

# In search of topological states with half quantum vortices.

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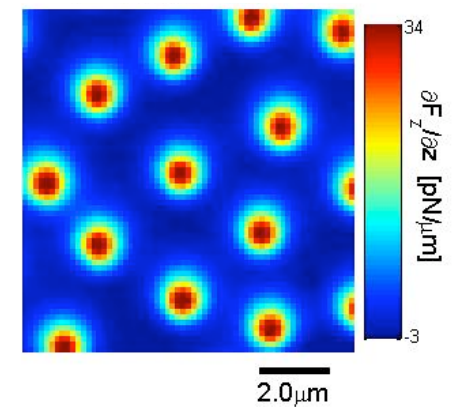






YBCO single crystal @ 22.3K.

Scan height 420nm





# In search of topological states with half quantum vortices

- Topological order and fractionalization
- $1/2$  QV's
- Stability of  $1/2$ -QV's in SrRuO
- $1/2$  QV lattices



# Ground state degeneracy

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# Ground state degeneracy

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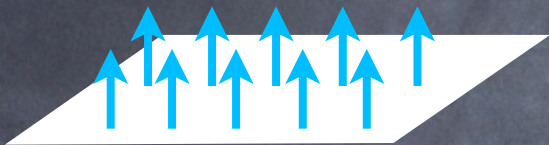
Conventional order



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- Symmetry of the underlying Hamiltonian.



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- Local measurements

↔ order parameter



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- Gapped spectrum



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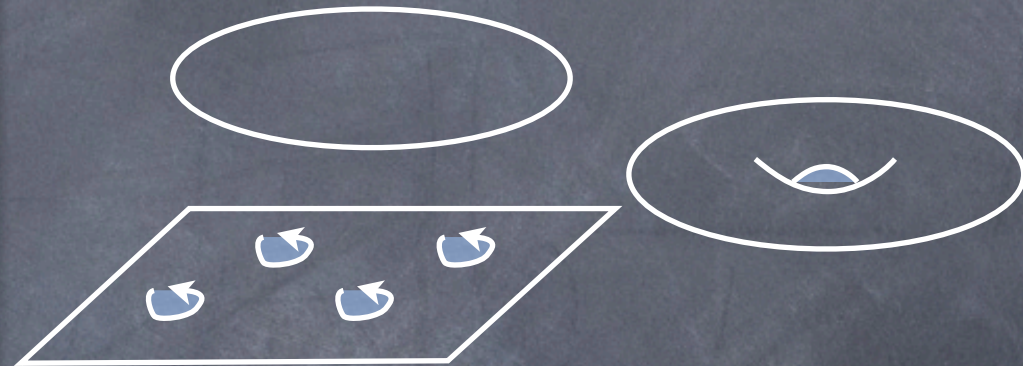
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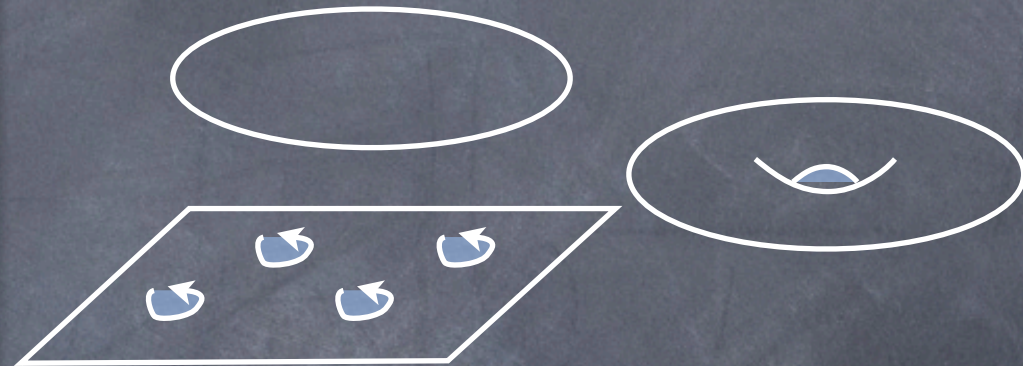
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- Gapped spectrum
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↔ emergent symmetry
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# Sweet Topology

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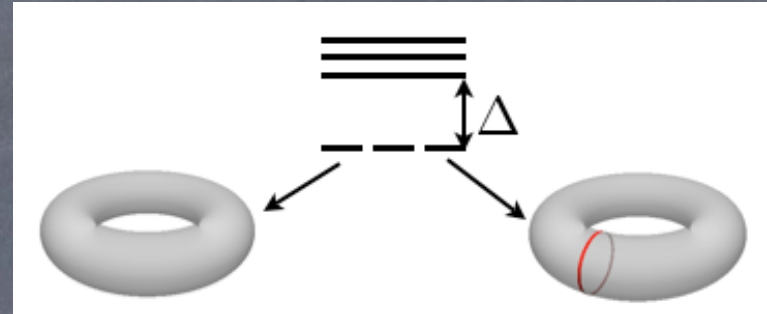




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① Fractional charge  $e^* = e/q$

$N_g = q^2$  e.g.,  $N_1 = 3$



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Hansson, Oganesyan, Sondhi  
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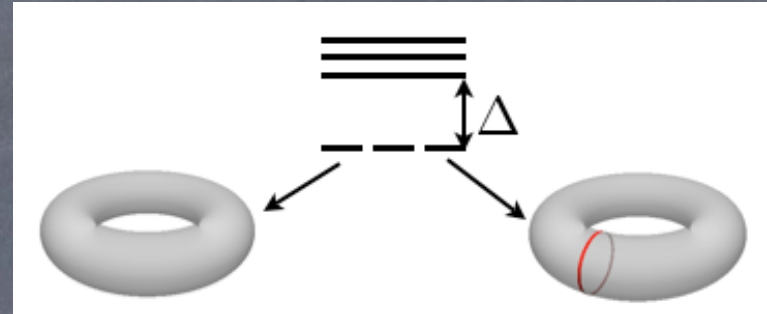
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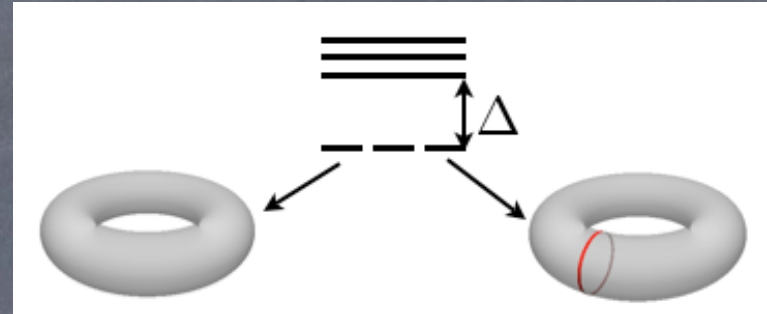
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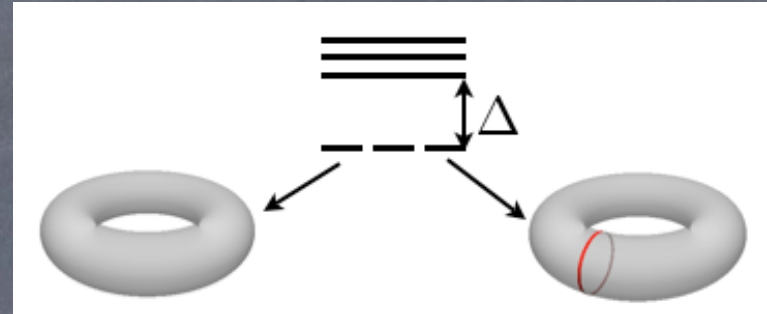
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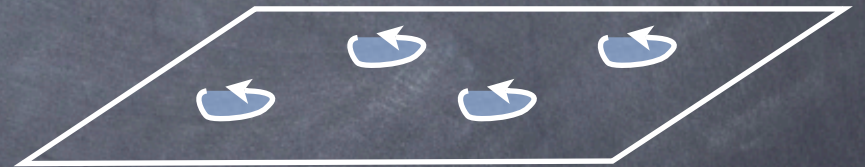
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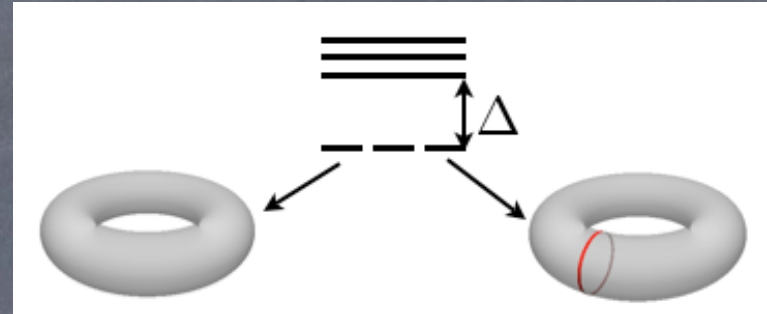
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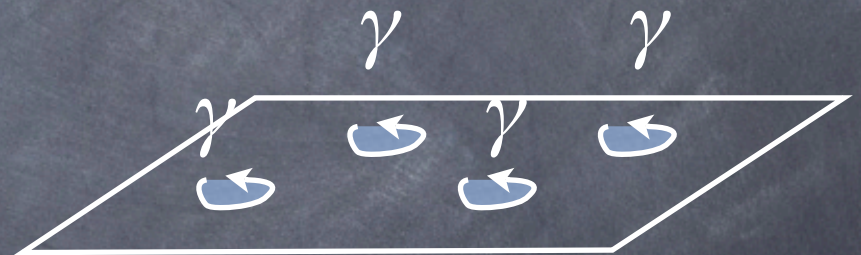
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# Abelian statistics

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• n-abelian vortex states

$$\Psi(x_1, \dots, x_n) = c\text{-number}$$

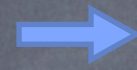


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exchange of qp's:  
phase multiplication  
to a complex number

$$\Psi(x_1 \leftrightarrow x_3) = e^{i\theta} \Psi$$

$$\Psi(x_1 \leftrightarrow x_2) = e^{i\theta} \Psi$$



# Nonabelian statistics

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A. Leggett, RMP 1975   Sigrist & Ueda RMP 1991



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🌀 Gap function

$$\Delta_{ss'}(\mathbf{k}) = - \sum_{\mathbf{k}', s_3, s_4} V_{s'ss_3s_4}(\mathbf{k}, \mathbf{k}') \langle a_{\mathbf{k}'s_3} a_{-\mathbf{k}'s_4} \rangle$$



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👁 Triplet gap matrix

$$\begin{aligned} \hat{\Delta}(\mathbf{k}) &= i(\mathbf{d}(\mathbf{k}) \cdot \hat{\sigma}) \hat{\sigma}_y \\ &= \begin{bmatrix} -d_x(\mathbf{k}) + id_y(\mathbf{k}) & d_z(\mathbf{k}) \\ d_z(\mathbf{k}) & d_x(\mathbf{k}) + id_y(\mathbf{k}) \end{bmatrix} \end{aligned}$$



# p+ip SC

## ① T-breaking (ABM)

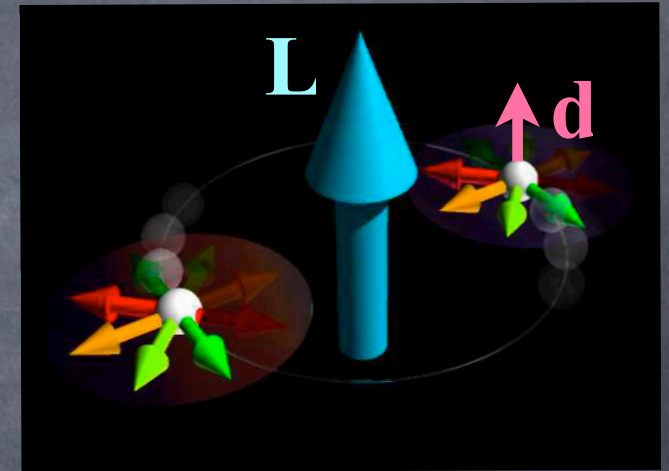
$$\Delta(\hat{\mathbf{k}}) = \Delta_0(T)(p_x \pm ip_y) \begin{pmatrix} -d_x + id_y & d_z \\ d_z & d_x + id_y \end{pmatrix}$$

where  $\hat{\mathbf{d}}$  is a real unit vector

## ② In plane $\hat{\mathbf{d}}$

$d_z=0$  i.e.,  $\mathbf{d} = (\cos\alpha, \sin\alpha, 0)$

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1/2 QV with in-plane  $\hat{d}$

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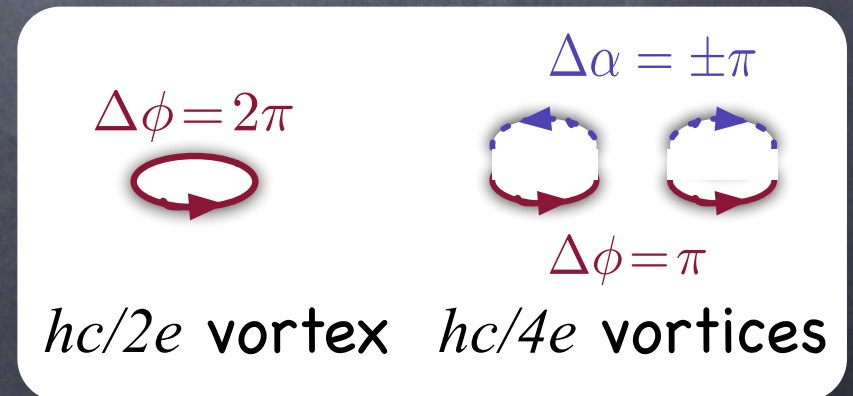
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$\longleftrightarrow$   $2\pi$  winding for only **one spin** component

$\longleftrightarrow$   $\pi$  winding of order parameter phase  $\phi$   
+  $\pi$  rotation of  $\mathbf{d}$  vector





# Why? Exotic nature of $1/2$ QV in $p+ip$

---

⦿ Vortices of  $p+ip$  SF  $\rightarrow$  zero modes at the core

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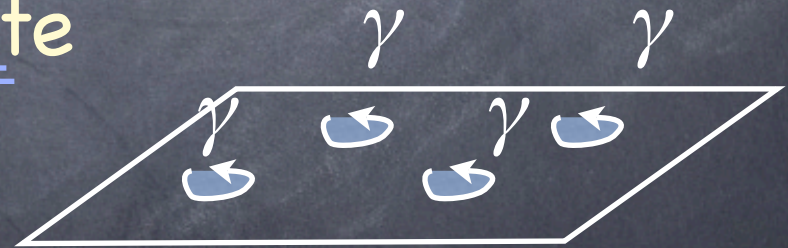
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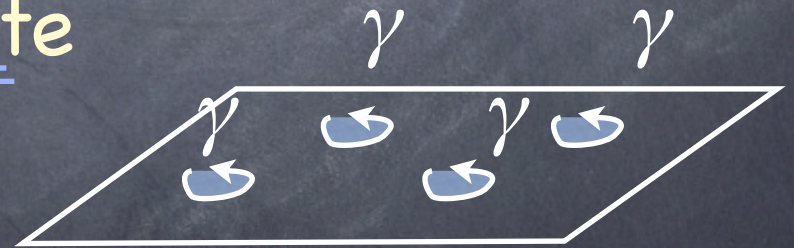
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1/2 QV's: single Majorana zero mode



$5/2$  state described as  $p+ip$  paired  
state of composite fermion

Pfaffian is real space many body BCS  
wave function of  $p+ip$  SF

HQV is equivalent to  $1/4$  qp

Moore & Read (91) Read & Green (00)

Schrieffer, p 48



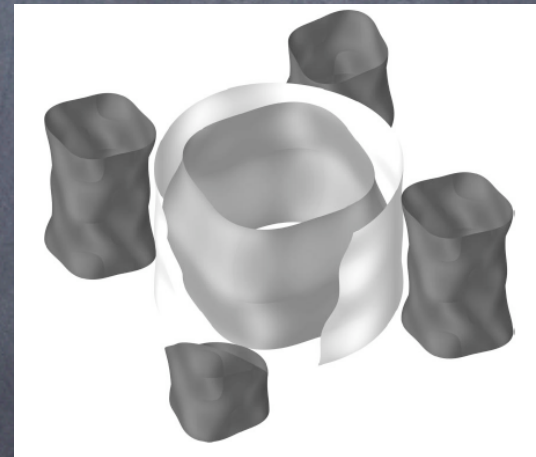
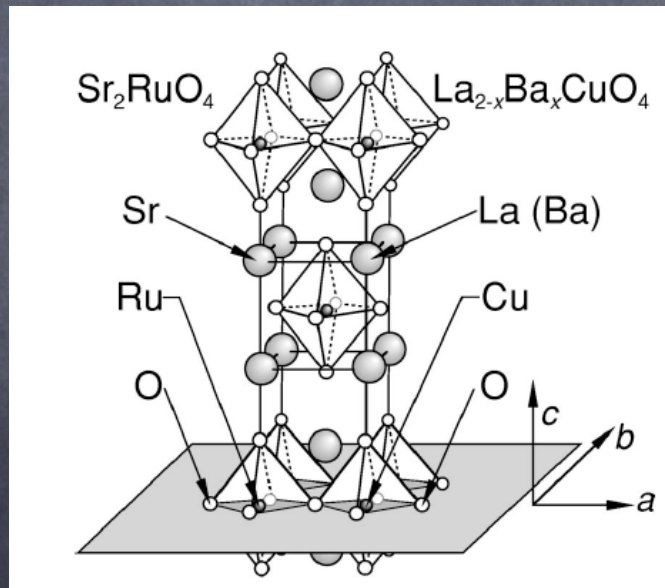
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K. Ishida et al, Nature (1998)

# Spin-triplet superconductivity in $\text{Sr}_2\text{RuO}_4$ identified by $^{17}\text{O}$ Knight shift





# Experiments?

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- ① NMR on  $^3\text{He}$ -A thin films: X Hakonen et al. Physica (89)
- ① Small angle neutron scattering: X Riseman et al. Nature (98)
- ① Scanning SQUID imaging: X  
Dolocan et al, PRL (05), Bjorsson et al, PRB (05)
- ① NMR in the presence of  $\mathbf{H} \perp ab$ 
  - ▶  $\mathbf{d} // ab$ : for  $H_{\perp} \approx 200 \text{ G}$ , Murakawa et al, PRL (04)



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- ① Spin current energy diverges logarithmically!



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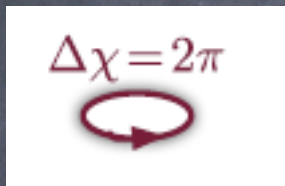
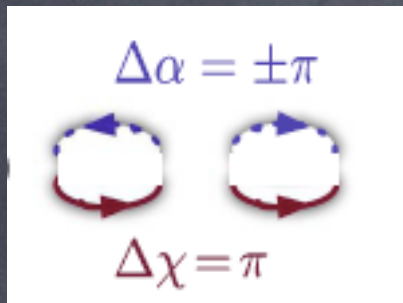
$$f_{\text{grad}}^{2\text{D}} = \frac{1}{2} \left( \frac{\hbar}{2m} \right)^2 \left[ \rho_s \left( \nabla_{\perp} \phi - \frac{2e}{\hbar c} \mathbf{A} \right)^2 + \rho_{\text{sp}} (\nabla_{\perp} \alpha)^2 \right] + \frac{1}{8\pi} (\nabla \times \mathbf{A})^2$$

- Spin current energy diverges logarithmically!

$$\epsilon_{\text{sp}} = \frac{\pi}{4} \left( \frac{\hbar}{2m} \right)^2 \rho_{\text{sp}} \ln \left( \frac{R}{\xi} \right)$$



# stability of 1/2 QV

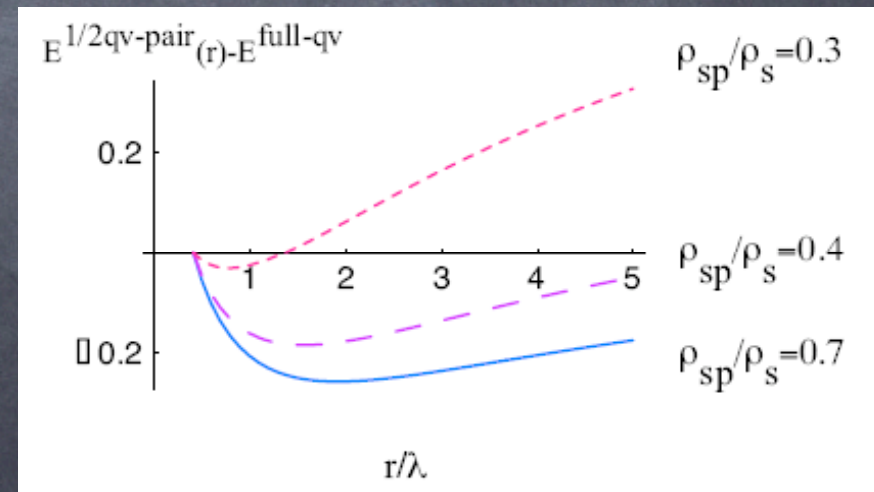


$$E_{\text{pair}}^{\text{half}}(r_{12}) = \frac{1}{2} \frac{\Phi_0^2}{16\pi^2 \lambda^2} \left[ \ln\left(\frac{\lambda}{\xi}\right) + K_0\left(\frac{r_{12}}{\lambda}\right) + \frac{\rho_{\text{sp}}}{\rho_s} \ln\left(\frac{r_{12}}{\xi}\right) \right]$$

$$E^{\text{full}} = \pi \left(\frac{\hbar}{2m}\right)^2 \rho_s \ln\left(\frac{\lambda}{\xi}\right) = \frac{\Phi_0^2}{16\pi^2 \lambda^2} \ln\left(\frac{\lambda}{\xi}\right)$$

- Competition between screened magnetic repulsion and unscreened spin attraction

- Finite equilibrium size for **small**  $\rho_{\text{sp}}/\rho_s$

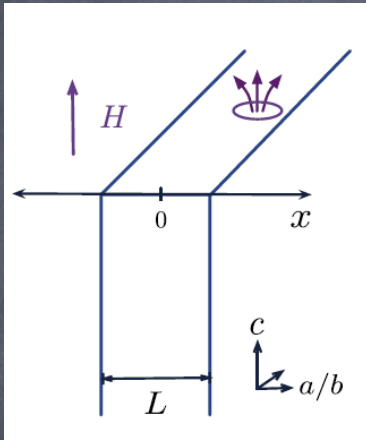


Leggett RMP 75

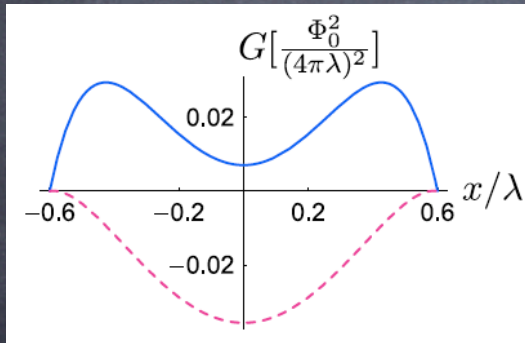
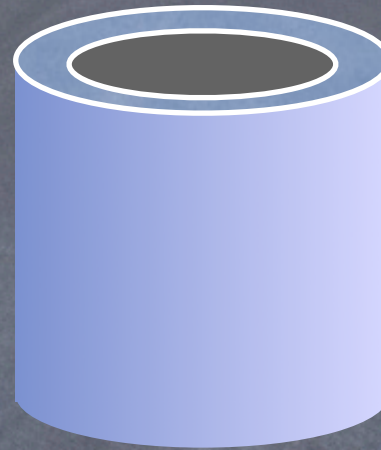


# Mesoscopic sample

- Sample of size  $\sim \lambda$  a few micron



$$L = 2\lambda$$



Underway in Budakian lab

- Sample of size  $\sim \lambda$  a few micron



# In search of topological states with fractionalized excitations.

- Topological order and fractionalization
- $1/2$ QV's
- Stability of  $1/2$ -QV's in SrRuO
- $1/2$  QV lattices



1/2 QV Lattice?

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# 1/2 QV Lattice?

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- Natural way to stabilize 1/2 QV



# 1/2 QV Lattice?

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- Natural way to stabilize 1/2 QV
- Good track record for full QV lattice
  - Agterberg, PRB (98) predicted square lattice
  - T. Riseman at el , Nature (98) confirmed square lattice



# 1/2 QV Lattice?

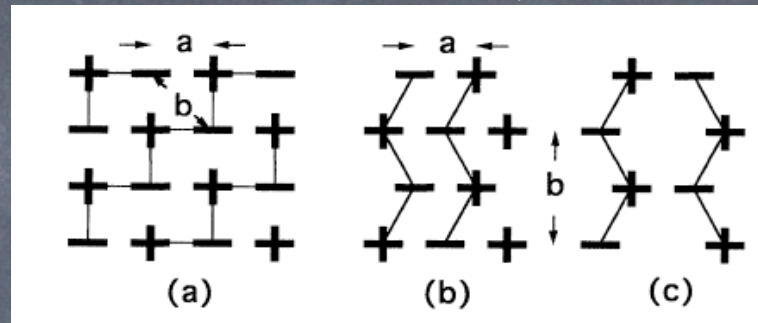
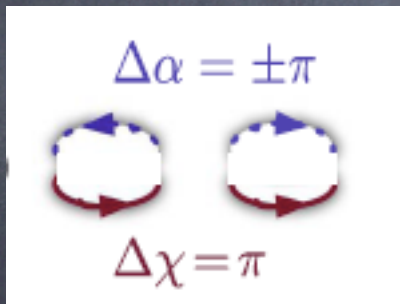
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- Natural way to stabilize 1/2 QV
- Good track record for full QV lattice
  - Agterberg, PRB (98) predicted square lattice
  - T. Riseman et al, Nature (98) confirmed square lattice
- Potential of tuning  $\rho_{sp}/\rho_s$ 
  - Knowledge exist for  $\rho_{sp}/\rho_s$  as a function of Fermi liquid parameters
  - p-wave Feshbach resonance



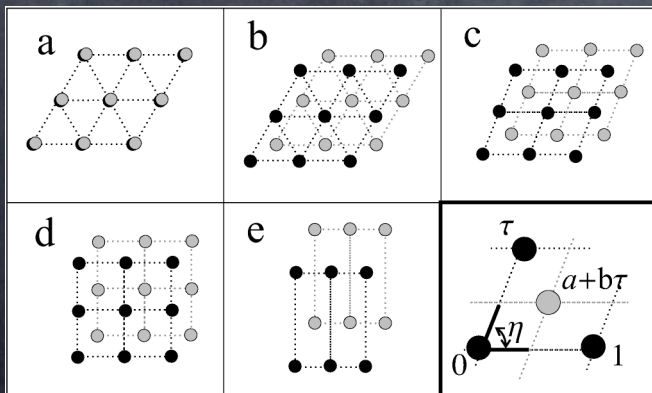
# 1/2 QV Lattice?

- SC (SF) with additional U(1) symmetry due to  $\hat{d}$  rotation
- Interlacing lattices of two types of vortices



–Different geometry depending on density and LL mixing

- Similar case arise in spinor condensate



Muller & Ho(02)

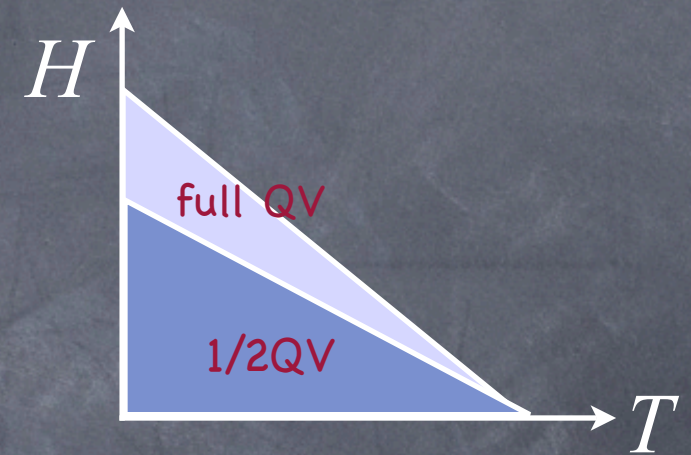
Barnett, Mukerjee & Moore(08)



# Prediction

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- ① Minimize GL free energy to determine the VL structure
- ② Quartic terms in the free energy determine the structure

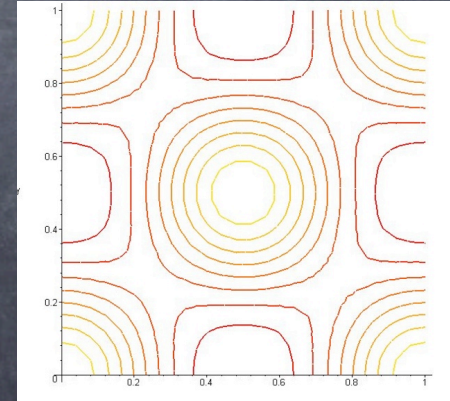
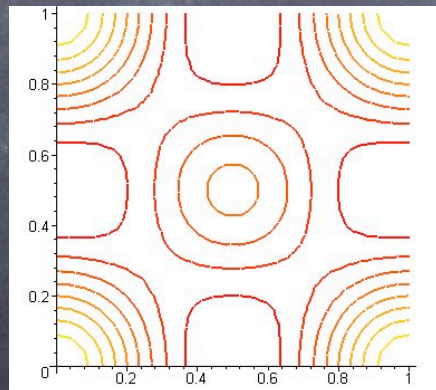
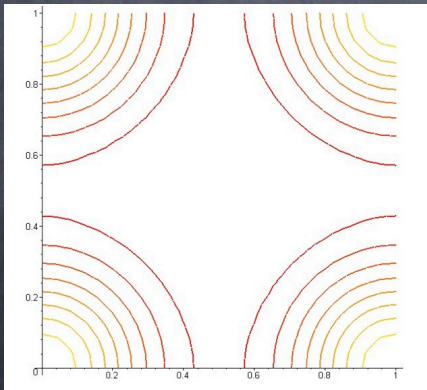
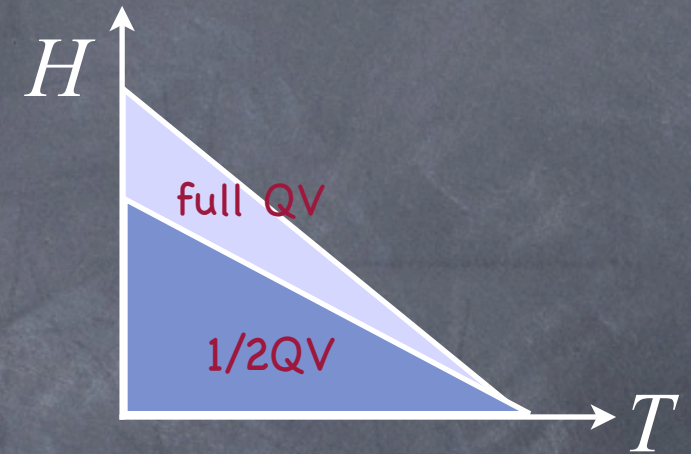


Field distribution as can be measured by neutron



# Prediction

- Minimize GL free energy to determine the VL structure
- Quartic terms in the free energy determine the structure



Field distribution as can be measured by neutron



# Stiffness engineering?

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- p-wave Feshbach resonance can allow for tuning for Fermi liquid parameters

$$H = \sum_{\mathbf{p}} \epsilon(\mathbf{p}) a_{\mathbf{p}}^{\dagger} a_{\mathbf{p}} + \sum_{\mathbf{p}, \alpha} \left[ \epsilon_{\alpha} + \frac{\epsilon(\mathbf{p})}{2} \right] b_{\mathbf{p}\alpha}^{\dagger} b_{\mathbf{p}\alpha} + \frac{1}{\sqrt{V}} \sum_{\mathbf{p}, \mathbf{q}, \alpha} g_{\mathbf{p}} p_{\alpha} \left( b_{\mathbf{q}\alpha} a_{\mathbf{p}+\frac{\mathbf{q}}{2}}^{\dagger} a_{-\mathbf{p}+\frac{\mathbf{q}}{2}}^{\dagger} + \text{h.c.} \right)$$

Gurarie, L. Radzihovsky, & A. V. Andreev (05)

- Hope to arrive at a PD where  $\rho_{sp}/\rho_s$  can be tuned as a function of microscopic parameter



In search of topological states with  
half quantum vortices



# In search of topological states with half quantum vortices





# In search of topological states with half quantum vortices



①  $1/2$  QV's are not stable in bulk systems



# In search of topological states with half quantum vortices



- ①  $1/2$  QV's are not stable in bulk systems
- ② **Mesoscopic** samples could favor  $1/2$  QV's



# In search of topological states with half quantum vortices



- $1/2$  QV's are not stable in bulk systems
- **Mesoscopic** samples could favor  $1/2$  QV's
- **$1/2$  QV Vortex Lattice** can be pursued and detected