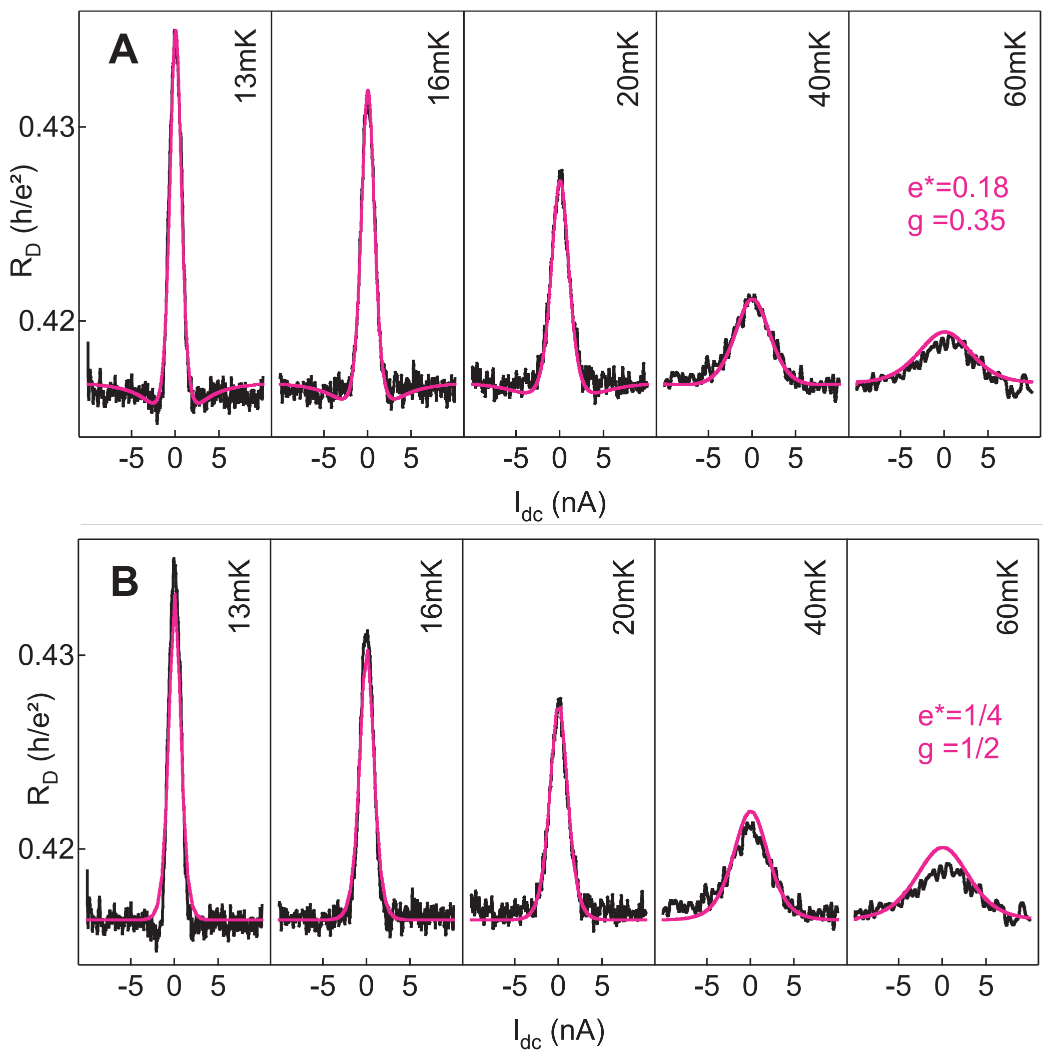
for Laughlin states: Wen ‘91

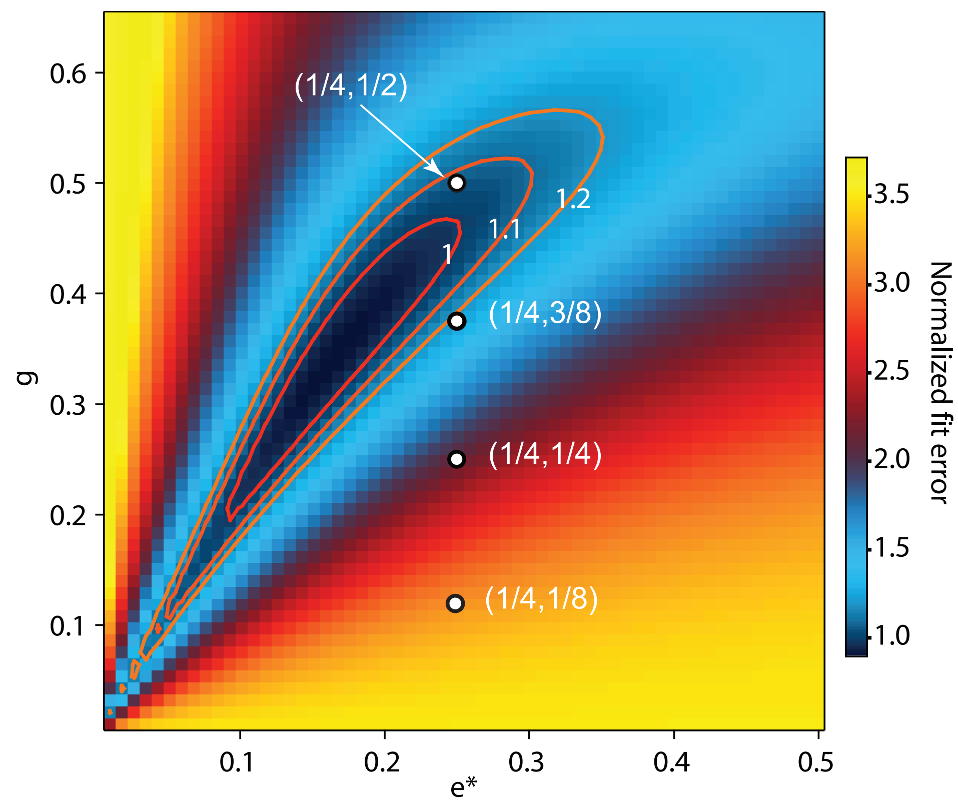
Since T dependence is difficult to measure at very low temps., it is useful to have both T and V.

Radu et al. ‘08:

Point contact in 5/2 state







Fits $\overline{\text{Pf}}$ reasonably well.

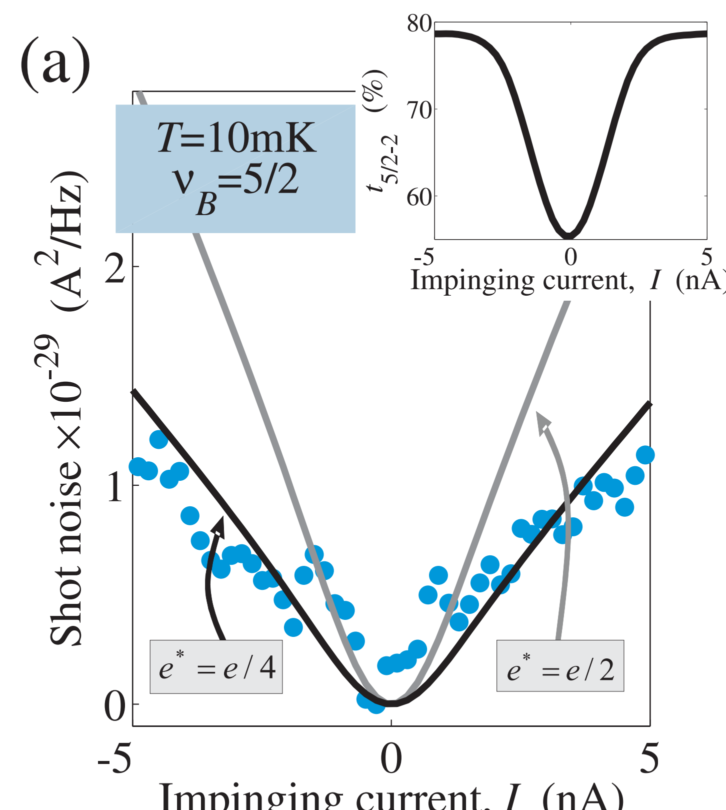
Shot Noise

- The ratio between the low-freq. noise and the tunneling current at a point contact is the qp. charge:

$$\langle S(\omega)\rangle = \frac{e^*}{2} I_t[|1-\omega/\omega_0|^{\delta-1} + |1+\omega/\omega_0|^{\delta-1}]$$

Laughlin states: Kane and Fisher '93;
Chamon, Freed, Wen '95
Expts on LLL: Samindayar *et al.* '97;
de Picciotto *et al.* '97.

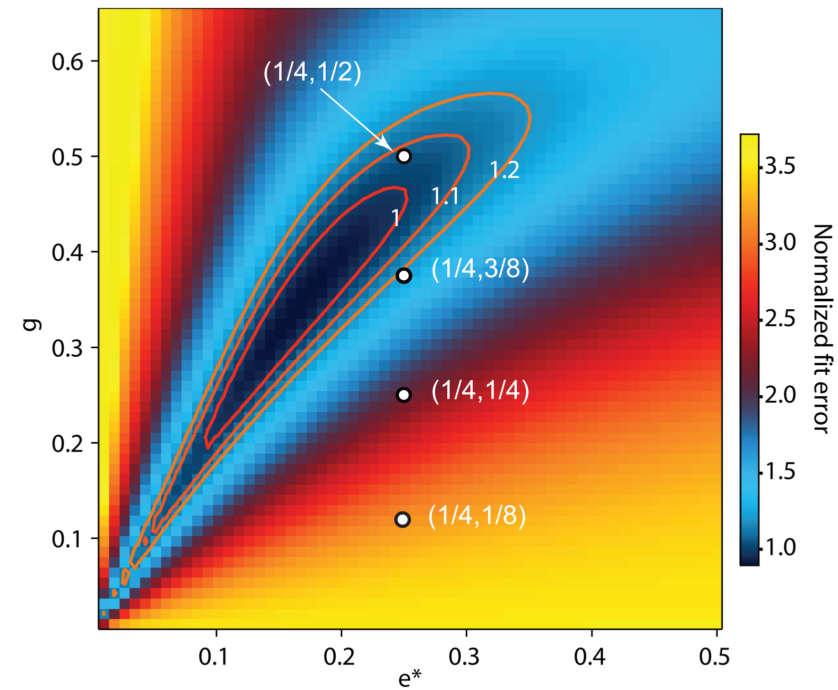
Pfaffian state: Bena and Nayak '06

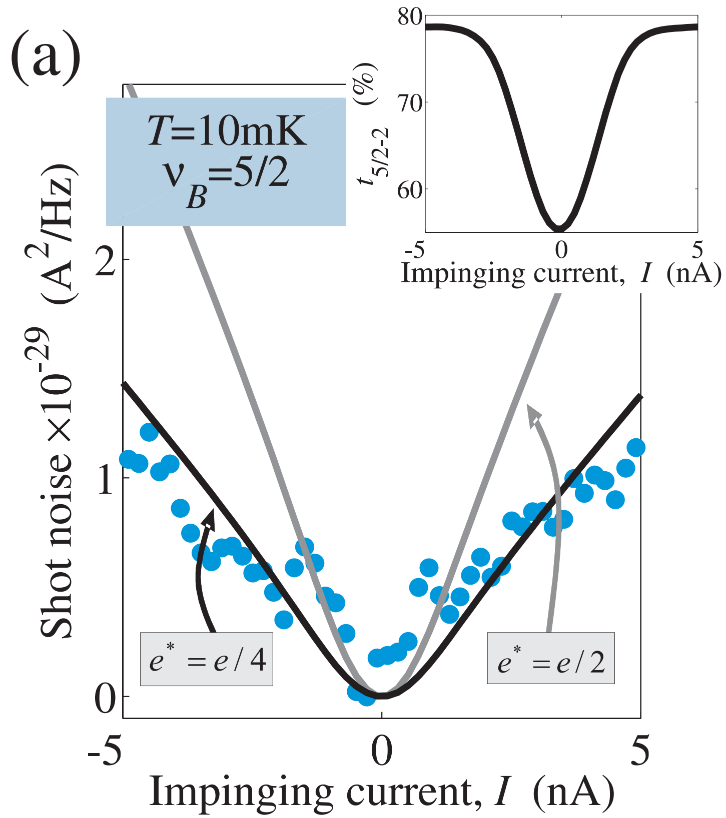


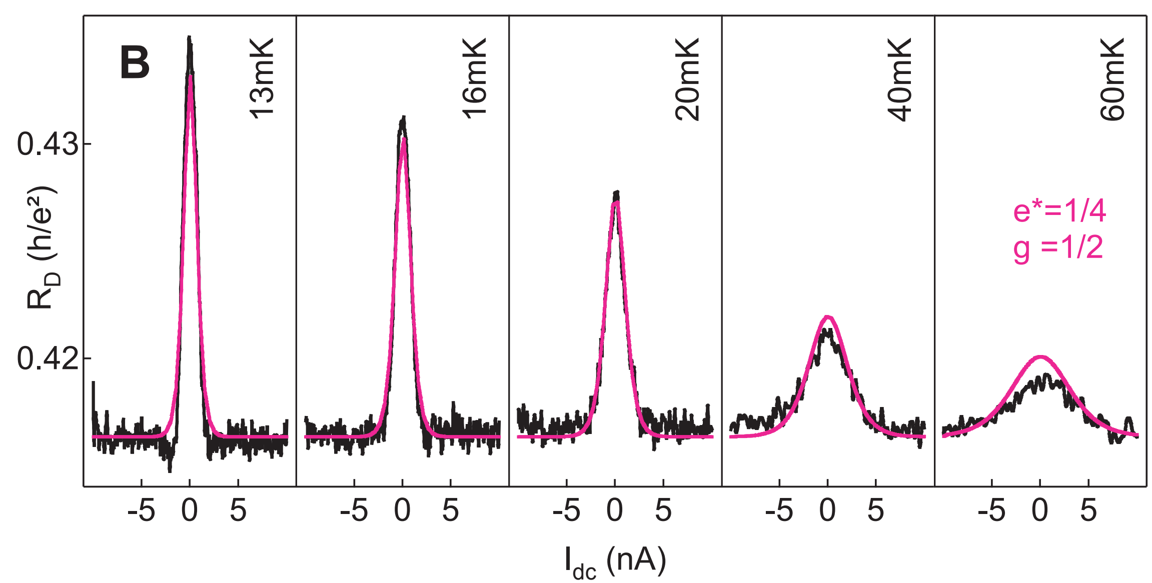
- Recent Dolev *et al.* expt. is consistent with $e/4$.

Taken together, these two experiments point towards e/4 qps with point contact I-V curves consistent with the anti-Pfaffian state.

But only a true probe of topological properties will be definitive.

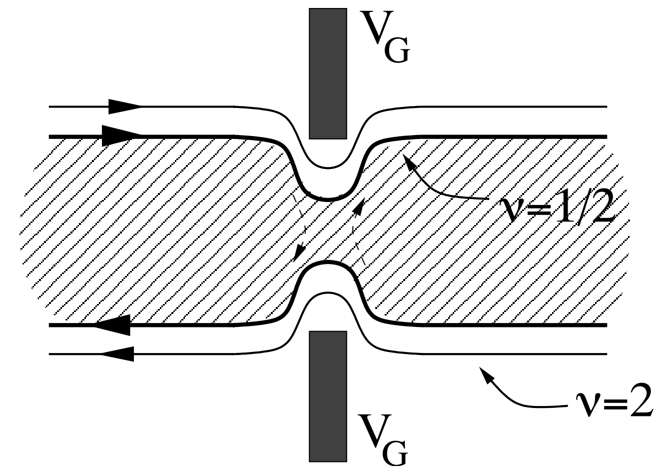




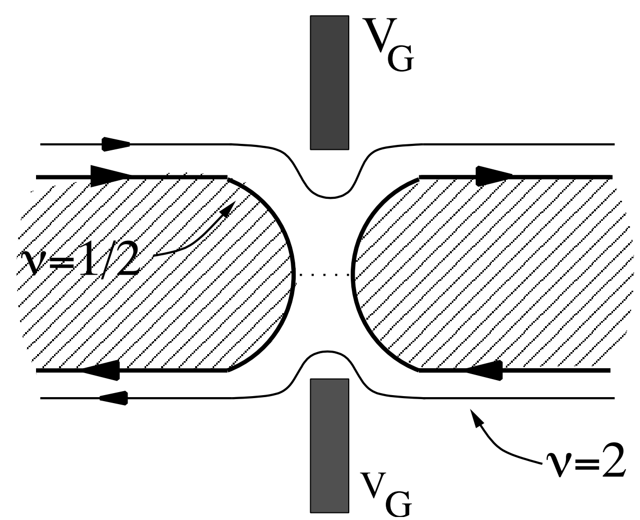


Beyond the Weak-BS. Limit

As temperature or voltage is decreased, tunneling becomes effectively stronger.



Eventually, the droplet will pinch off and there will be weak hopping from left to right.



Experiments are probably somewhere in the crossover regime.

Non-Perturbative Treatment

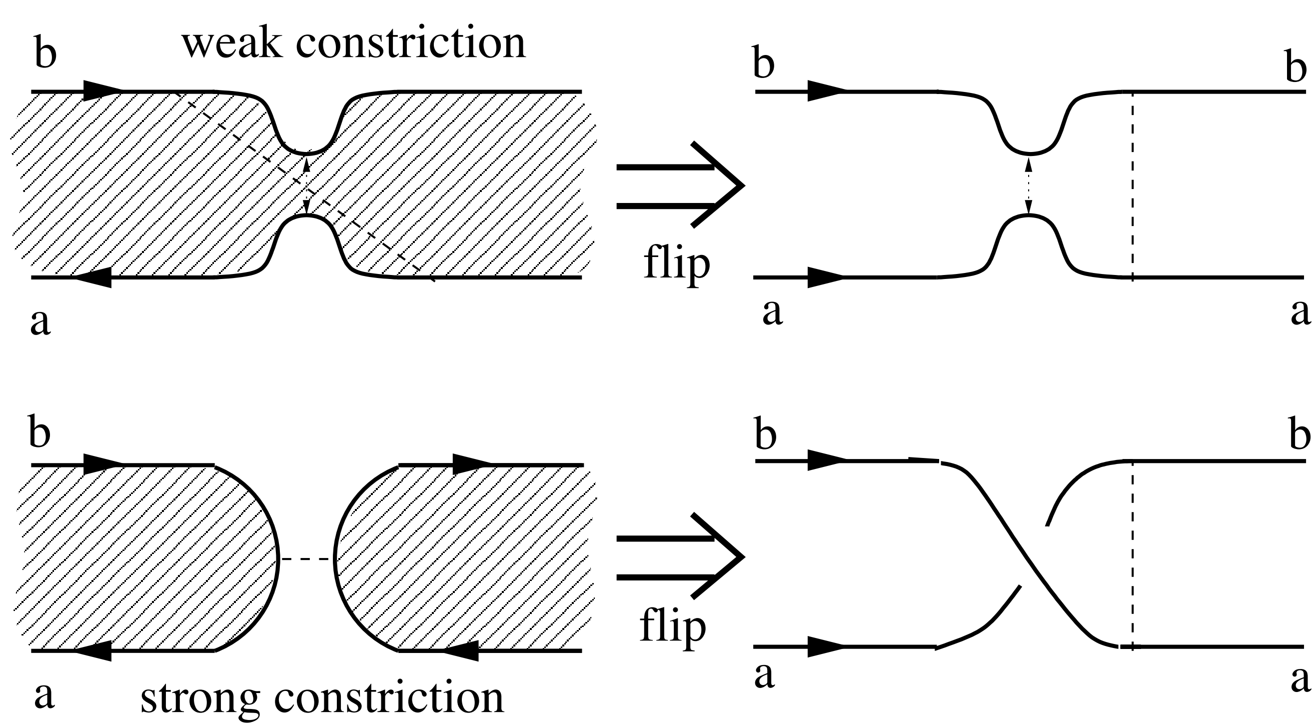
$$\mathcal{L}^{\rm edge} = \frac{1}{4\pi}\partial_x\phi_c(\partial_t + v_c\partial_x)\phi_c + \frac{1}{2\pi}\,\psi(\partial_t + v_n\partial_x)\psi$$

$$\begin{aligned}\mathcal{L}^{\rm tun} \quad &= \quad \lambda_{1/4}\,\sigma_\alpha\sigma_\beta\, e^{i(\phi_{c\alpha}-\phi_{c\beta})/2\sqrt{2}} \;+\; \text{h.c.} \\ &\quad + \lambda_{1/2}\, e^{i(\phi_{c\alpha}-\phi_{c\beta})/\sqrt{2}} + \text{h.c.} \;+\; i\lambda_1\psi_\alpha\psi_\beta\end{aligned}$$

1. Redraw with one edge flipped.

2. Combine the two Majorana fermions into a single Dirac fermion.

3. Bosonize.



Standard bosonization:

$$\psi_a + i\psi_b \sim e^{i\phi}$$
$$i\psi_a\psi_b \sim \partial\phi$$

However, to recover the pert. expansion of e/4 tunneling:

$$\sigma_a\sigma_b \sim S^+ e^{-i\phi/2} + S^- e^{i\phi/2}$$

bookkeeping → S^+

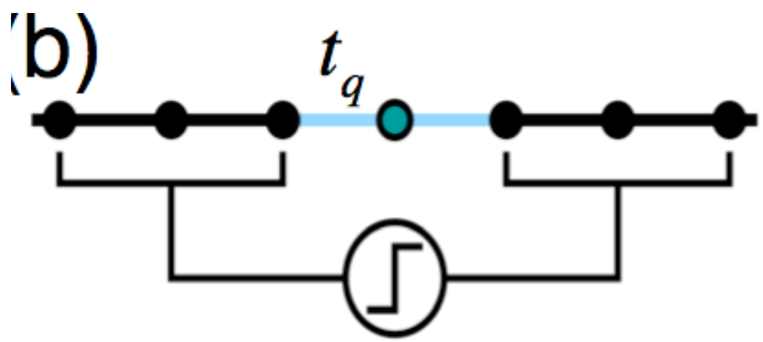
Fendley, Fisher, Nayak PRL, PRB '06.

$$\mathcal{H}_{5/2} = \int dx \left(\frac{v_c}{2\pi} \left(\partial_x \phi_\rho\right)^2 + \frac{v_n}{2\pi} \left(\partial_x \phi_\sigma\right)^2 \right)$$
$$+ \lambda_{1/4} \left(S^+ e^{-i\phi_\sigma(0)/2} + S^- e^{i\phi_\sigma(0)/2} \right) \cos(\phi_\rho(0)/2)$$
$$+ \ \lambda_{1/2} \cos \phi_\rho(0) \ + \ \frac{\lambda_{\psi,0}}{2\pi} \partial_x \phi_\sigma(0),$$

Crossover from Weak to Strong Tunneling

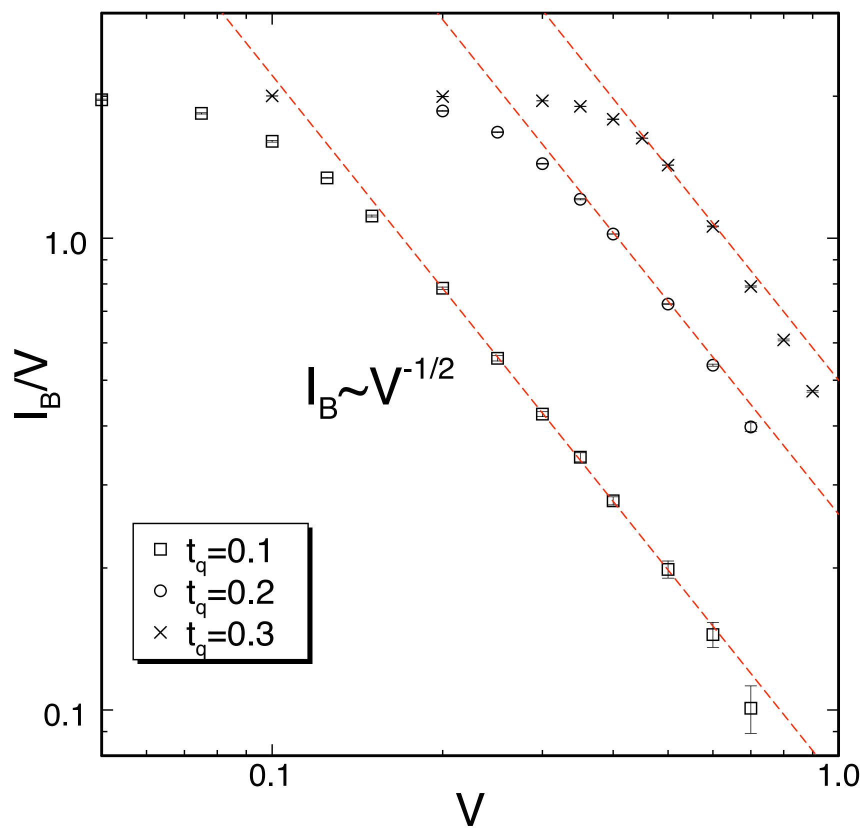
- Pf. point contact can be rewritten as resonant tunneling between Luttinger liquids

$$\mathcal{H}_{\rm res} = \int_0^\infty dx\, \frac{v}{2\pi} \left((\partial_x \phi_a)^2 + (\partial_x \phi_b)^2 \right)$$
$$+ t\, d^\dagger e^{i\phi_a(0)/\sqrt{g}} + t\, d^\dagger e^{i\phi_b(0)/\sqrt{g}} + \text{h.c.}$$



- Tunneling current can be computed by time-dependent DMRG.

- Agrees with perturbative calculations around the weak- and strong-backscattering limits. Only way to compute the current in the crossover regime.
  Agrees with Bethe ansatz for 1/3 point contact.



Feiguin, Fendley, Fisher, Nayak '08

- Future: time-dep. DMRG for anti-Pfaffian, 331.

What about 12/5?

- One Abelian candidate, several non-Abelian ones: BS = Pf + hierarchy, $\overline{\text{BS}}$ = $\overline{\text{Pf}}$ + hierarchy
  $\overline{\text{RR}}$ = p/h conjugate of RR.

- BS: $R_{xx} \sim T^{-31/20}$

  $\overline{\text{BS}}$: $R_{xx} \sim T^{-21/20}$

  Abel. 2/5: $R_{xx} \sim T^{-6/5}$

  $\overline{\text{RR}}$: $R_{xx} \sim T^{-6/5}$

2e/5 at low-T (Abel. 2/5)

e/5 and 2e/5 at low-T ($\overline{\text{RR}}$)

Bishara, Fiete, Nayak '08

- Thermal Hall conductance distinguishes these states, and also Pfaffian and anti-Pfaffian.

Summary

- Quasiparticle tunneling at point contacts is a good probe of the topological character of possible non-Abelian quantum Hall states.

- Measurements of current, noise through a pt. contact at 5/2 indicate e/4 qps., consistent with anti-Pfaffian.

- Single point contact in MR Pfaffian: closely related to 2-channel Kondo.
  Current can be computed outside weak-backscatt. regime by time-dep. DMRG.

- Multiple point contacts enable quasiparticle interferometry.