Majorana Fermions in Topological Insulators

I. Introduction
   - Topological band theory and topological insulators

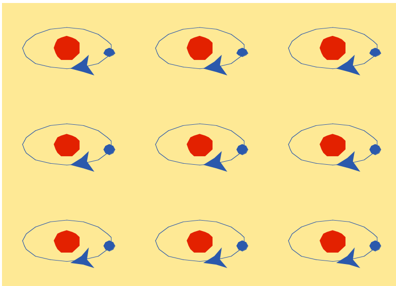
II. Two Dimensions : Quantum Spin Hall Effect
   - Time reversal symmetry and edge states
   - Experiment: Transport in HgTe quantum Wells

III. Three Dimensions : Topological Insulators
   - Topological Insulator Surface States
   - Experiment: Photoemission on Bi_xSb_{1-x}

IV. Superconducting proximity effect and Majorana fermions
   - Majorana fermion bound states:
     A platform for topological quantum computing?
   - Fractional Josephson effect in a S-QSHI-S junction

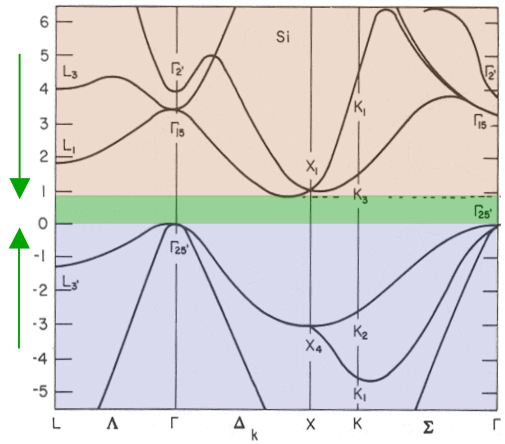
Thanks to Gene Mele, Liang Fu, Jeffrey Teo

The Insulating State

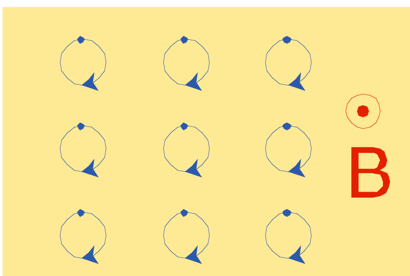


e.g. Silicon

$E_g \sim 1$ eV

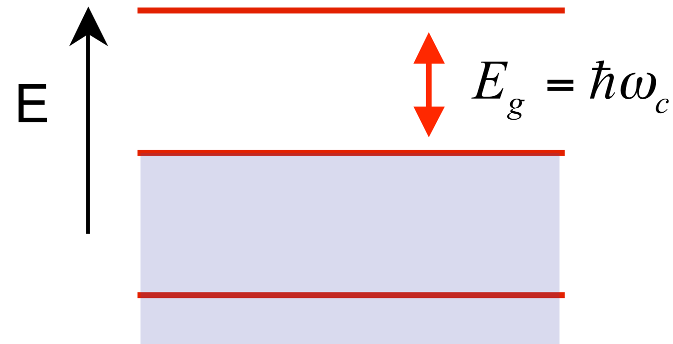


The Integer Quantum Hall State



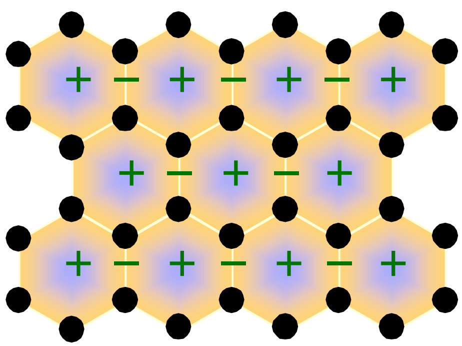
2D Cyclotron Motion,
Landau Levels

$\sigma_{xy} = e^2/h$



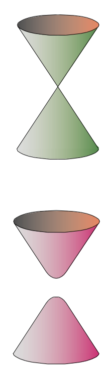
IQHE with zero net magnetic field

Graphene with a periodic magnetic field B(r) (Haldane PRL 1988)

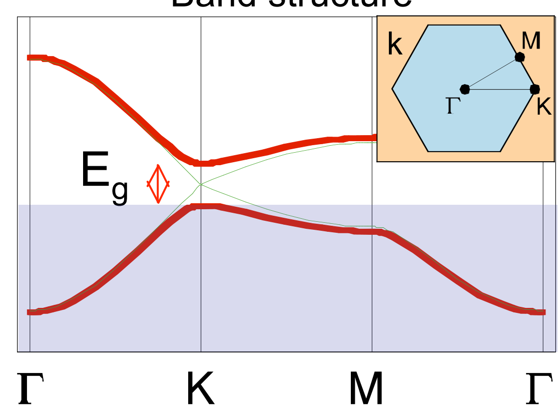


B(r) = 0
Zero gap,
Dirac point

B(r) ≠ 0
Energy gap
$\sigma_{xy} = e^2/h$



Band structure



Topological Band Theory

The distinction between a conventional insulator and the quantum Hall state is a topological property of the manifold of occupied states

$$|\Psi(\vec{k})\rangle \ : \ \text{Brillouin zone } (T^2) \mapsto \text{Hilbert space}$$

Classified by the TKNN (or Chern) topological invariant (Thouless et al, 1984)

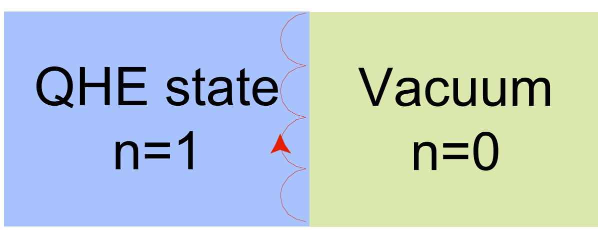
$$n = \frac{1}{2\pi i}\int_{BZ} d^2\mathbf{k} \cdot \langle \nabla_{\mathbf{k}} u(\mathbf{k}) | \times | \nabla_{\mathbf{k}} u(\mathbf{k}) \rangle$$

Insulator : $n = 0$

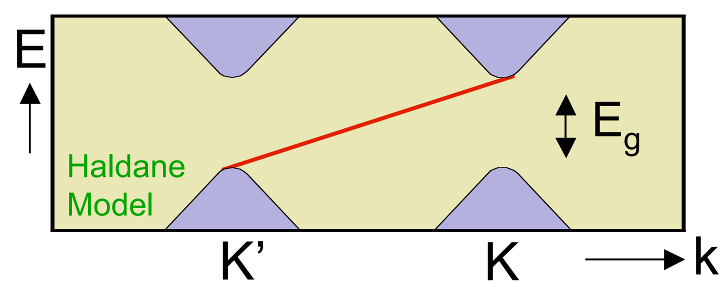
IQHE state : $\sigma_{xy} = n\, e^2/h$

The TKNN invariant can only change when the energy gap goes to zero

Edge States at a domain wall



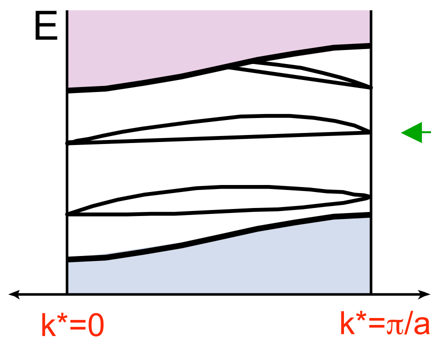
Gapless Chiral Fermions



Topological Insulator : A New B=0 Phase

2D Time reversal invariant band structures have a **Z_2 topological invariant, ν = 0,1**

ν=0 : Conventional Insulator



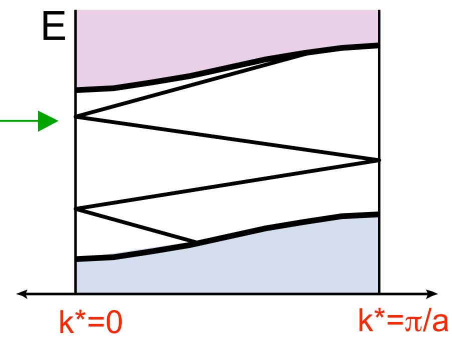
Edge States

Kramers degenerate at

time reversal invariant momenta

k* = –k* + G

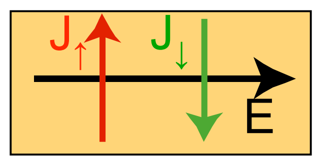
ν=1 : Topological Insulator



ν is a property of bulk bandstructure. Easiest to compute if there is extra symmetry:

1. S_z conserved : independent spin Chern integers : $n_\uparrow = - n_\downarrow$ (due to time reversal)

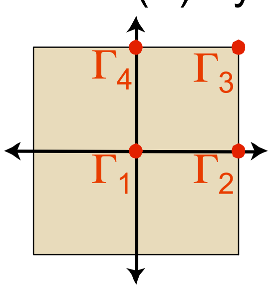
Quantum spin Hall Effect :



$$\nu = n_{\uparrow,\downarrow} \mod 2$$

2. Inversion (P) Symmetry : determined by Parity of occupied 2D Bloch states at $\Gamma_{1,2,3,4}$

Bulk Brillouin Zone



$$P|\psi_n(\Gamma_i)\rangle = \xi_n(\Gamma_i)|\psi_n(\Gamma_i)\rangle$$

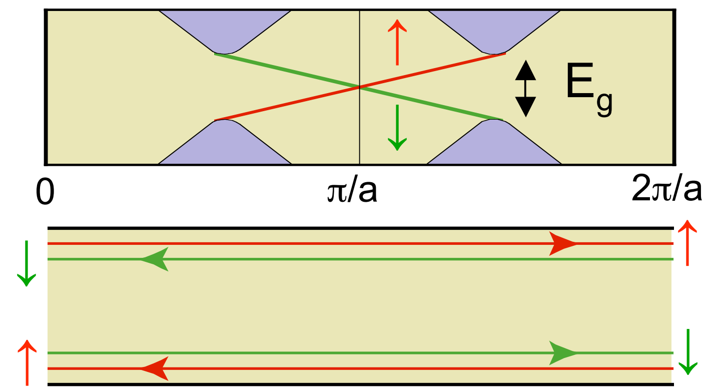
$$\xi_n(\Gamma_i) = \pm 1$$

$$(-1)^\nu = \prod_{i=1}^{4} \prod_n \xi_{2n}(\Gamma_i)$$

Two dimensions : Quantum Spin Hall Insulator

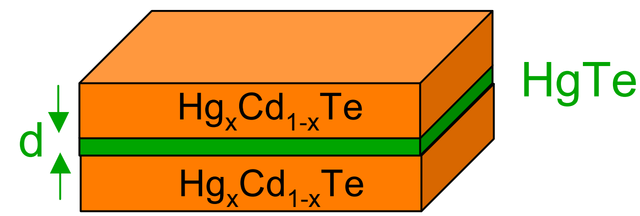
I. Graphene Kane, Mele PRL '05

- Intrinsic spin orbit interaction
  $\Rightarrow$ small (~10mK-1K) band gap
- S_z conserved : "| Haldane model $|^2$"
- Edge states : $G = 2\ e^2/h$



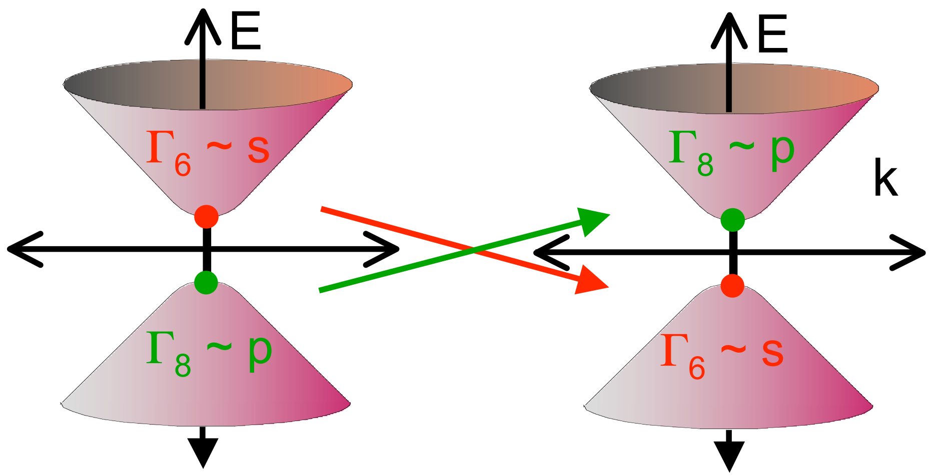
II. HgCdTe quantum wells

Theory: Bernevig, Hughes and Zhang, Science '06
Experiement: Konig et al. Science '07



d < 6.3 nm
Normal band order

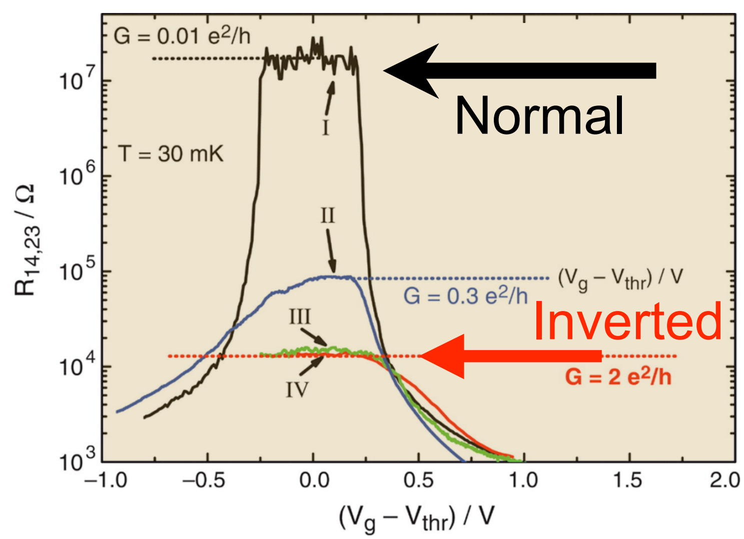
d > 6.3 nm:
Inverted band order



Conventional Insulator

QSH Insulator

$G \sim 2e^2/h$ in QSHI

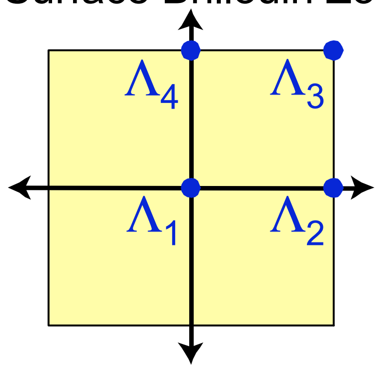


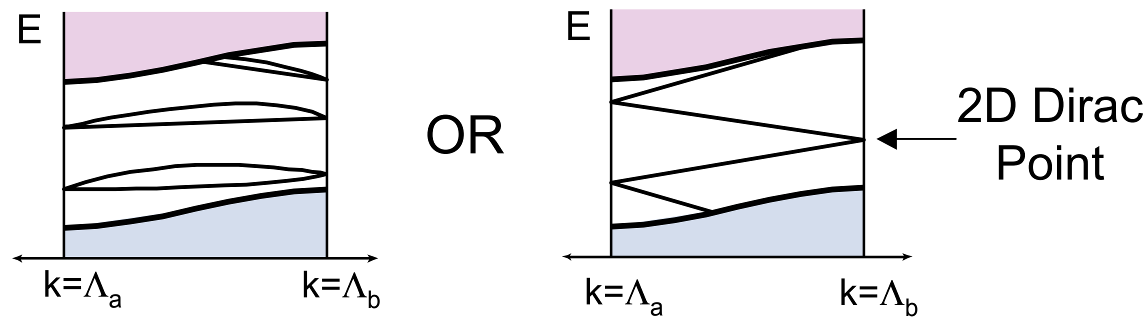
Three Dimensional Topological Insulators

In 3D there are 4 Z_2 invariants: $(\nu_0 ; \nu_1\nu_2\nu_3)$ characterizing the bulk. These determine how surface states connect.

Fu, Kane & Mele PRL 07
Moore & Balents PRB 07
Roy, cond-mat 06

Surface Brillouin Zone



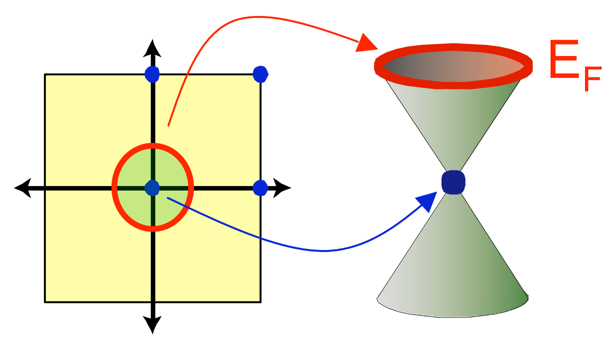


ν_0 = 1 : Strong Topological Insulator

Fermi surface encloses odd number of Dirac points

Topological Metal

- Berry's phase π around Fermi surface
- Robust to disorder (antilocalization)

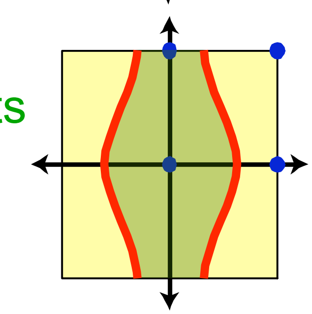


ν_0 = 0 : Weak Topological Insulator

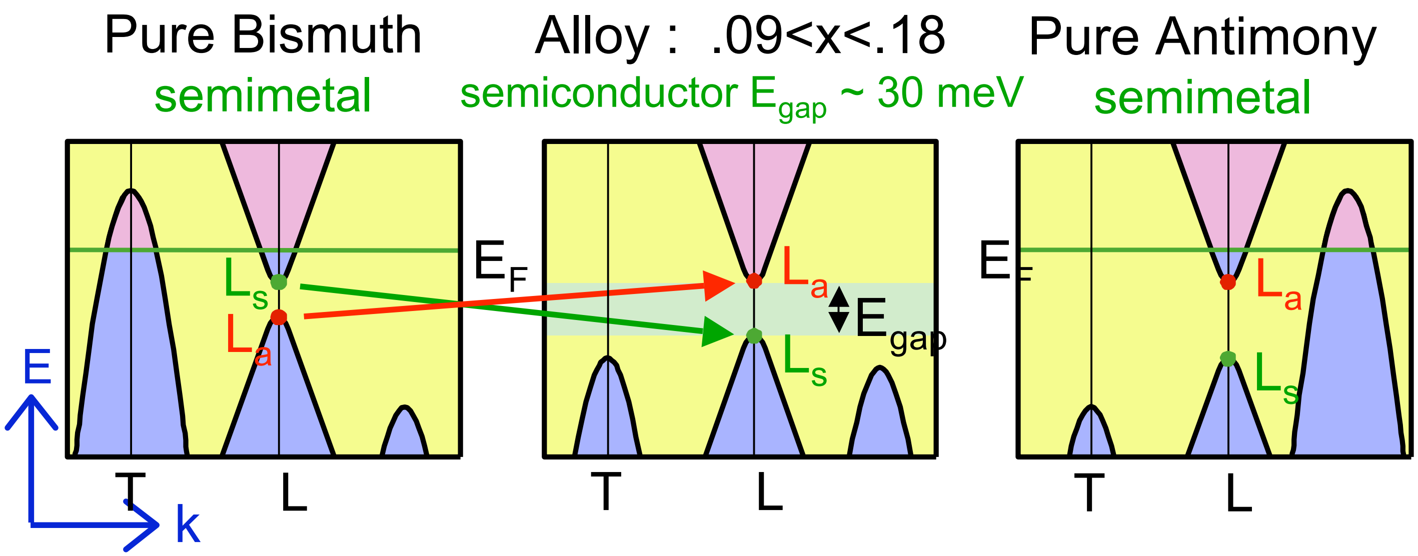
Fermi surface encloses even number of Dirac points

Normal Metal

- Berry's phase 0, less robust.
- Equivalent to layered 2D QSHI

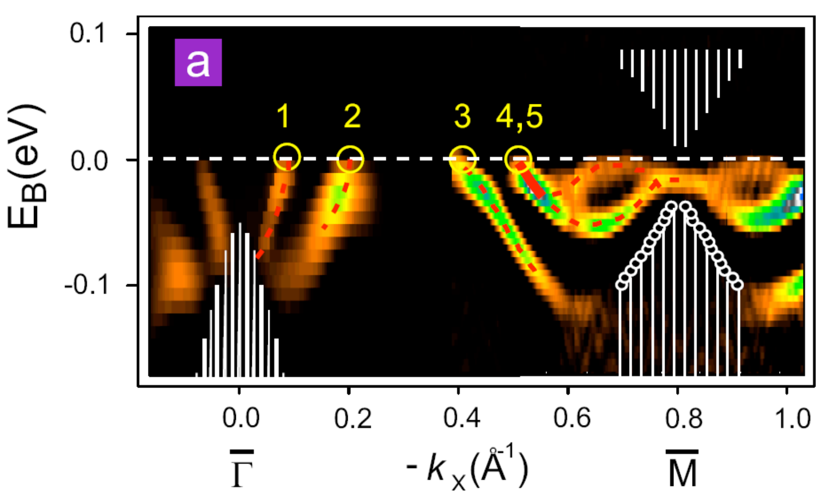


$Bi_{1-x}Sb_x$



Theory: Predict $Bi_{1-x}Sb_x$ is a topological insulator by exploiting inversion symmetry of Bi, Sb (Fu,Kane PRL'07)

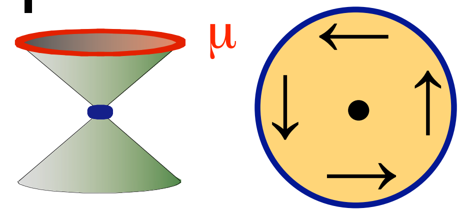
Experiment: ARPES (Hsieh et al. Nature '08)



- 5 surface state bands cross E_F between Γ and M
- Proves that $Bi_{1-x}Sb_x$ is a Strong Topological Insulator

Proximity Effects, Energy Gaps

Minimal surface state model: $$H_0 = \psi^\dagger(-iv\vec{\sigma}\cdot\vec{\nabla} - \mu)\psi$$

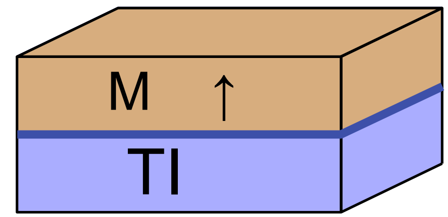


"half" a 2DEG

Two Gapped Phases :

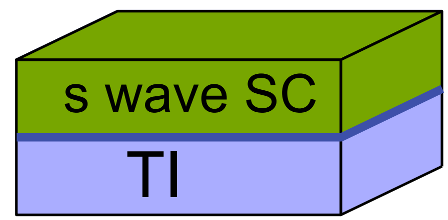
Magnetic $$V_M = M\psi^\dagger\sigma_z\psi$$

Gap for $M>M_c(\mu)$
Broken time reversal symmetry
"half quantized" QHE $\sigma_{xy} = e^2/2h$



Superconducting $$V_S = \Delta\psi_\uparrow^\dagger\psi_\downarrow^\dagger + \Delta^*\psi_\downarrow\psi_\uparrow$$

Similar to spinless p_x+ip_y SC
No broken time reversal



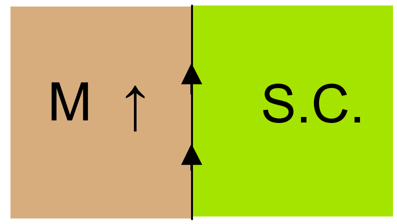
Bogoliubov Spectrum (μ=0) $$E_\pm(k) = \sqrt{\left(|\Delta| \pm |M|\right)^2 + v^2k^2}$$

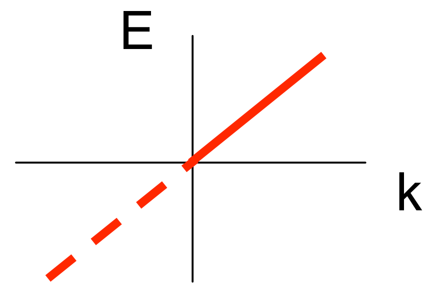
$|\Delta|>|M|$: Superconducting phase
$|\Delta|<|M|$: Insulating phase
$|\Delta|=|M|$: Critical : 2D gapless Majorana

Majorana Fermions

S-TI-M interface

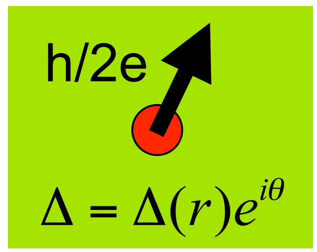
Gapless 1D chiral
Majorana fermions
bound to domain wall





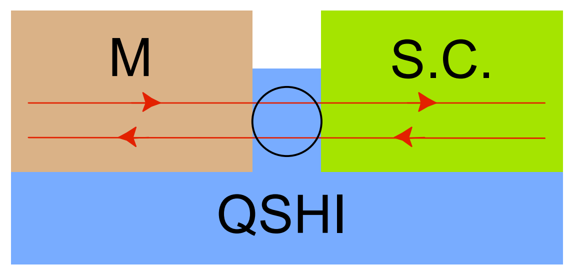
Vortex in 2D SC :

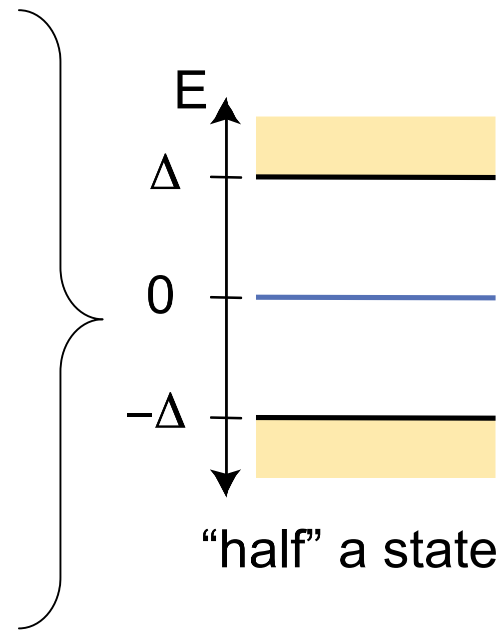
Zero energy
Majorana bound state
at vortex



S-QSHI-M junction

Zero energy
Majorana bound state
at junction



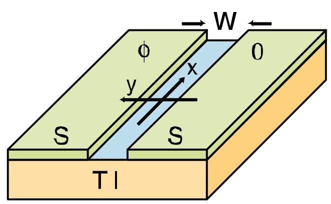


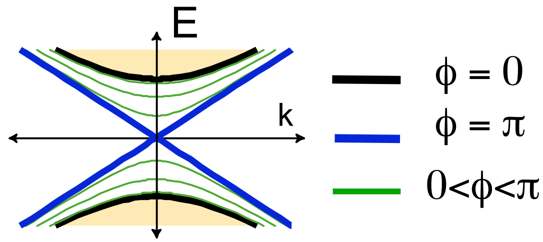
"half" a state

Kitaev 2003 : 2N Majoranas = N qubits: fault tolerant quantum memory
Braiding : Quantum computation

Manipulation of Majorana Fermions

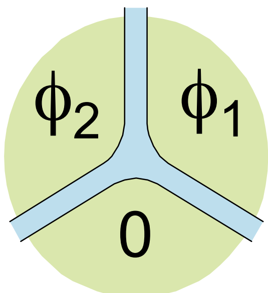
S-TI-S Line Junction : A 1D "wire" for Majorana fermions

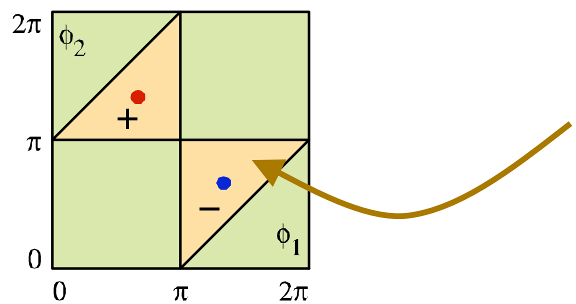




Gapless 1D Majorana Fermions for $\phi = \pi$

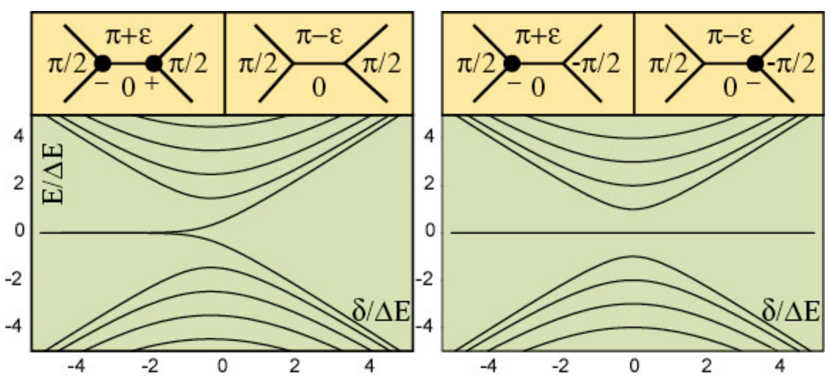
S-TI-S Tri-Junction :

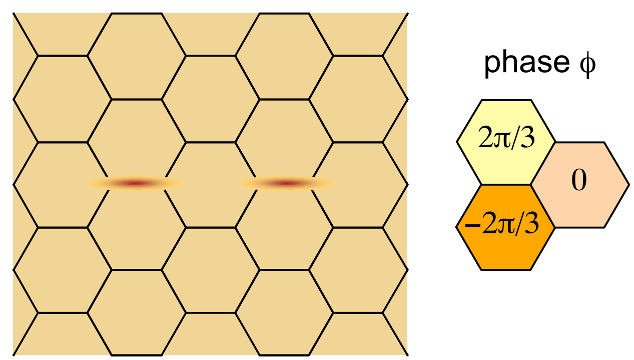




Majorana bound state present at tri-junction

Create, Transport and Fuse Majorana fermions along line junctions





$$\left(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle\right) / \sqrt{2}$$

The challenges :

- Find suitable topological insulator ($Bi_{1-x} Sb_x$? E_g ~ 30 meV)
- Find suitable superconductor which makes good interface (Nb ?)
- Optimize proximity induced gap and discrete Andreev bound states
- Control the superconducting phases with Josephson junctions
- Measure current difference when Majoranas are fused ………

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Anomalous Proximity Effect in the Nb-BiSb-Nb Junctions

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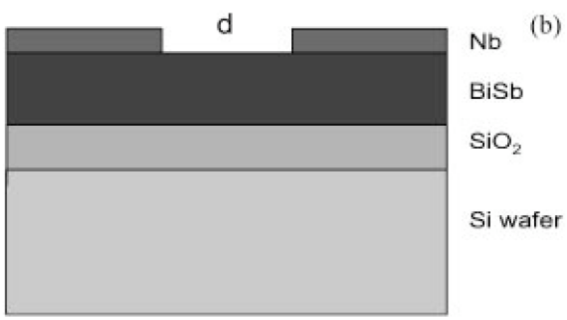
(Received 20 November 1995)



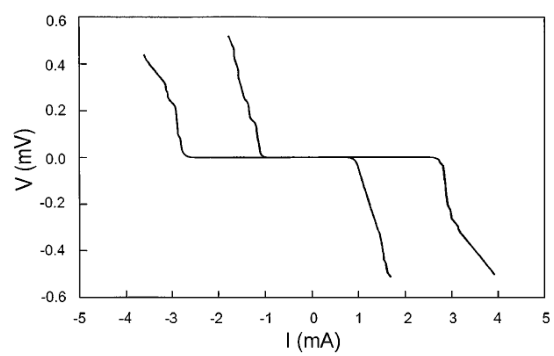
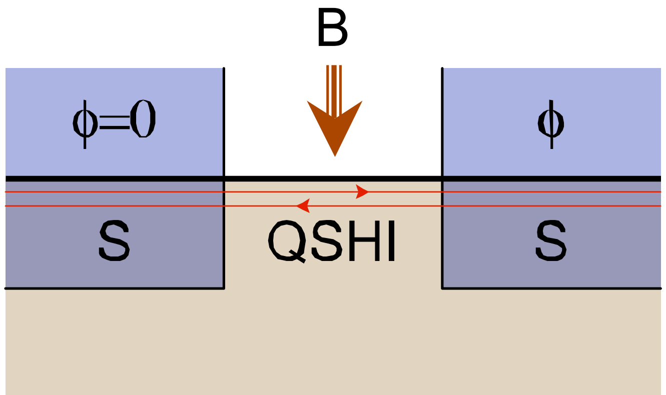


FIG. 2. *I-V* characteristics for the junction at 4.2 K. I_c = 1 mA for 0.6 μm slit junction; I_c = 2.5 mA for 1.2 μm slit junction.

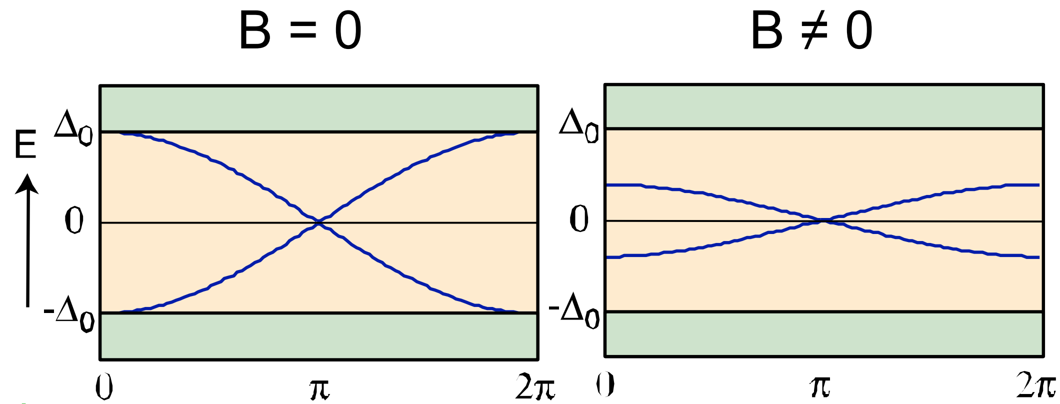
Evidence for good contact between BiSb and Nb : minimal Shottky barrier

S-QSHI-S Josephson Junction

Fu, Kane cond-mat/08



Andreev bound states in the junction

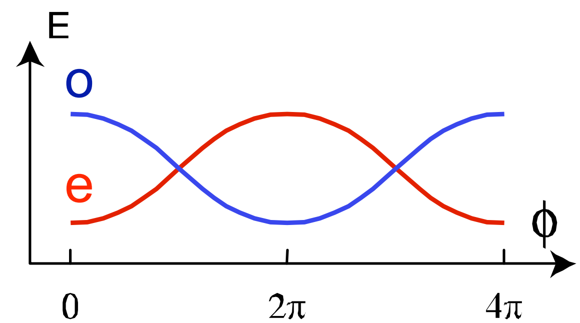


B = 0 : "Half" a perfectly transmitting superconducting quantum point contact

B ≠ 0 : "Fractional Josephson Effect"

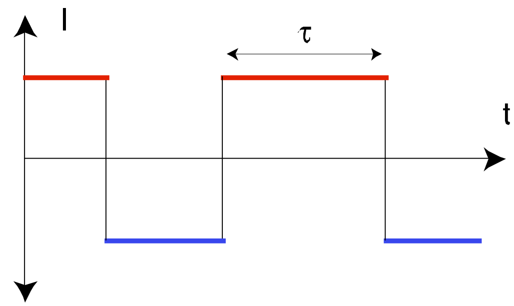
(Kitaev 2001; Kwon, Sengupta, Yakovenko, 2004)

- 4π periodicity of $E(\phi)$ protected by local conservation of fermion parity.
- Two coupled Majoranas



"Telegraph noise" at $\phi \sim \pi$

- Switch fermion parity by inelastic scattering of thermally activated quasiparticles
- $\tau \sim \exp(\Delta_0/T)$
- Noise $S(\omega \to 0) \sim \exp(\Delta_0/T)$



Observation of Maximum Supercurrent Quantization in a Superconducting Quantum Point Contact

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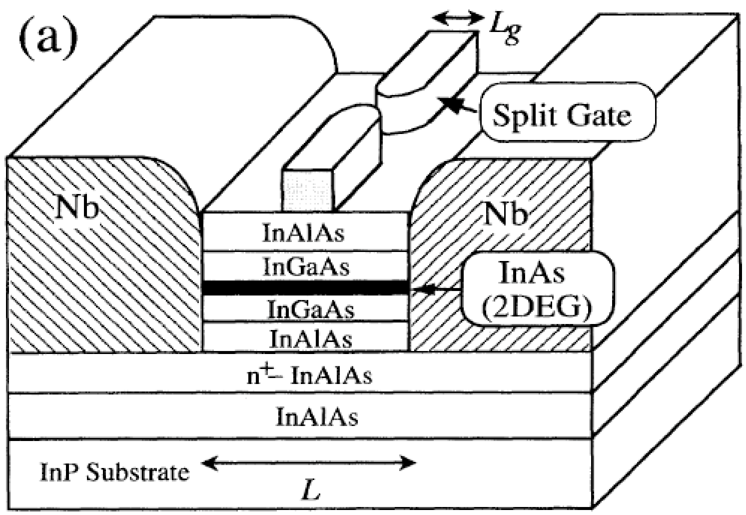
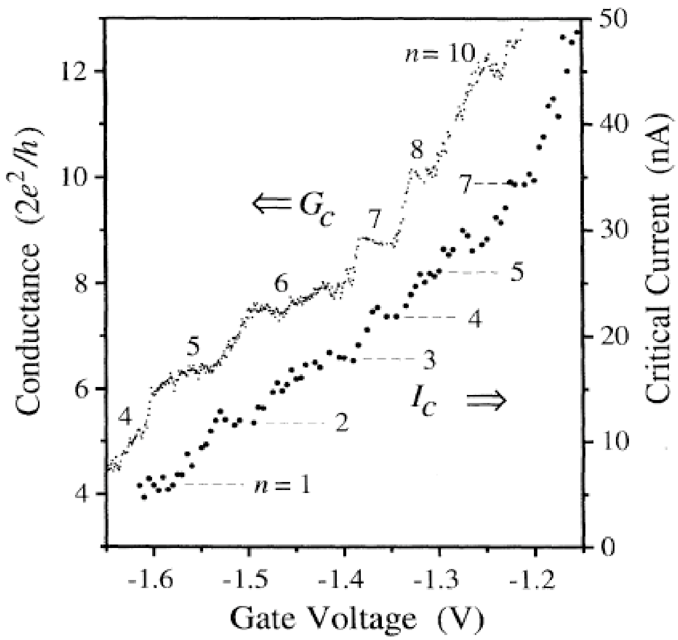
(Received 2 June 1995)





Conclusion

- A new electronic phase of matter has been predicted and observed

  - 2D : Quantum spin Hall insulator in HgCdTe QW's
  - 3D : Strong topological insulator in $Bi_{1-x}Sb_x$

- Experimental Challenges

  - Spin dependent Transport Measurements
  - Transport and magneto-transport measurements on $Bi_{1-x}Sb_x$
  - Superconducting proximity effect :
    - BiSb-Nb ? HgCdTe-Nb ??
    - Characterize S-TI-S junctions
    - Create the Majorana bound states
    - Detect the Majorana bound states

- Theoretical Challenges

  - Effects of disorder, interactions on surface states, superconductivity and critical phenomena

  - Other Materials?