Signatures For Coulomb Blockade and Aharonov-Bohm Interference in Electronic Fabry-Perot Interferometers

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Motivation

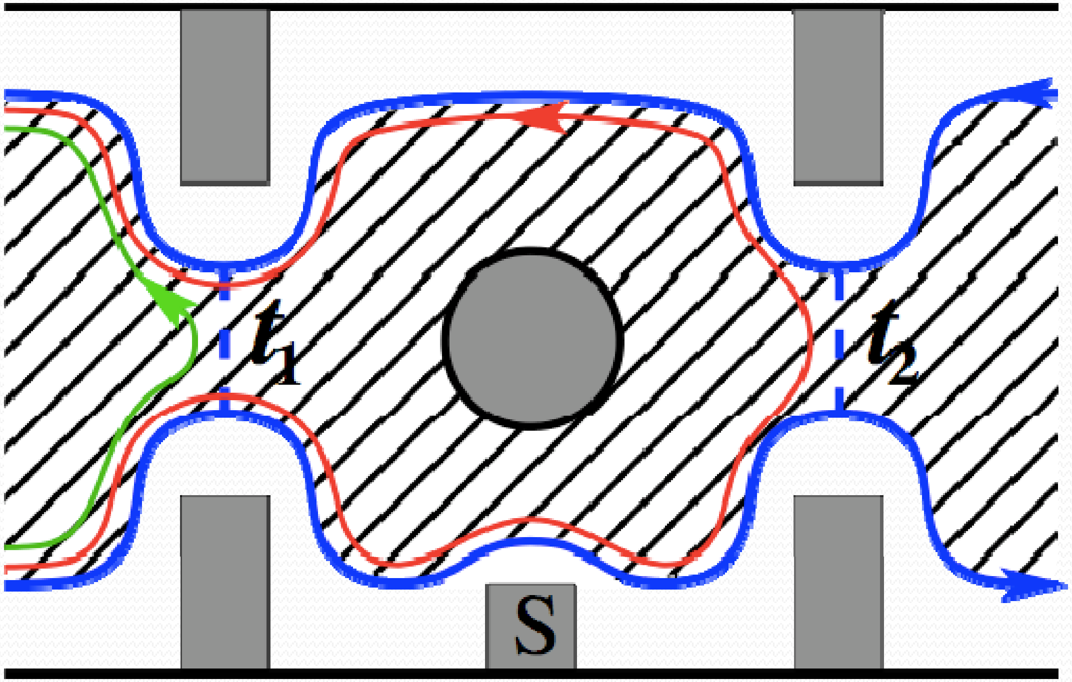
- In 2D, clockwise exchange and counter-clockwise exchange are distinct, leading to Anyonic statistics:

$$\psi(\vec{r_i}, \vec{r_j}) \rightarrow e^{i\theta}\psi(\vec{r_j}, \vec{r_i}) \qquad e^{i\theta} \neq e^{-i\theta}$$

  Therefore, θ can be any value, hence called Anyons
- Fractional statistics may show up in the FQHE regime: Ex. 1/3, 2/5, etc.
- Non-Abelian statistics is also possible in 2D, and may be found in some FQHE states: Ex. 5/2, 12/5
- Proposed experiments to detect fractional/non-Abelian statistics have made use of Fabry-Perot interferometers

Motivation

- Fabry-Perot interferometer in QH regime



Bonderson, *et al.*, PRL 97, 016401 (2006)

$$I_{back} \propto |t_1|^2 + |t_2|^2 + 2\eta |t_1 t_2| \cos(\phi)$$

Motivation

- For detecting fractional statistics

PHYSICAL REVIEW B VOLUME 55, NUMBER 4 15 JANUARY 1997-II

Two point-contact interferometer for quantum Hall systems

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(Received 29 July 1996)

- At constant density: $\Delta B = \Phi_0/A$, $\Phi_0 = h/e$
- At constant filling factor: $\Delta B = \Phi_0^*/A$, $\Phi_0^* = h/e^*$

Motivation

- For detecting non-Abelian statistics

PRL **96,** 016802 (2006) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2006

Proposed Experiments to Probe the Non-Abelian $\nu = 5/2$ Quantum Hall State

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PRL **96,** 016803 (2006) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2006

Detecting Non-Abelian Statistics in the $\nu = 5/2$ Fractional Quantum Hall State

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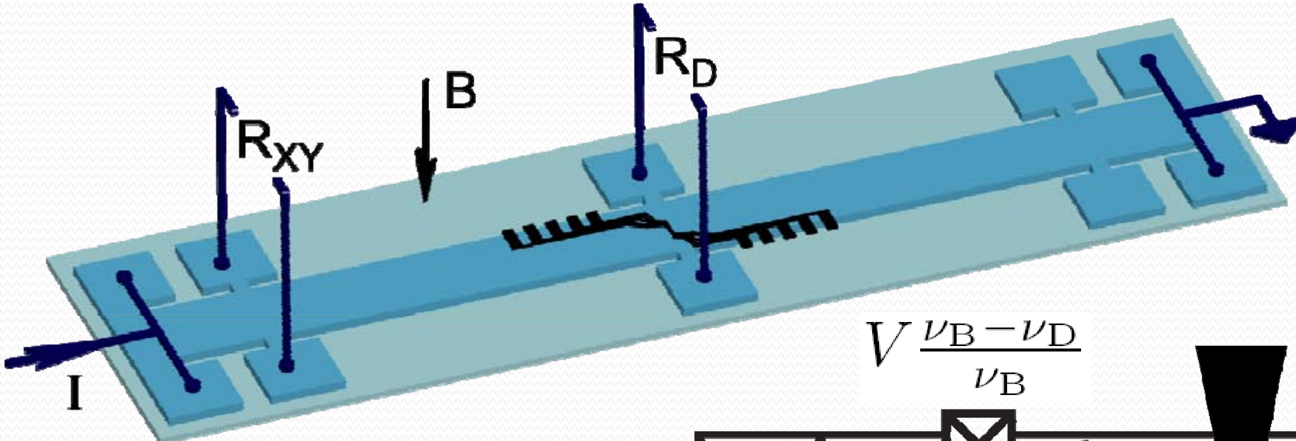
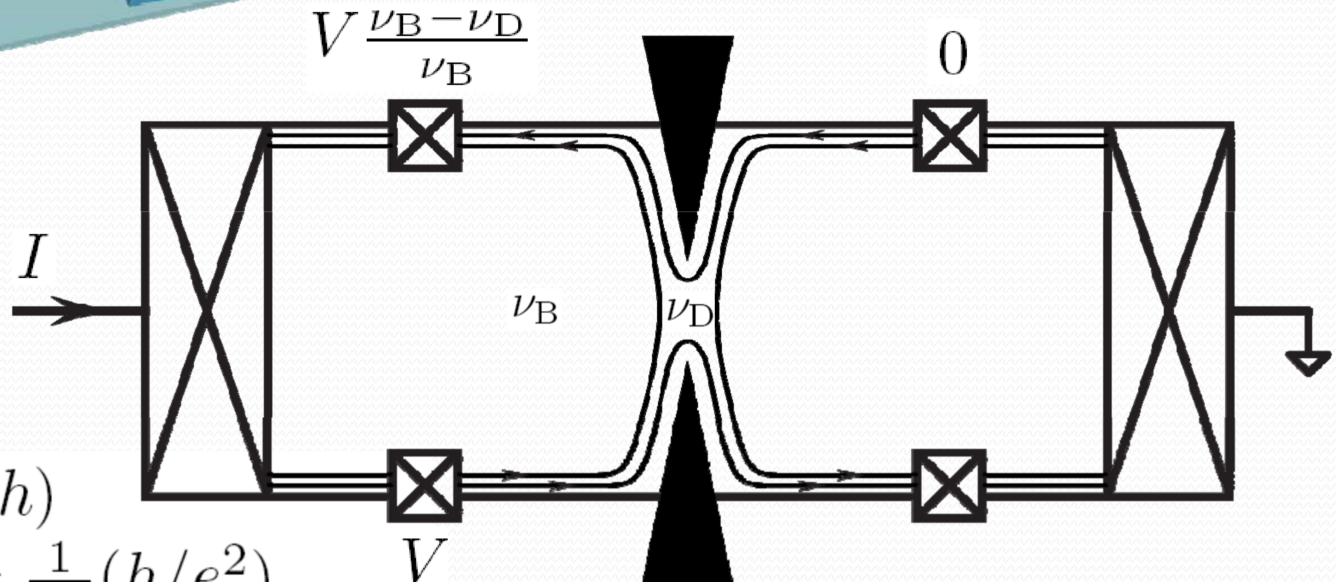
- Odd number of quasi-particles inside: no interference
- Even number of quasi-particles inside: has interference

Measurement Setup



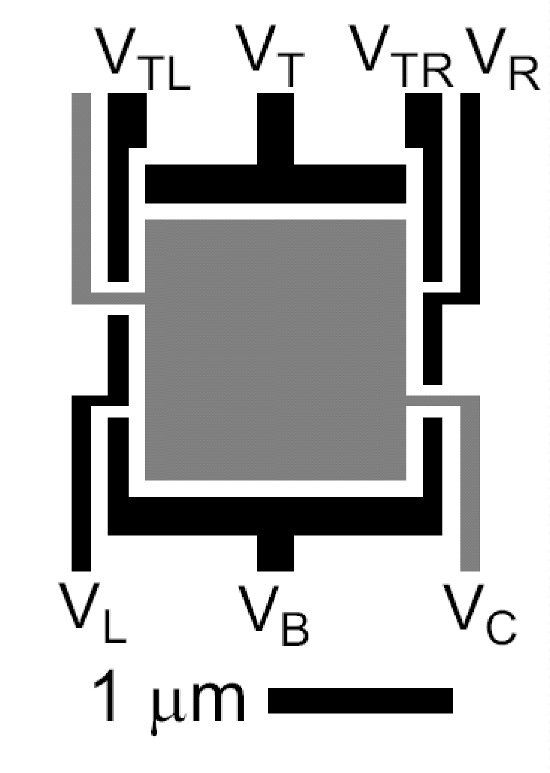


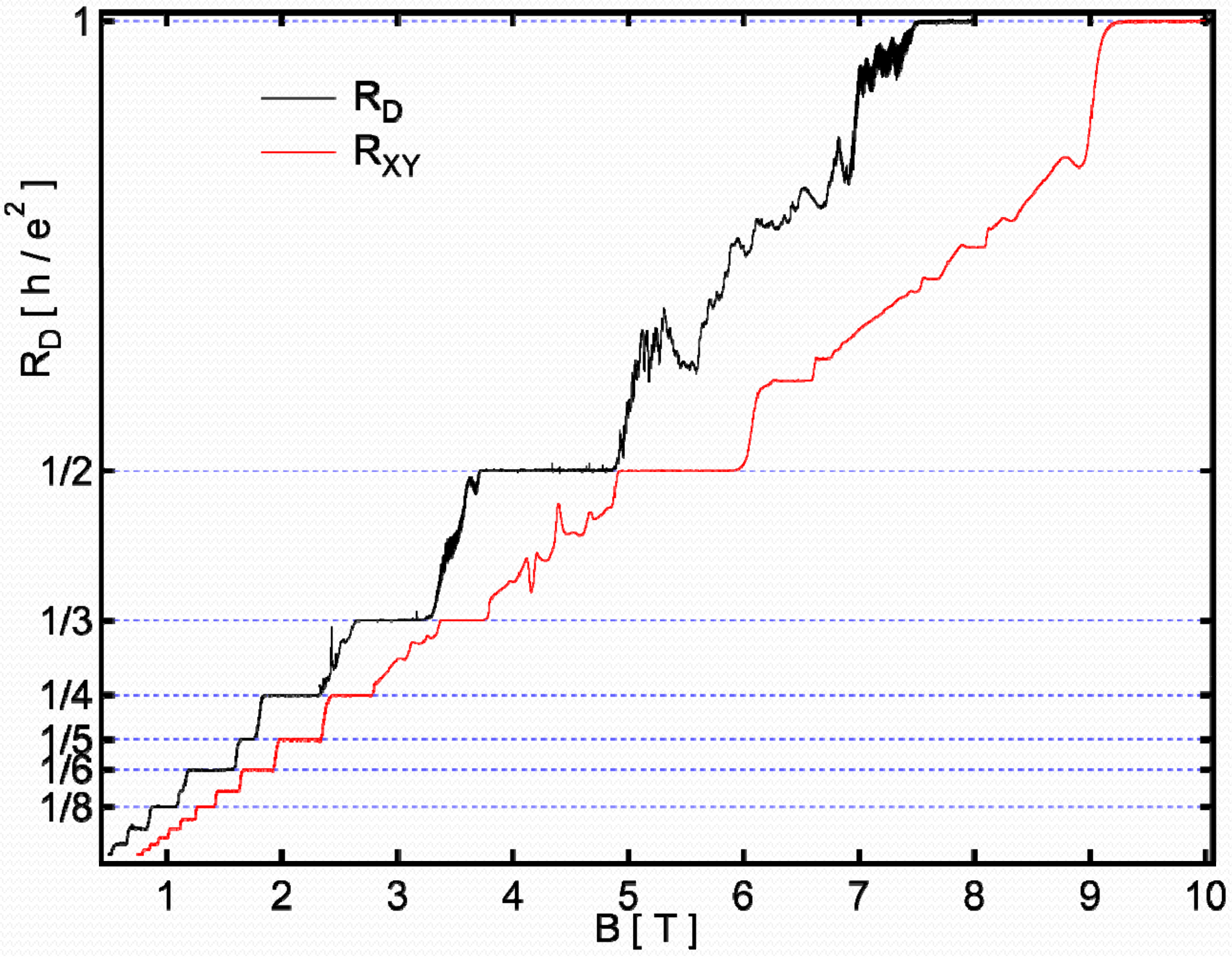
$$I = V \cdot \nu_D (e^2/h)$$

$$V_D = V,\ R_D = \frac{1}{\nu_D}(h/e^2)$$

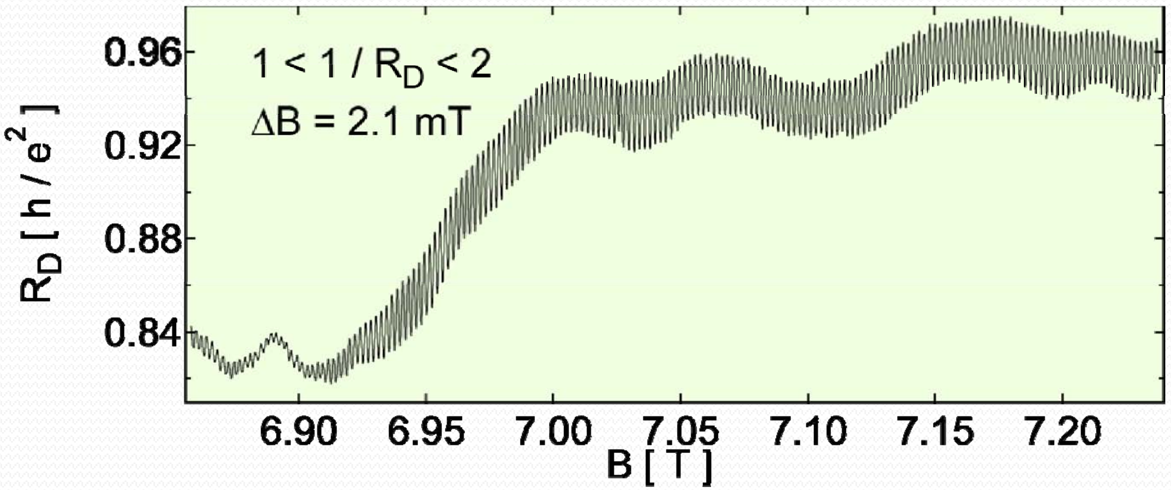
$$V_{XY} = V\frac{\nu_D}{\nu_B},\ R_{XY} = \frac{1}{\nu_B}(h/e^2)$$

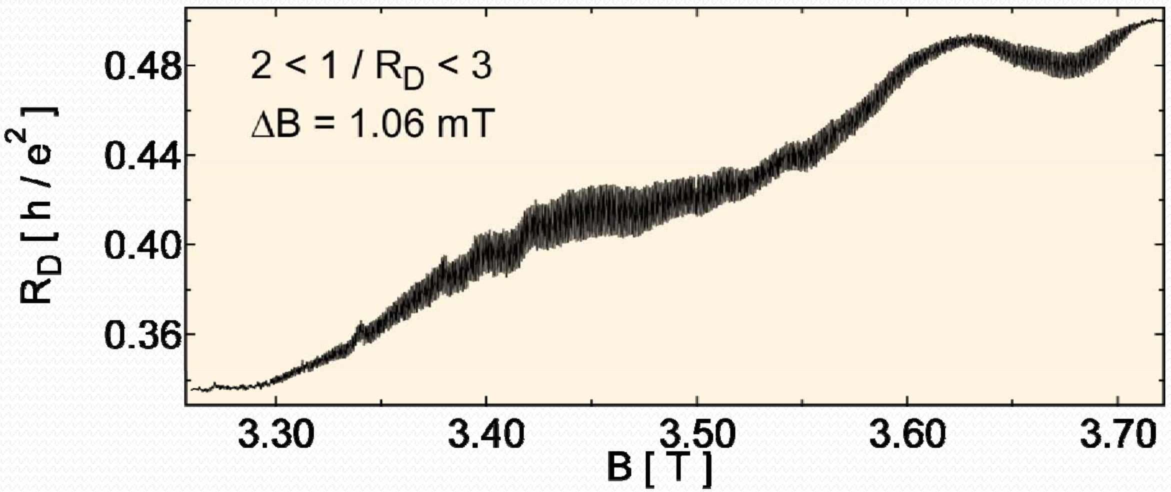
Data - 2 μm² device



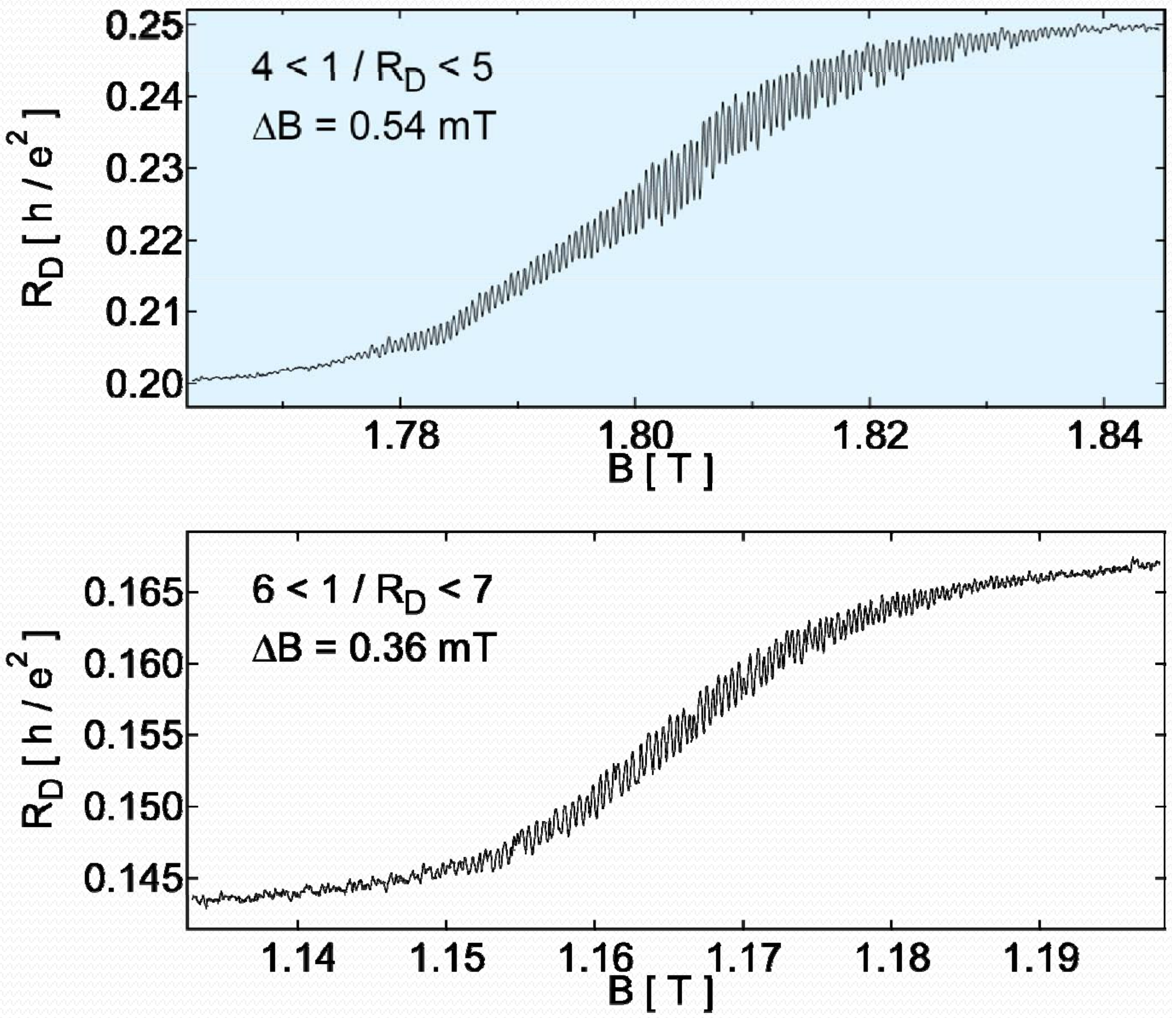


Data - 2 μm² device



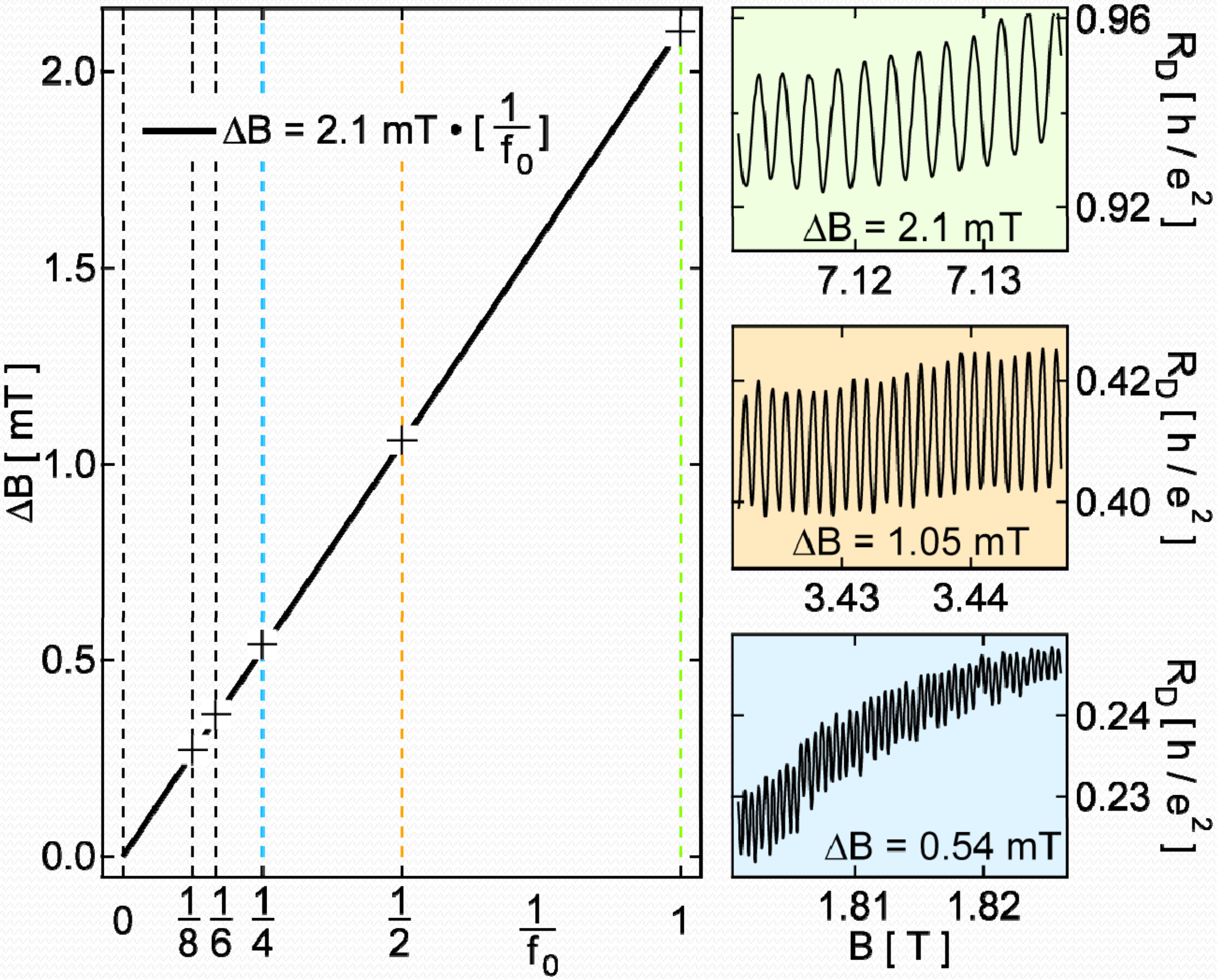


Data - 2 μm² device

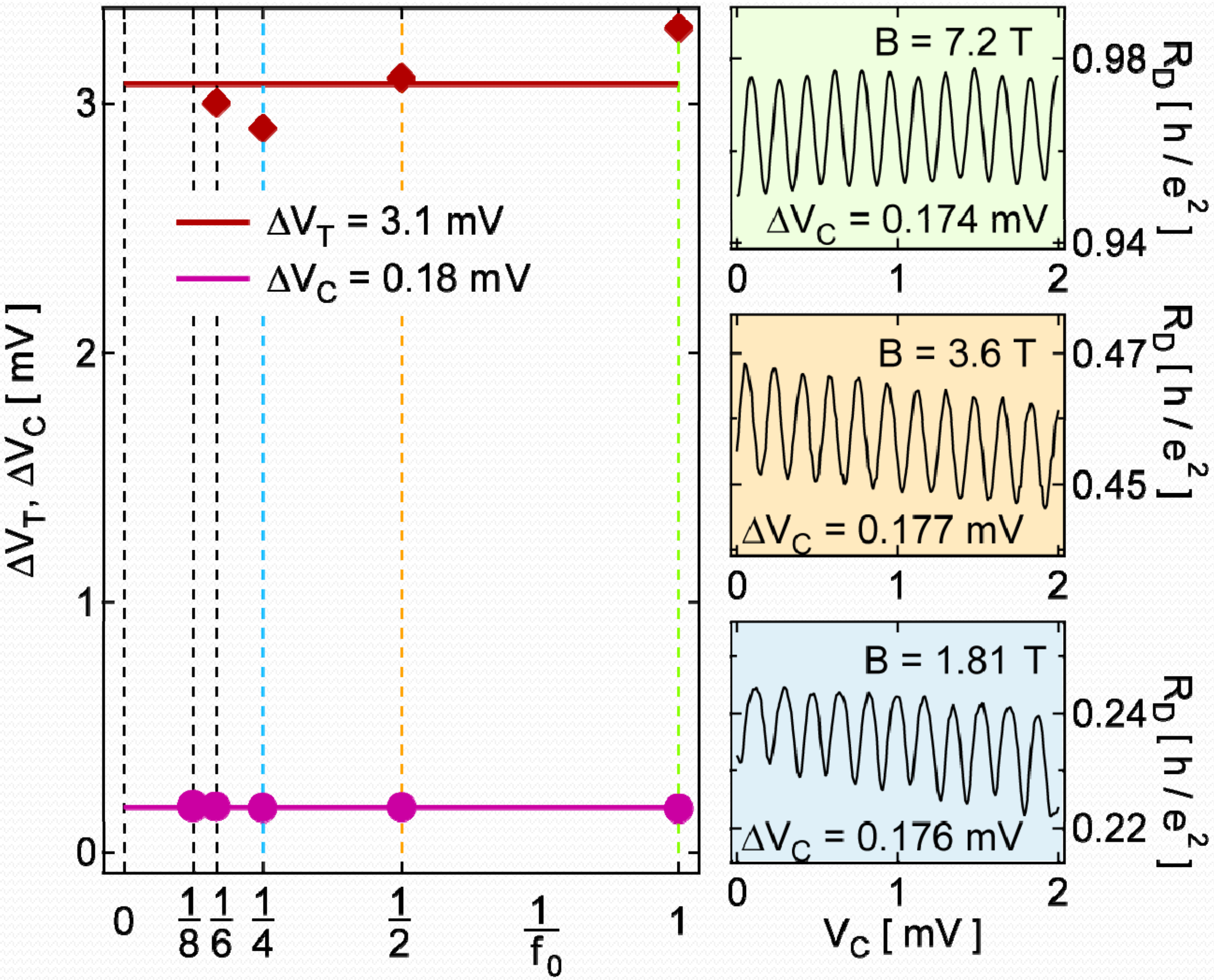


Data - 2 μm² device

$f_0 < 1/R_D < f_0 + 1$



Data - 2 μm² device



Previous Experiments

VOLUME 62, NUMBER 21 PHYSICAL REVIEW LETTERS 22 MAY 1989

Observation of Zero-Dimensional States in a One-Dimensional Electron Interferometer

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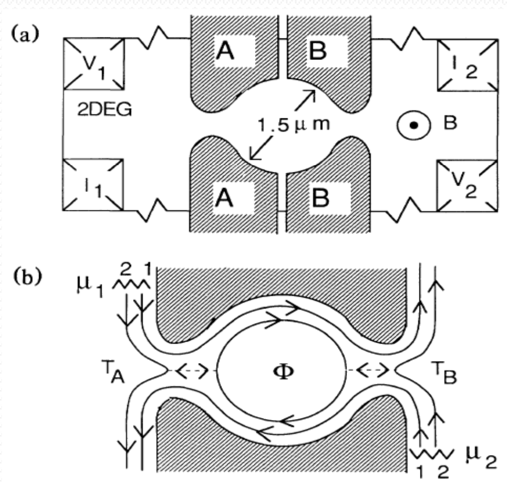
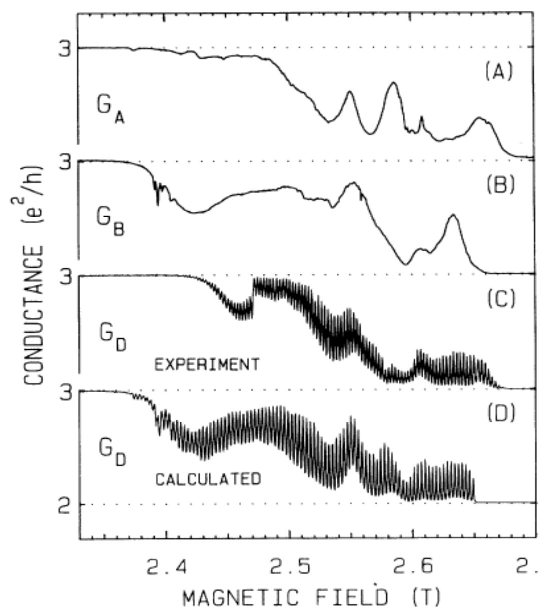
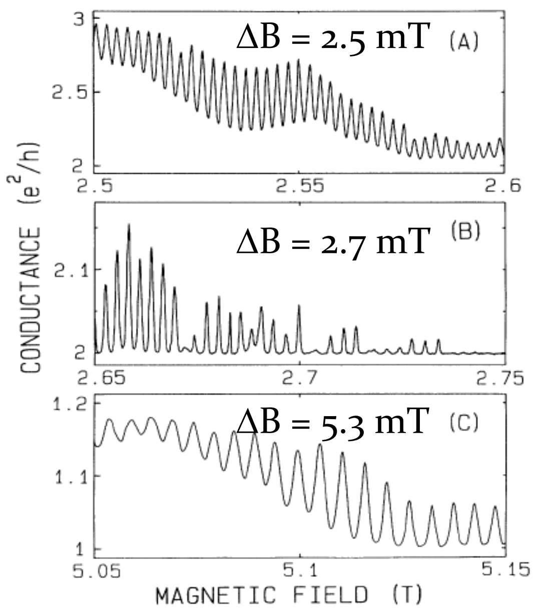
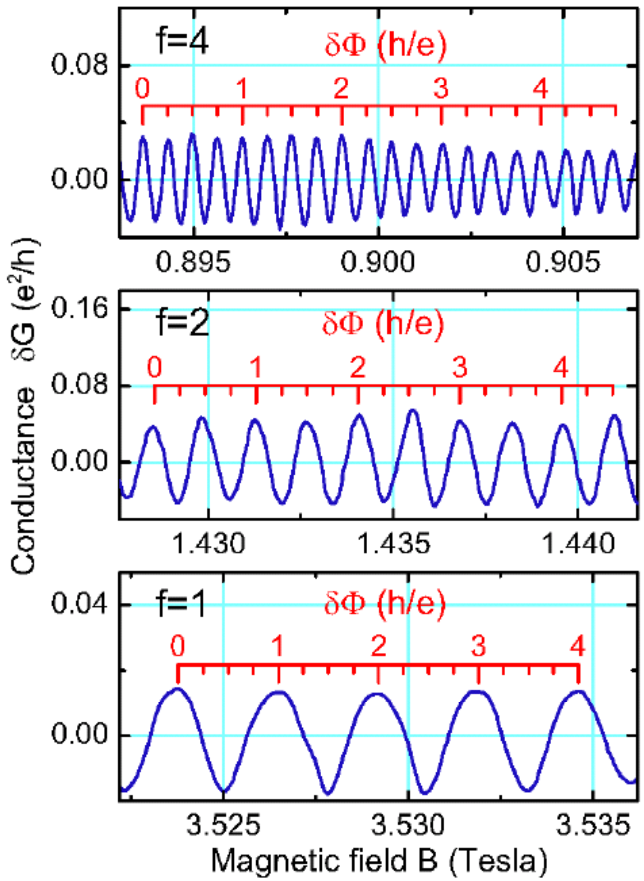
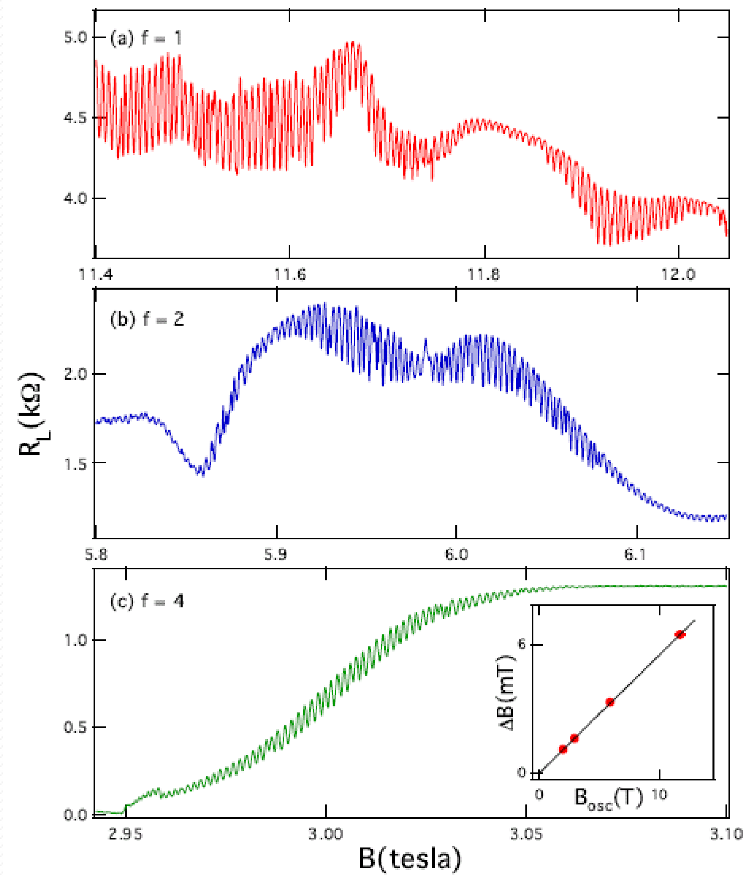






Previous Experiments





Device diameter: 2.1 μm
Camino, *et al.*, PRB 76, 155305 (2007)

Device diameter: 1.2 μm
Godfrey, *et al.*, arXiv: 0708.2448v1

Model – Coulomb blockade

PRL 98, 106801 (2007) PHYSICAL REVIEW LETTERS week ending 9 MARCH 2007

Influence of Interactions on Flux and Back-Gate Period of Quantum Hall Interferometers

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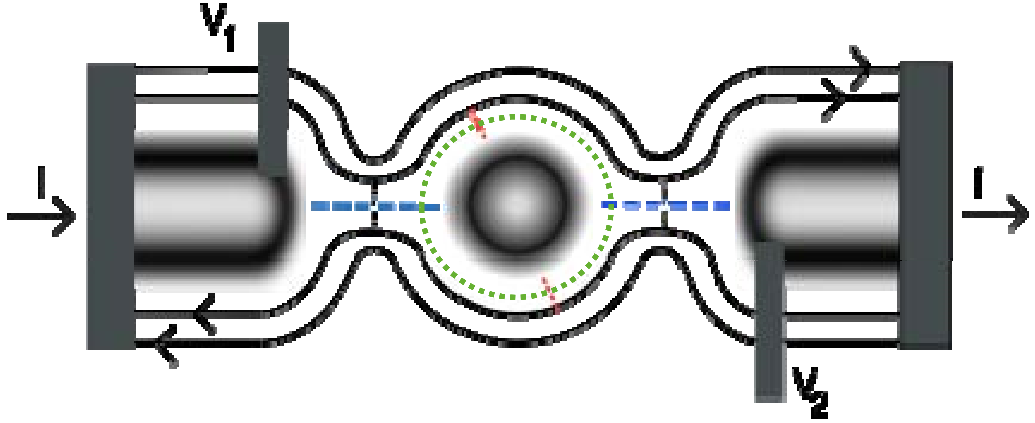
(Received 3 November 2006; published 7 March 2007)



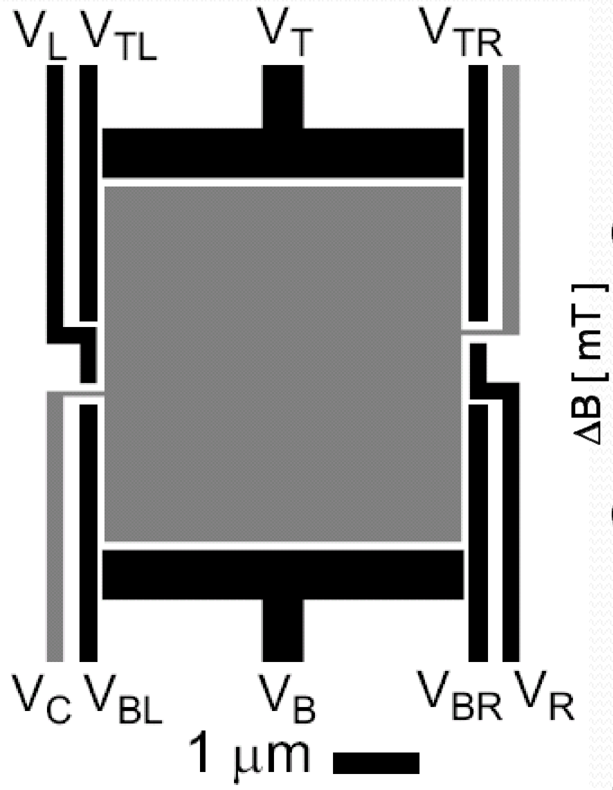
Model – Coulomb blockade

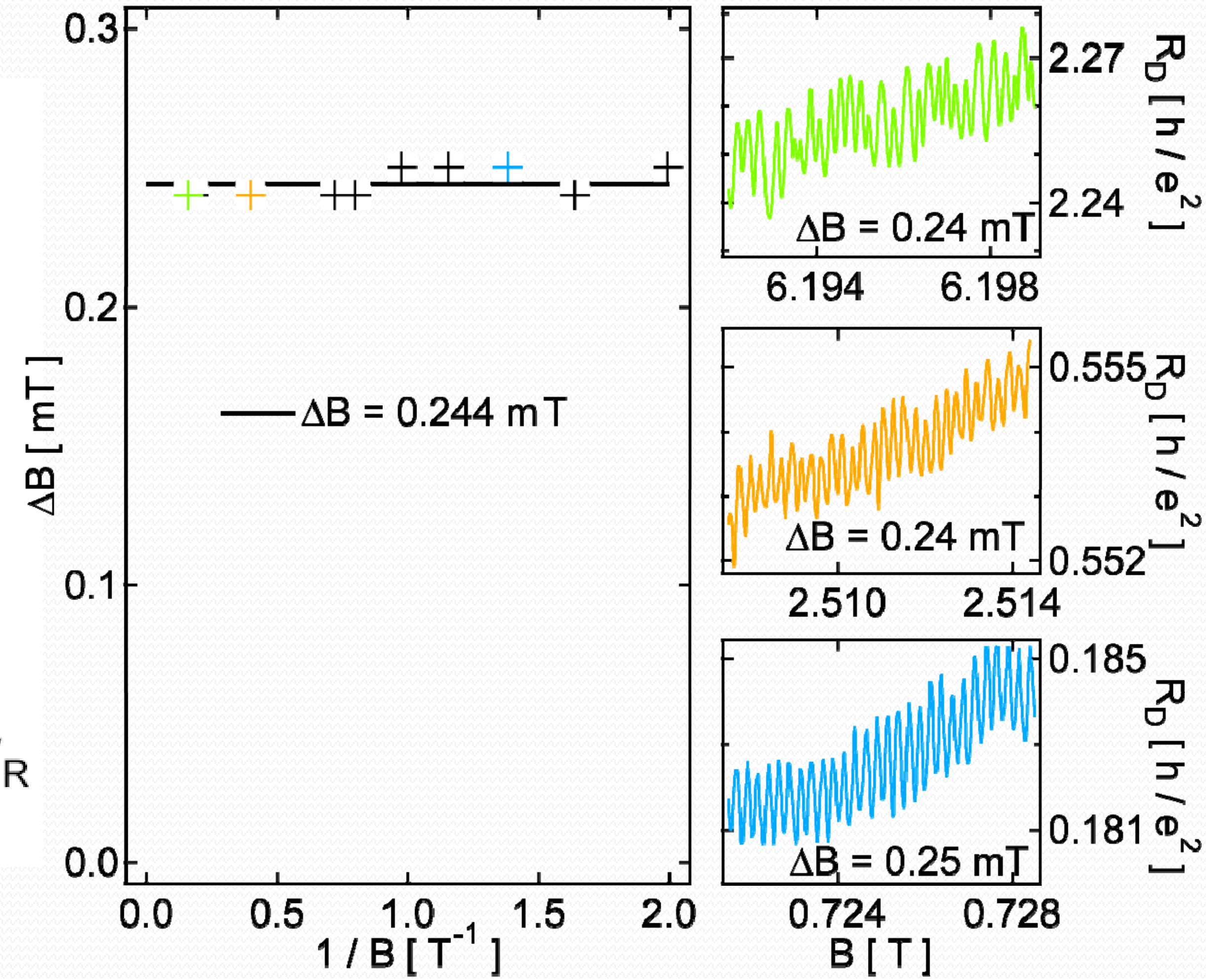
- Charging Energy:

$$E = \frac{e^2}{2C}(f_0 \cdot \delta BA/\Phi_0 + N - \alpha V_{\mathrm{gate}})^2$$

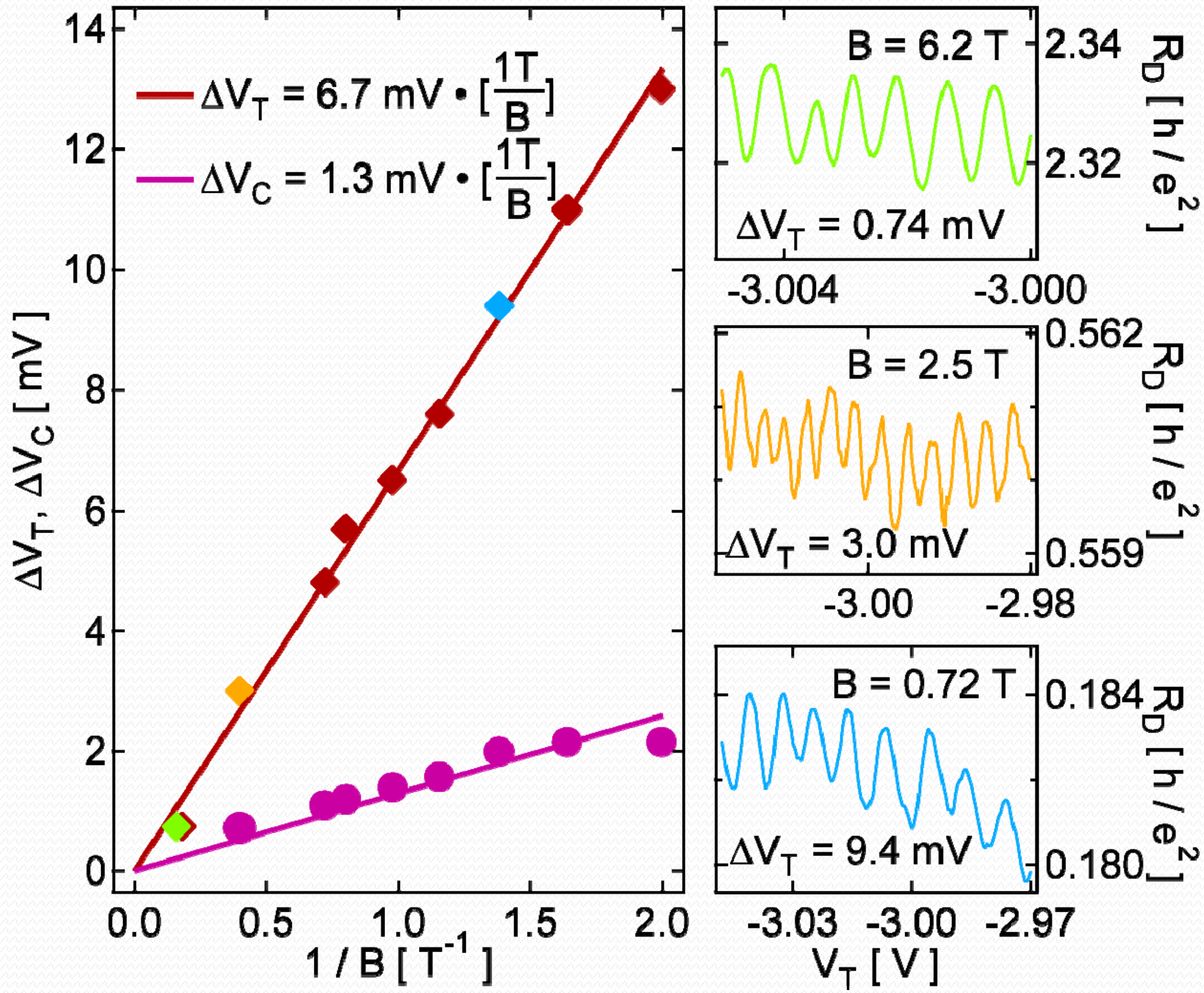
- Due to Coulomb blockade, *N* is an integer
- Expect: $\Delta B = \frac{1}{f_0}\frac{\Phi_0}{A}$ and $\Delta V_{\mathrm{gate}} = \mathrm{constant}$
- In excellent agreement with experiments

Data - 18 μm² device





Data - 18 μm² device



Interpretation – AB Interference

- The Aharonov-Bohm phase is given by:

$$\phi_{AB} = \frac{q}{\hbar}\oint \vec{A}\cdot d\vec{\ell} = \frac{q}{\hbar}\iint \vec{B}\cdot d\vec{s} = 2\pi\frac{\Phi}{h/q}$$

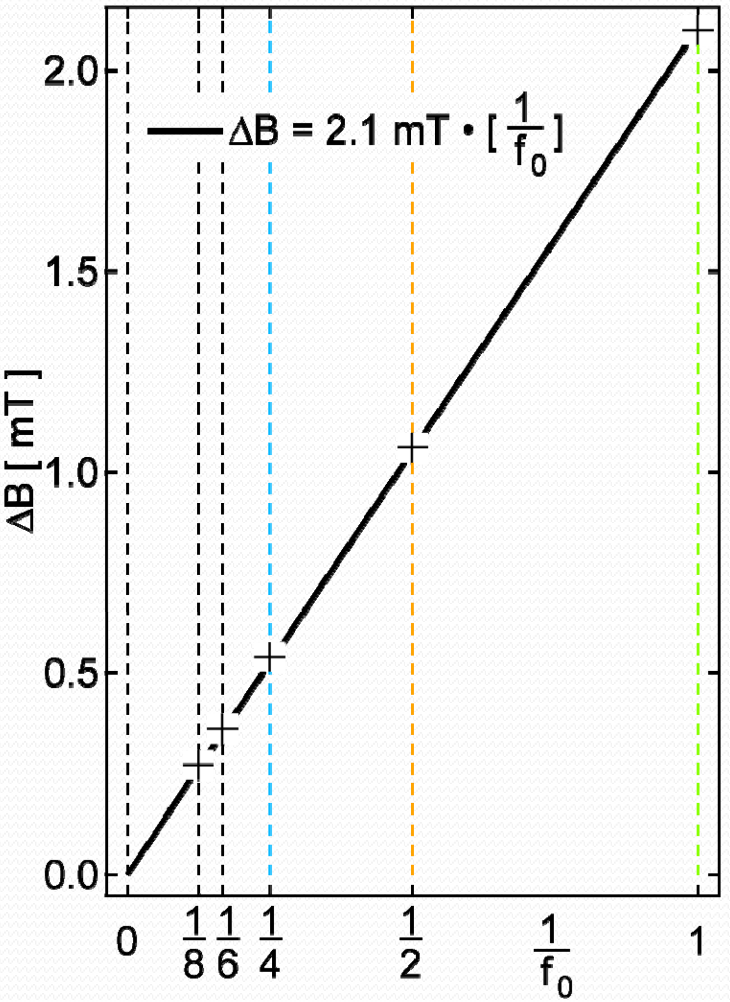
where $\Phi = BA$

- Therefore, $\Delta(BA) = \Phi_0$
  - With constant area: $\Delta B = \Phi_0/A$
  - With constant *B*: $\Delta A = \Phi_0/B$

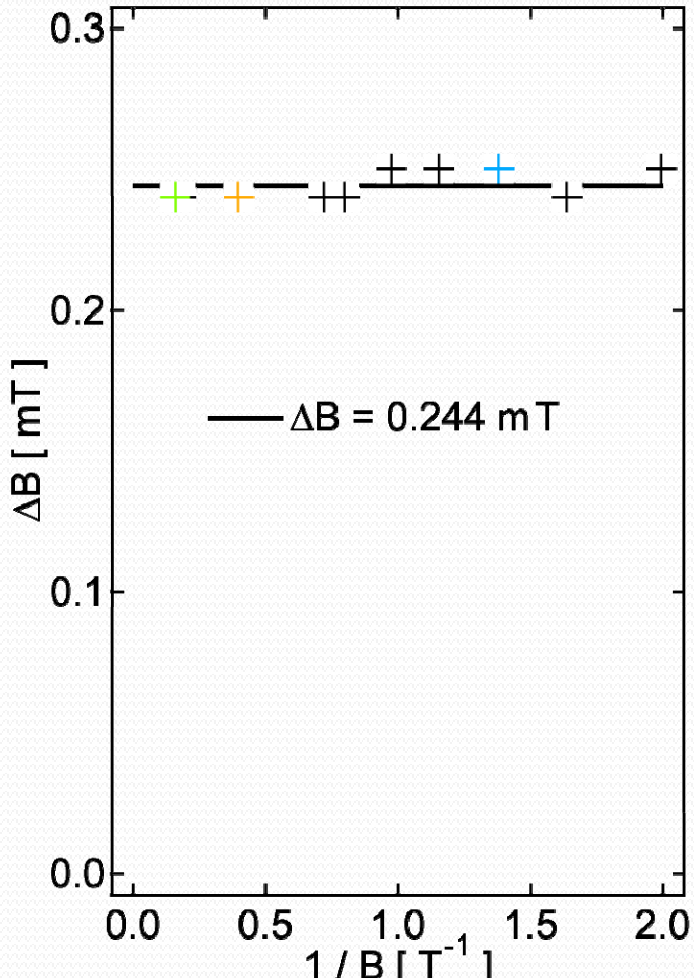
  Assuming $\Delta V_{\text{gate}} \propto \Delta A$: $\Delta V_{\text{gate}} \propto 1/B$

Comparison Between CB and AB

Coulomb blockade

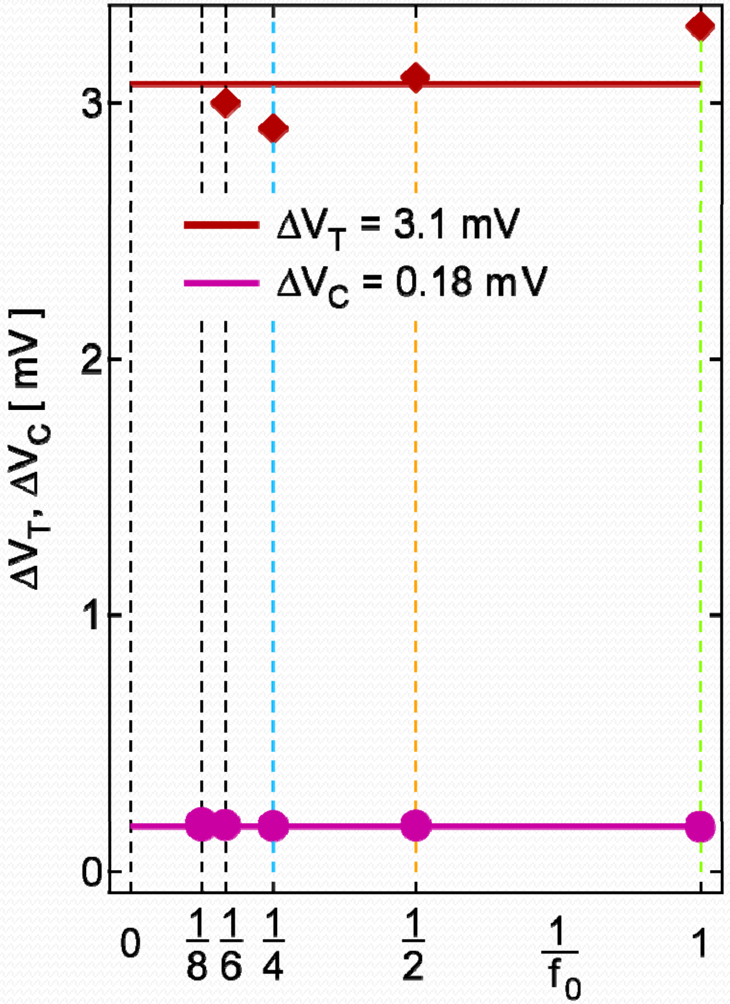


Aharonov-Bohm Interference

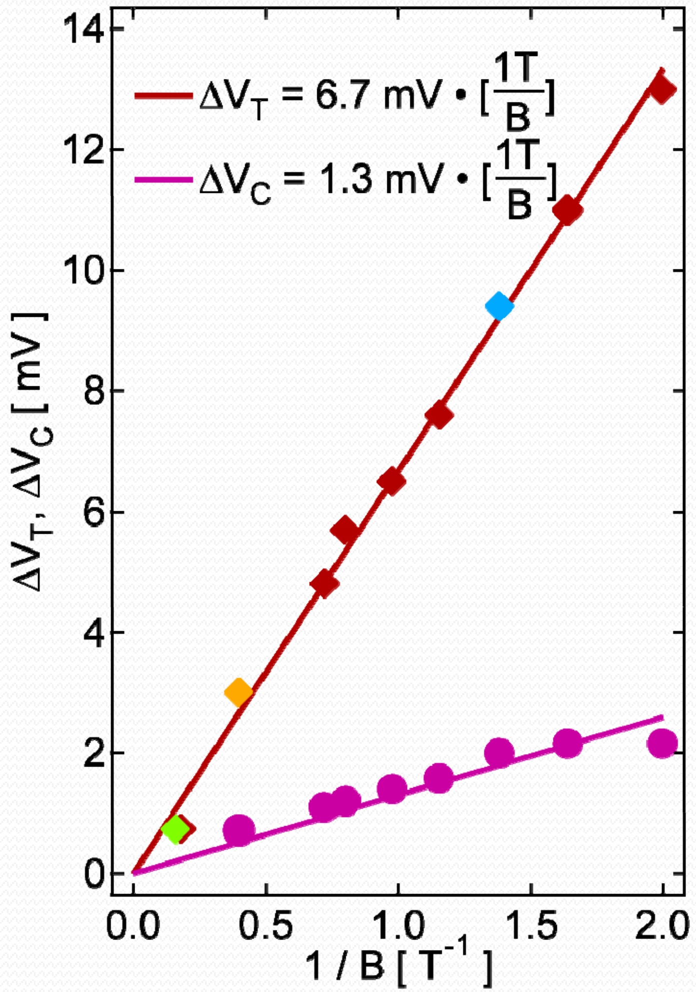


Comparison Between CB and AB

Coulomb blockade



Aharonov-Bohm Interference



Comparison Between CB and AB

There is one more predicted difference between them

- Recall for CB: $E = \frac{e^2}{2C}(f_0 \cdot \delta BA/\Phi_0 + N - \alpha V_{\mathrm{gate}})^2$

  Therefore, $\delta B \sim -\delta V_{\mathrm{gate}}$

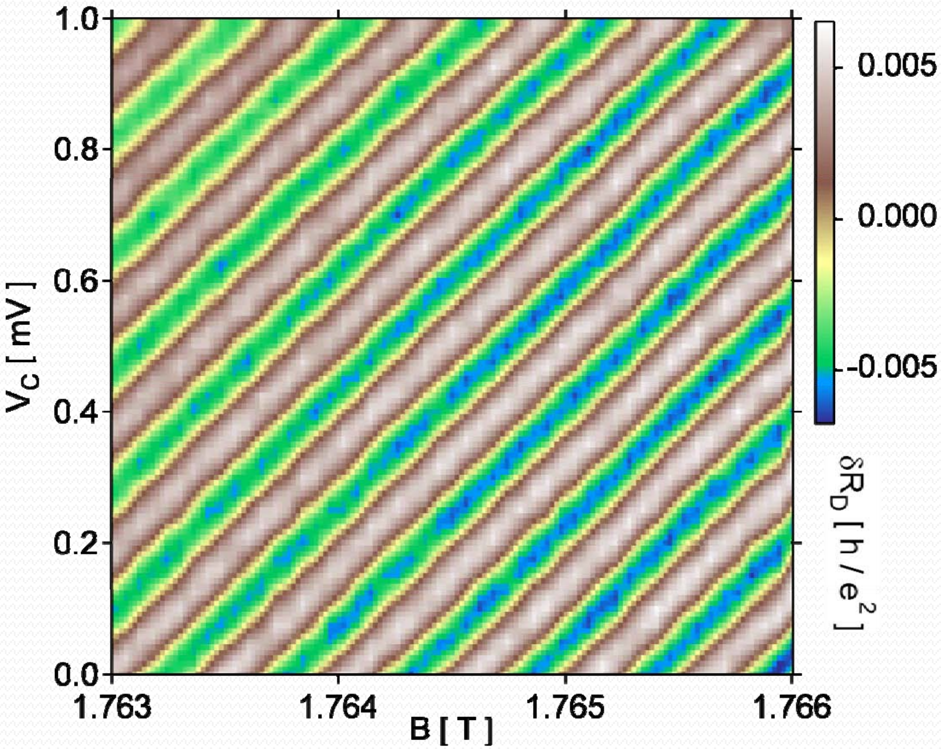
- For AB: $\phi_{AB} = 2\pi(BA)/\Phi_0$, and $\delta V_{\mathrm{gate}} \propto \delta A$

  Therefore, $\delta B \sim +\delta V_{\mathrm{gate}}$

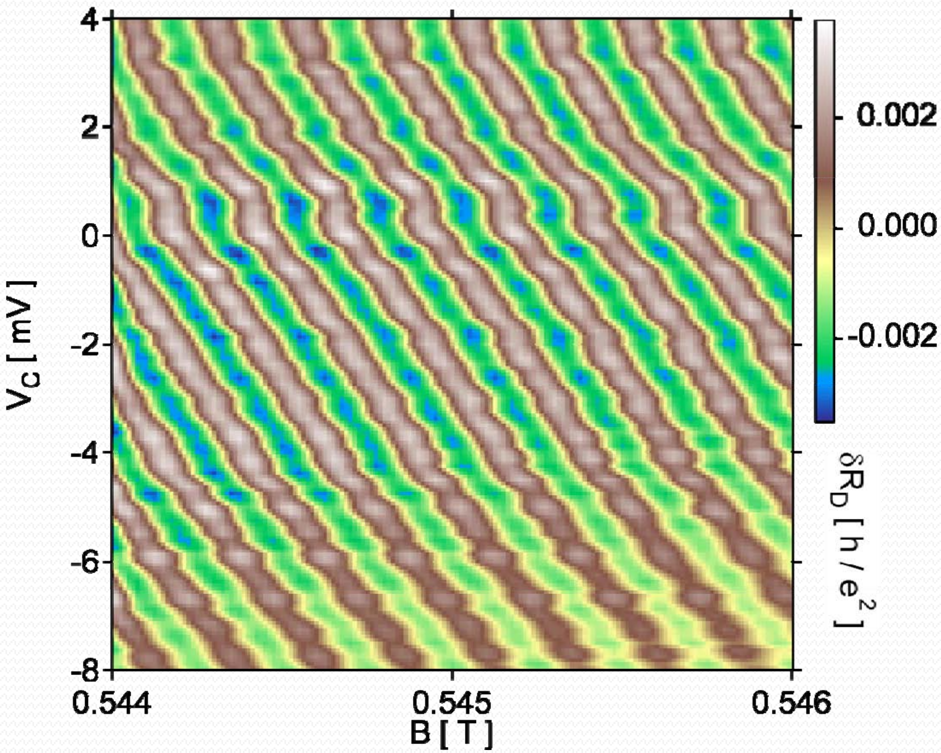
Comparion Between CB and AB

Measure in a 2D plane of *B* and V_C

Coulomb blockade



Aharonov-Bohm Interference



Previous Experiment in FQHE

PRL **98,** 076805 (2007) PHYSICAL REVIEW LETTERS week ending 16 FEBRUARY 2007

$e/3$ Laughlin Quasiparticle Primary-Filling $\nu = 1/3$ Interferometer

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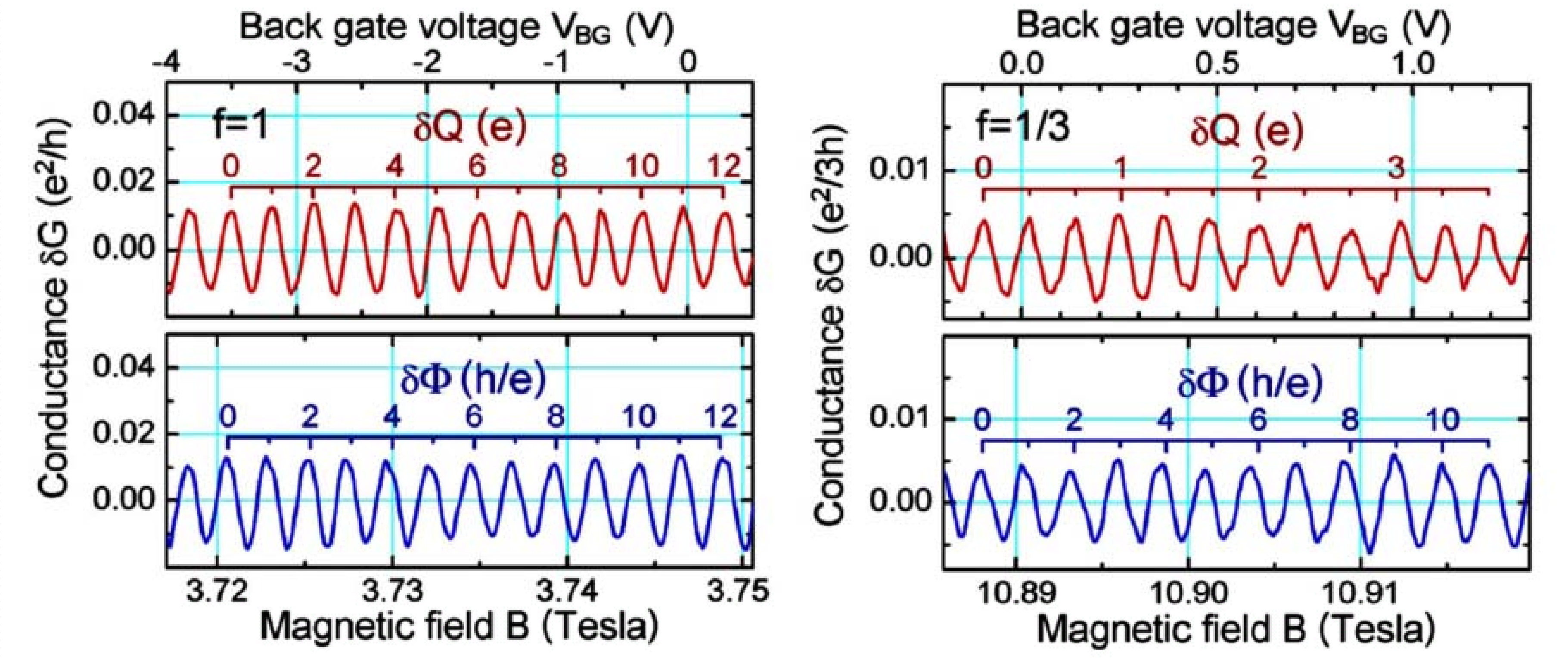
(Received 6 September 2006; published 15 February 2007)

We report experimental realization of a quasiparticle interferometer where the entire system is in $1/3$ primary fractional quantum Hall state. The interferometer consists of chiral edge channels coupled by quantum-coherent tunneling in two constrictions, thus enclosing an Aharonov-Bohm area. We observe magnetic flux and charge periods h/e and $e/3$, equivalent to the creation of one quasielectron in the island. Quantum theory predicts a $3h/e$ flux period for charge $e/3$, integer statistics particles. Thus, the observed periods demonstrate the anyonic braiding statistics of Laughlin quasiparticles.

Previous Experiment in FQHE



$$\Delta B(f = 1/3) = \Delta B(f = 1) = \Phi_0/A$$

$$\Delta V_{\rm BG}(f = 1/3) = (1/3)\Delta V_{\rm BG}(f = 1)$$

Previous Experiment in FQHE

At least two other possible interpretations:

- Integer Aharonov-Bohm interference

$\Delta B = \text{constant}$ $\Delta V_{\text{BG}} \propto 1/B$

- Coulomb blockade for 1/3 quasi-particles

$$E = \frac{1}{2C}(e f_0 \cdot \delta B A/\Phi_0 + N e^* - \alpha V_{\text{gate}})^2$$

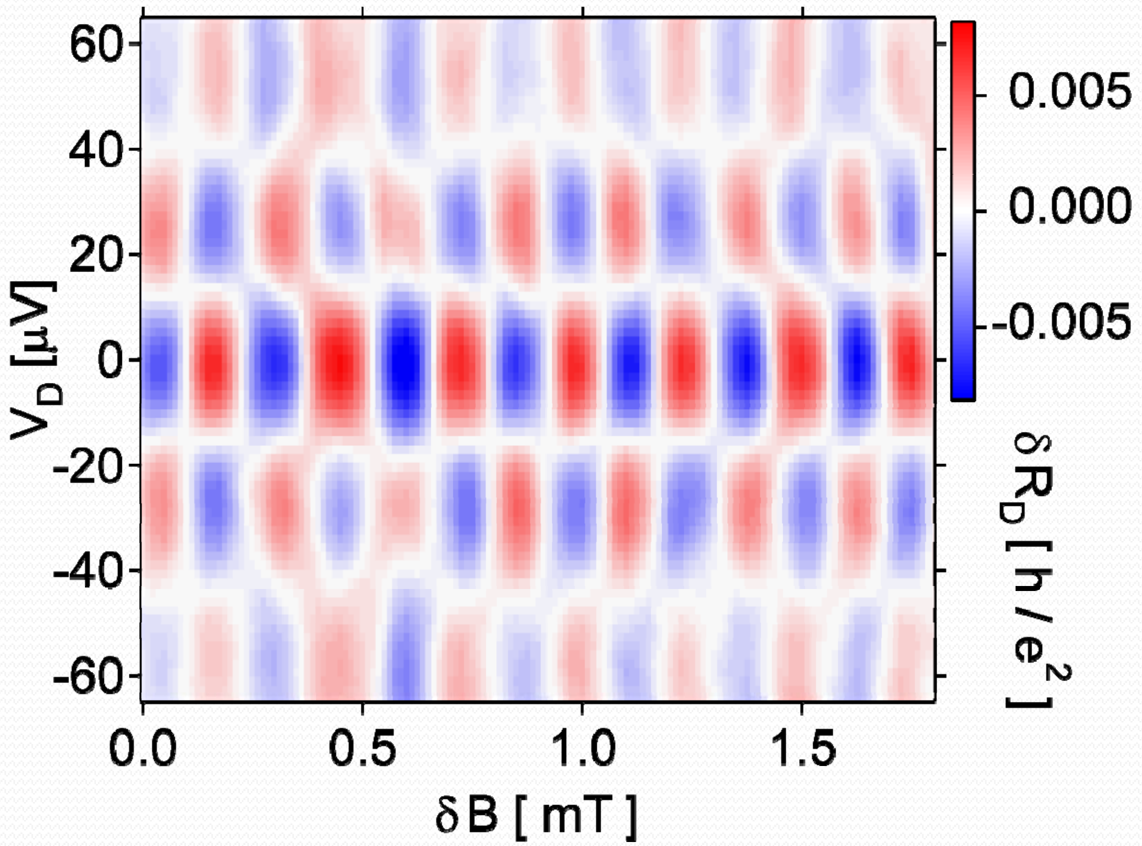
For $f_0 = 1/3$ and $e^* = e/3$, also expect:

$$\Delta B(f = 1/3) = \Delta B(f = 1) = \Phi_0/A$$
$$\Delta V_{\text{BG}}(f = 1/3) = (1/3)\Delta V_{\text{BG}}(f = 1)$$

➔ Need to know which type of oscillation here

Non-linear Regime

The 18 μm² device: checkerboard pattern



Non-linear Regime

PHYSICAL REVIEW B VOLUME 55, NUMBER 4 15 JANUARY 1997-II

Two point-contact interferometer for quantum Hall systems

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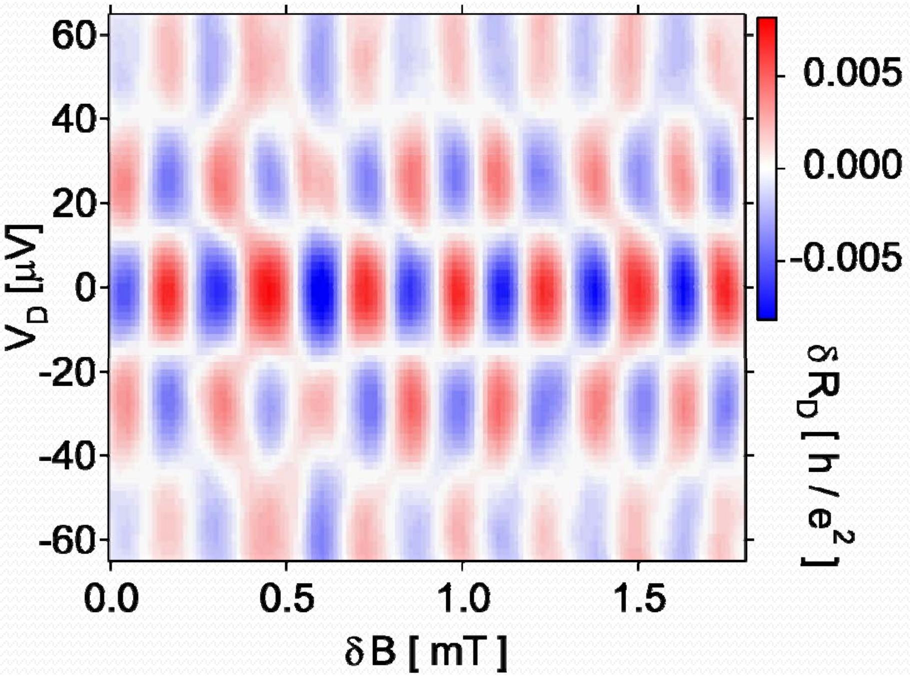
(Received 29 July 1996)

Non-linear Regime

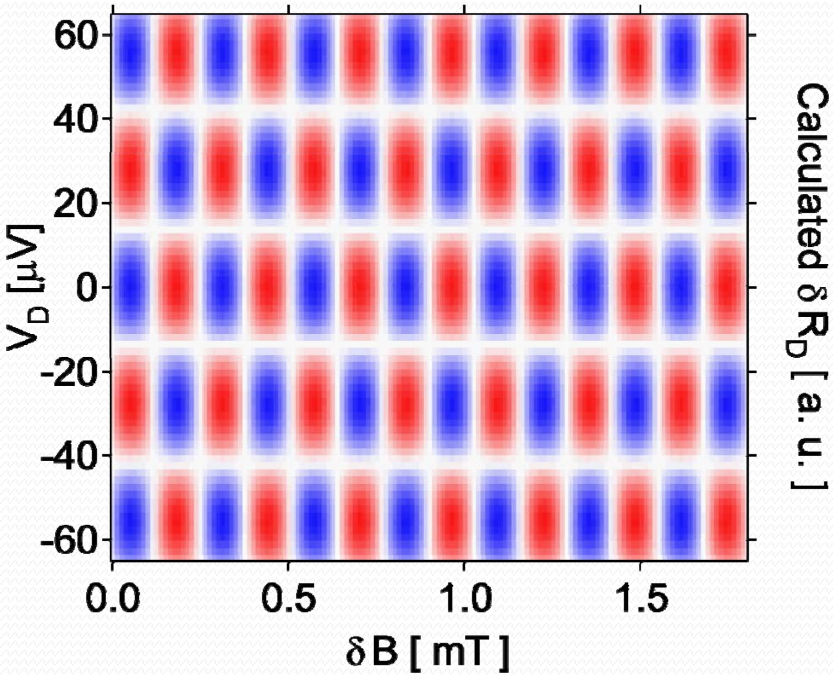
$$dI_t/dV_D \propto \cos(2\pi\delta BA/\Phi_0)\cos(2\pi V_D/\Delta V_D)$$

$$v = ae\Delta V_D/h$$

Data



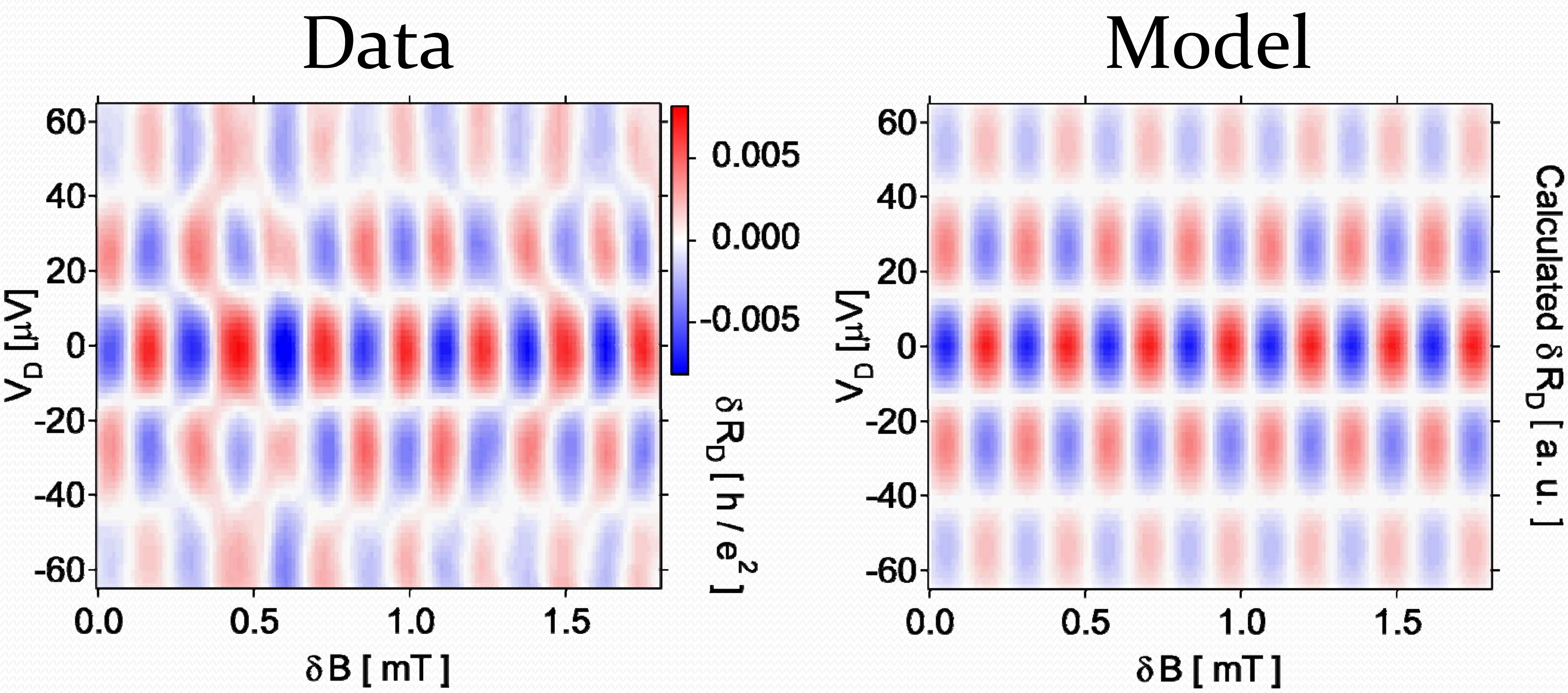
Model



Non-linear Regime

Add damping at high-bias:

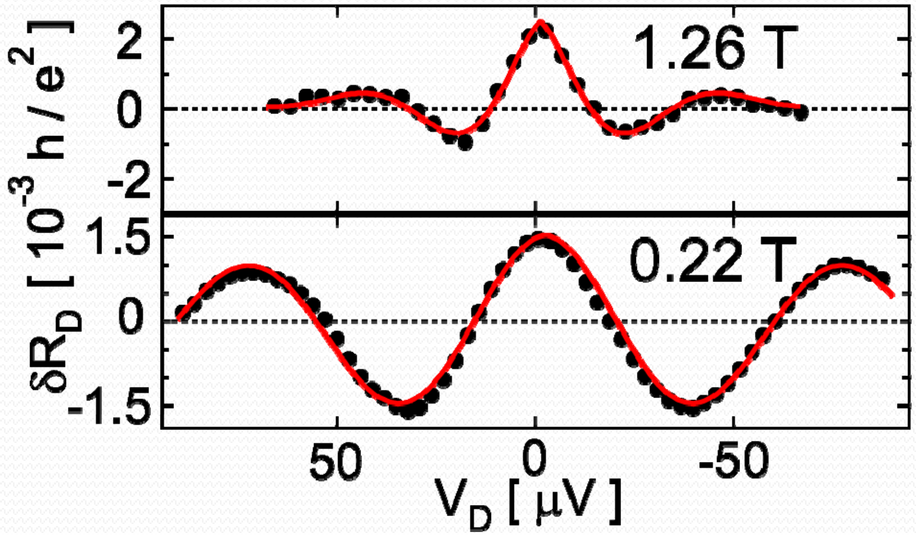
$$dR_{\mathrm{D}} \propto \cos(2\pi\delta BA/\Phi_0)\cos(2\pi V_{\mathrm{D}}/\Delta V_{\mathrm{D}})\exp(-2\pi\alpha|V_{\mathrm{D}}|/\Delta V_{\mathrm{D}})$$

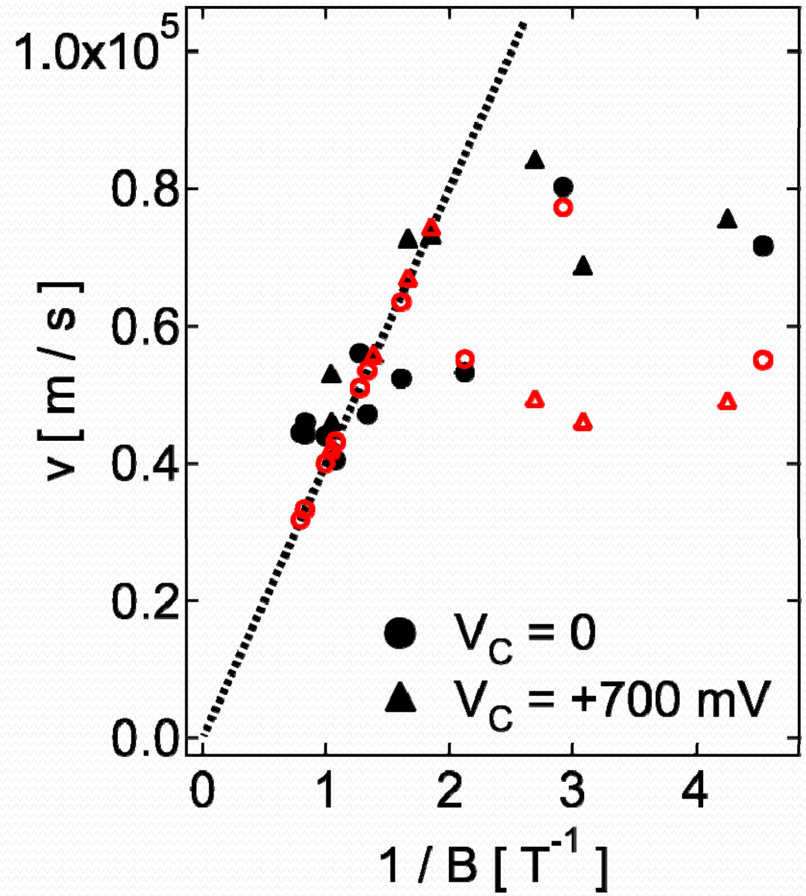


Non-linear Regime

$$dR_\mathrm{D} \propto \cos(2\pi V_\mathrm{D}/\Delta V_\mathrm{D})\exp(-2\pi\alpha|V_\mathrm{D}|/\Delta V_\mathrm{D})$$

$$v = ae\Delta V_\mathrm{D}/h$$





Non-linear Regime

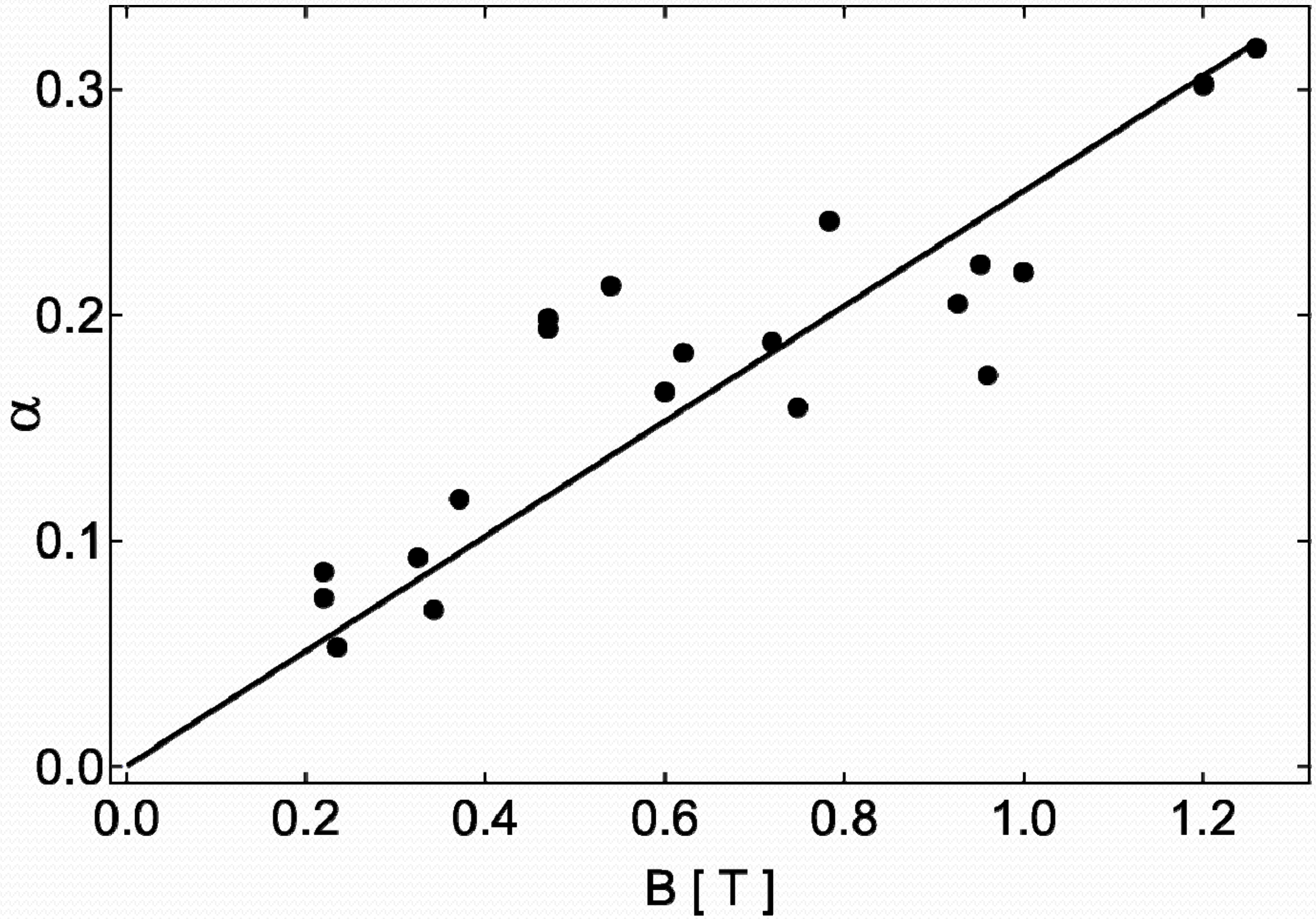
The simple model for velocity has considered only two limits:

- In the high-field limit, cyclotron radius is much smaller than the confining potential variation, thus edge velocity is the drift velocity: $v_{\mathrm{D}} = E/B$
- In the low-field limit, model the confining potential as hard wall, edge velocity is skipping-orbit velocity:

$$v_{\mathrm{S}} = \frac{\sqrt{2}\Gamma(N+1/2)(2N+1)}{\pi\Gamma(N+1)}\omega_{\mathrm{C}}\ell_{\mathrm{B}} \sim \omega_{\mathrm{C}} r_{\mathrm{C}}$$

Non-linear Regime

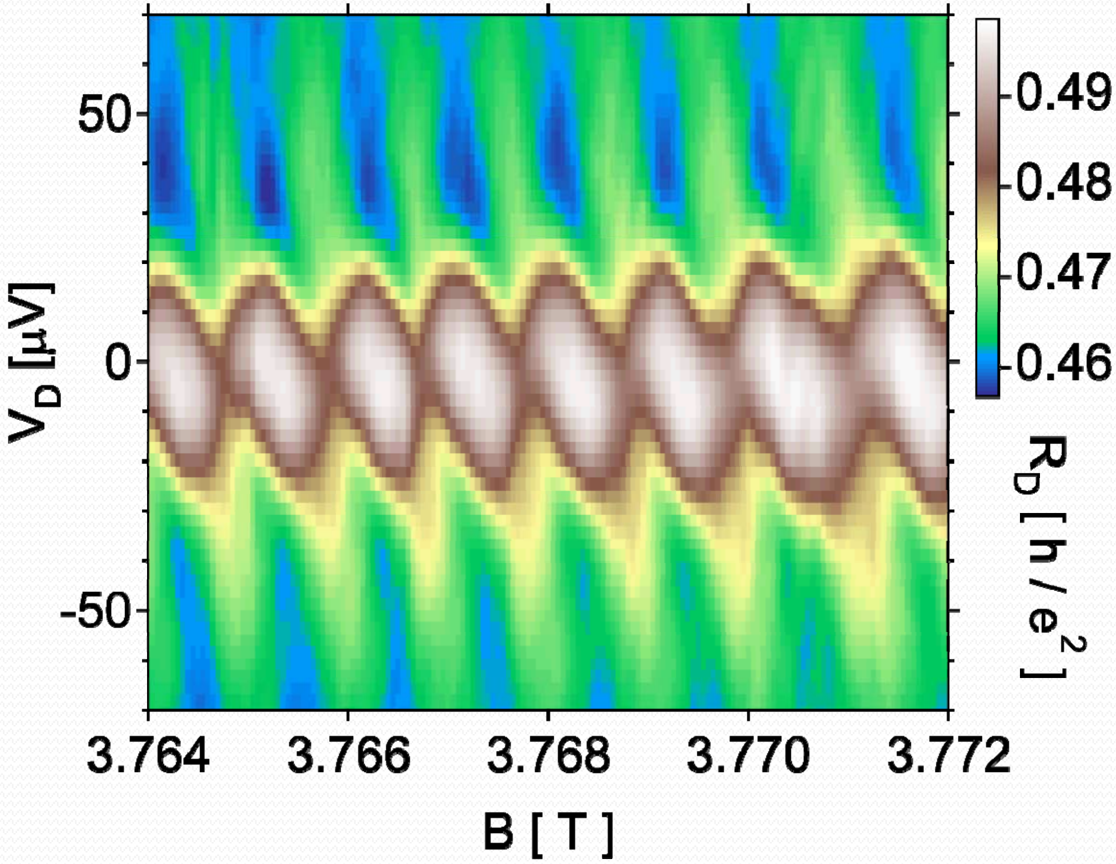
The damping factor α:



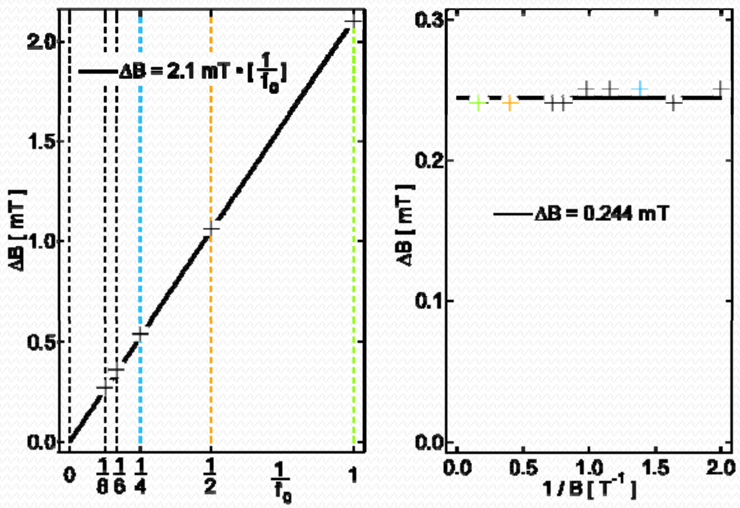
Model?

Non-linear Regime

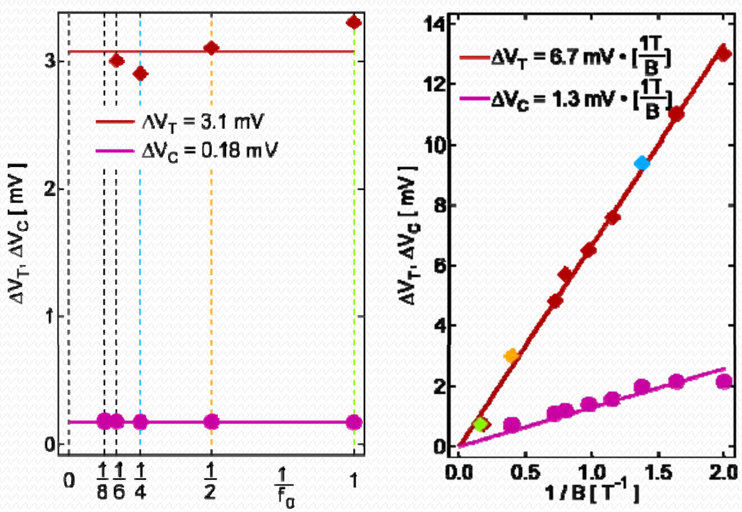
The 2 μm² device: diamond pattern



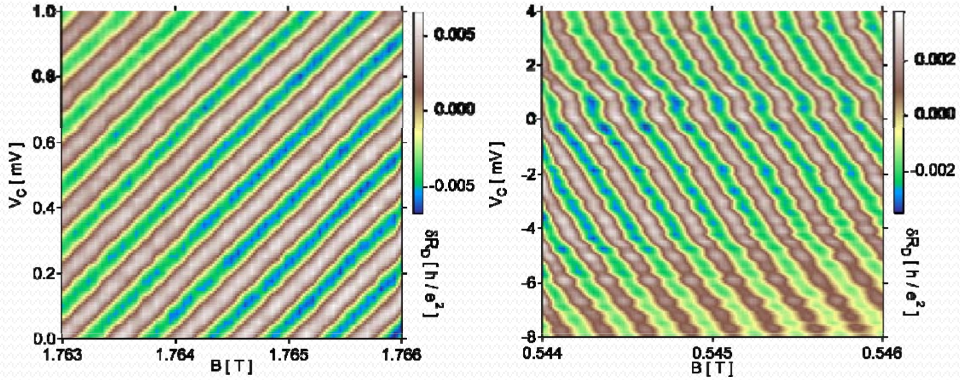
Conclusion



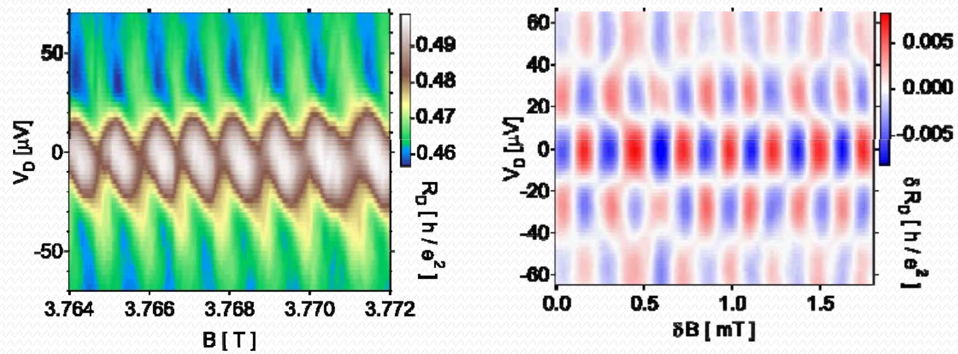
ΔB = 2.1 mT • [1/f0]
ΔB [mT]
1/f0
ΔB = 0.244 mT
ΔB [mT]
1 / B [T^-1]



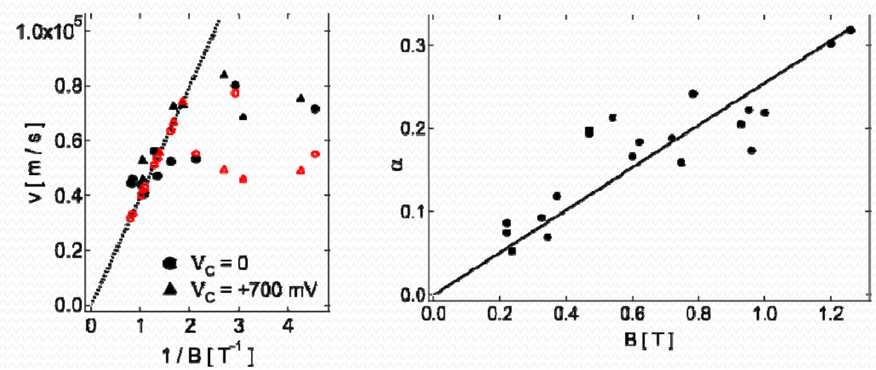
ΔVT = 3.1 mV
ΔVC = 0.18 mV
ΔVT, ΔVC [mV]
1/f0
ΔVT = 6.7 mV • [1T/B]
ΔVC = 1.3 mV • [1T/B]
ΔVT, ΔVC [mV]
1 / B [T^-1]



VC [mV]
B [T]
δRD [h / e^2]
VC [mV]
B [T]
δRD [h / e^2]



VD [μV]
B [T]
RD [h / e^2]
VD [μV]
δB [mT]
δRD [h / e^2]



v [m / s]
1 / B [T^-1]
VC = 0
VC = +700 mV
α
B [T]