Non-Abelian Hall Sates in High Landau Levels and Atomic Bose Gases

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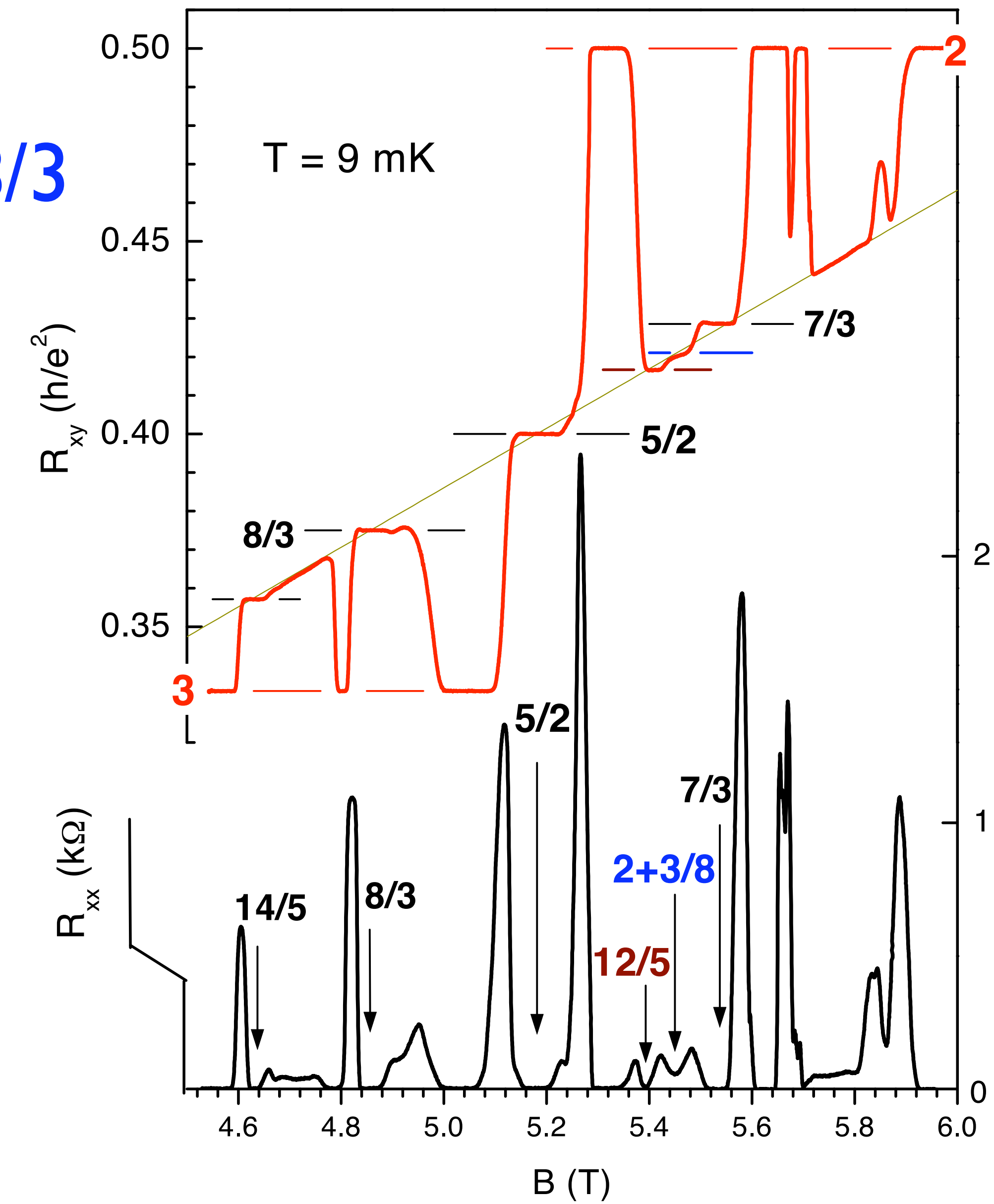
Outline

- Hall states in the first excited Landau level
- Moore-Read paired state for 5/2 (edge + bulk)
- generalizations of pairing to k-particle groupings (RR 1999)
- k=3 and k=4 in rapidly rotating atomic Bose gases with dipolar interactions
- k=3 for fermions

First excited Landau Level

- 5/2 is quantized
- Strongest or as strong as 8/3
- Not a 2nd generation or daughter state
- Reentrant phases
- Weak 12/5
- PH symmetry violation (LL mixing?)
- Apparent formation of 2+3/8

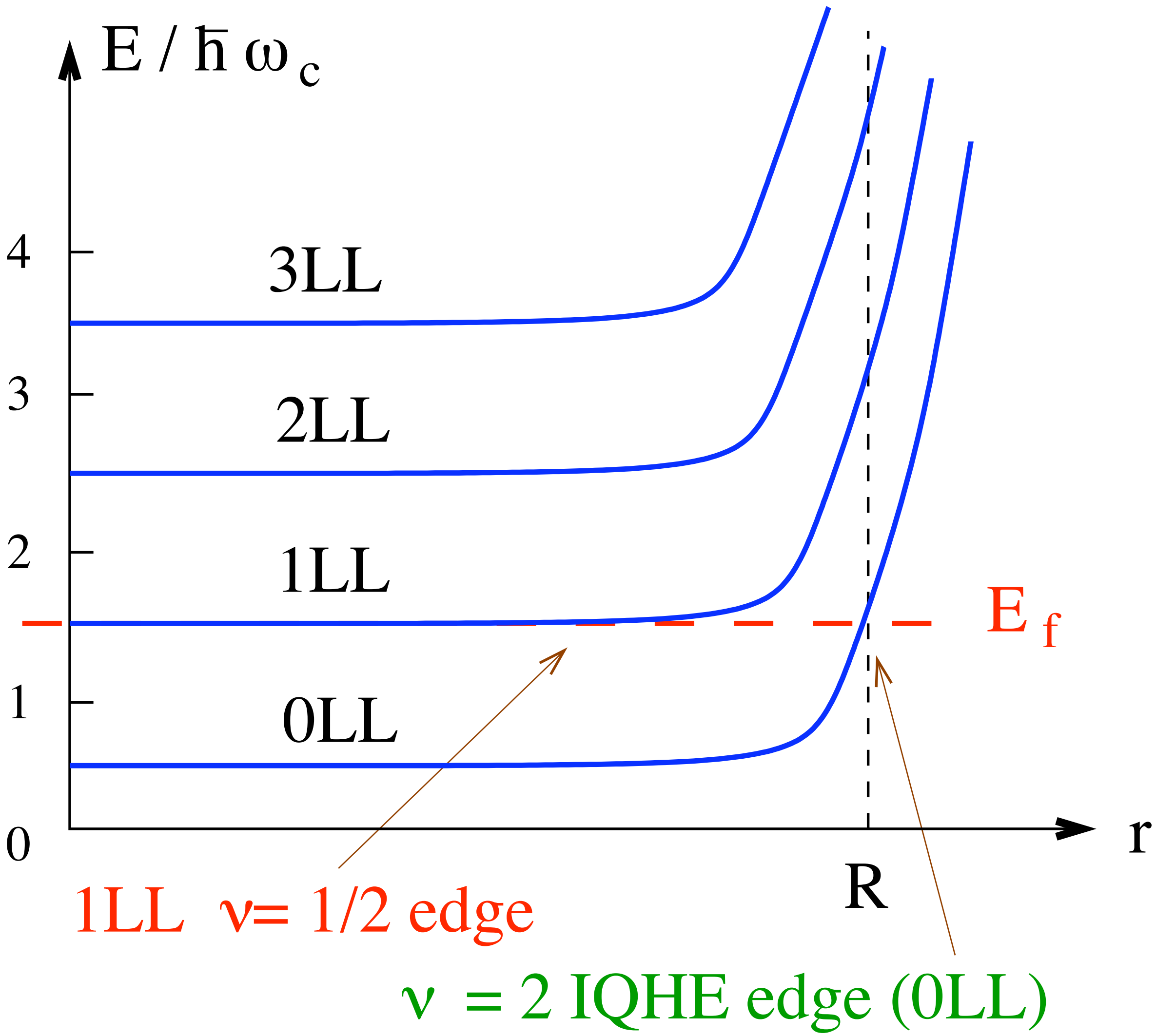


Data of Xia *et. al.*

Edge Modes of 5/2

with Wan, Hu and Yang

PRB 2008



MR wave functions

$$\prod_{i<j}(z_i - z_j)^2 \mathrm{Pf} \ \frac{1}{z_i - z_j} \qquad z = x + iy \quad M_{MR} = \frac{N(2N-3)}{2}$$

MR state on disk (drop the exponential factor)

$$\prod_{i<j}(u_i v_j - v_i u_j)^2 \mathrm{Pf}\frac{1}{u_i v_j - v_i u_j}$$

On the sphere

$$(u, v) = (\cos\theta/2 \exp i\phi/2, \sin\theta/2 \exp -i\phi/2)$$

$$\prod_{i<j}(z_i - z_j)^2 \mathrm{Pf}\frac{z_i + z_j}{z_i - z_j}$$

e/4 non-Abelian quasi-hole

$$\prod_i z_i \prod_{i<j}(z_i - z_j)^2 \mathrm{Pf} \ \frac{1}{z_i - z_j}$$

$e/2$ Laughlin quasi-hole

Generalizing pairs to groupings of k particles

$$\Psi_{LJ} = \prod_{i<j}^{N} (z_i - z_j)^2$$ Boson Laughlin state at 1/2

$$\Psi_{MR} = S\{\prod_{i<j}^{N/2} (u_i - u_j)^2 \prod_{i<j}^{N/2} (v_i - v_j)^2\}$$ at 1/2+1/2=1

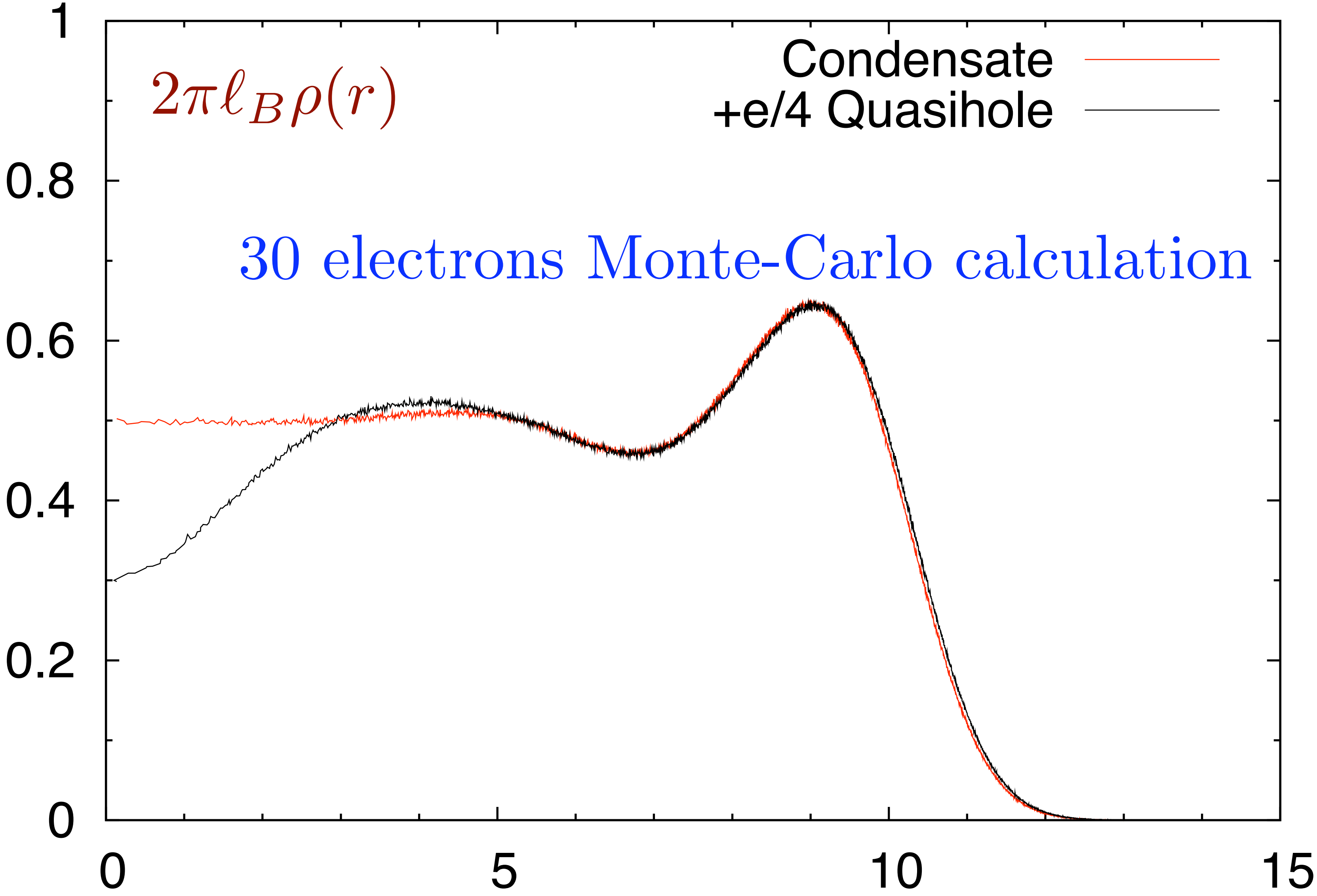
S symmetrizes *u*, *v* and *w*'s to z's

$$\Psi_{RR} = S\{\prod_{i<j}^{N/3} (u_i - u_j)^2 \prod_{i<j}^{N/3} (v_i - v_j)^2 \prod_{i<j}^{N/3} (w_i - w_j)^2\}$$

at filling 3/2

No more than k particles can have same coordinate
filling factor=k/2

$$\prod_{i<j}(z_i - z_j)^2 \mathrm{Pf}\frac{z_i + z_j}{z_i - z_j} \qquad M = M_{MR} + N/2$$



The Hamiltonian

$$H = \lambda H_{3-body} + (1-\lambda) H_{Coul} + H_{conf} + H_{probe}$$

$H_{3-body}|\Psi_{MR}\rangle = 0$ otherwise positive definite operator

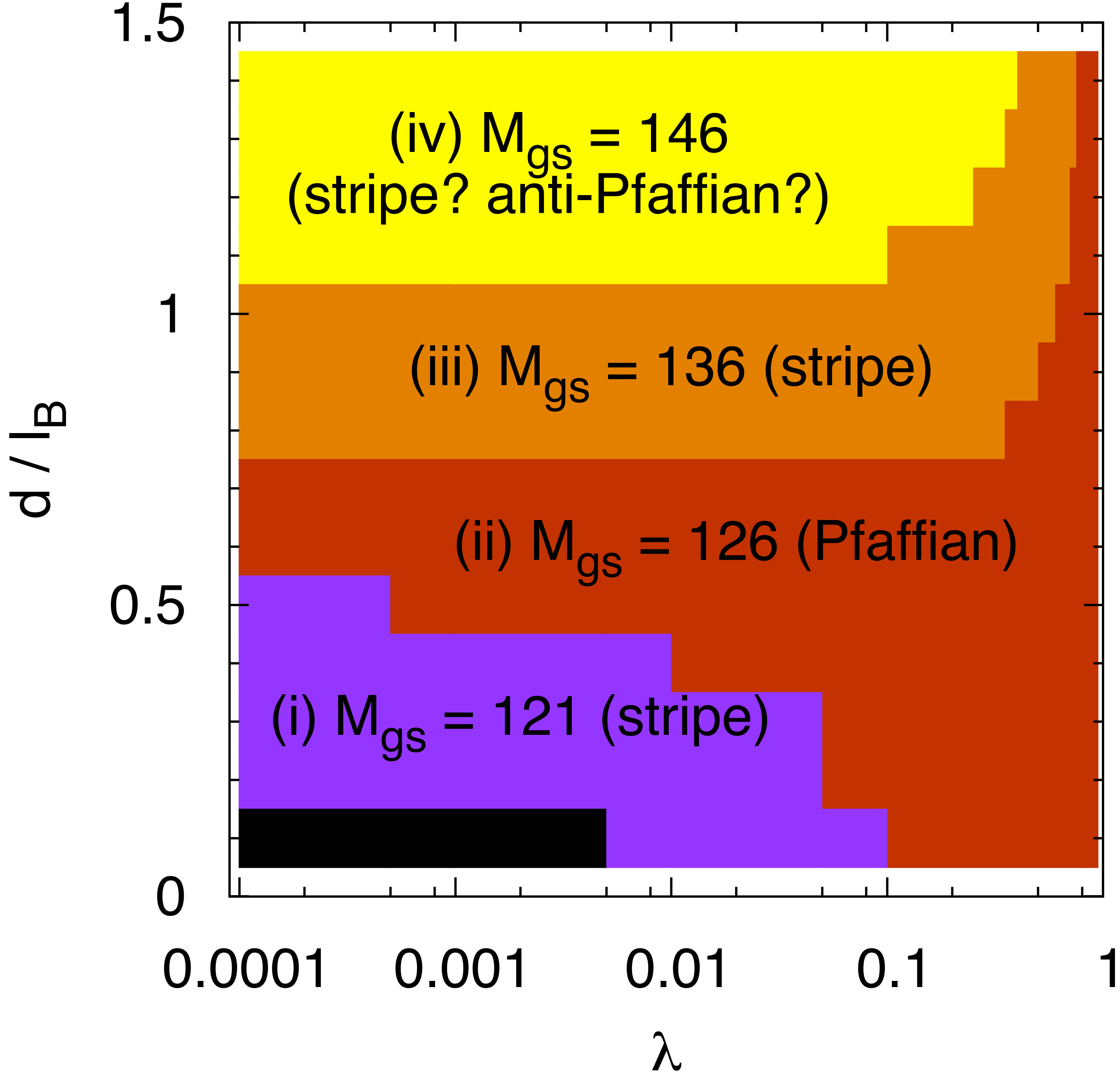
H_{Coul} is the Coulomb potential in the first excited LL

H_{conf} Neutralizing charge a distance d from the 2-d layer.
Edge confining potential

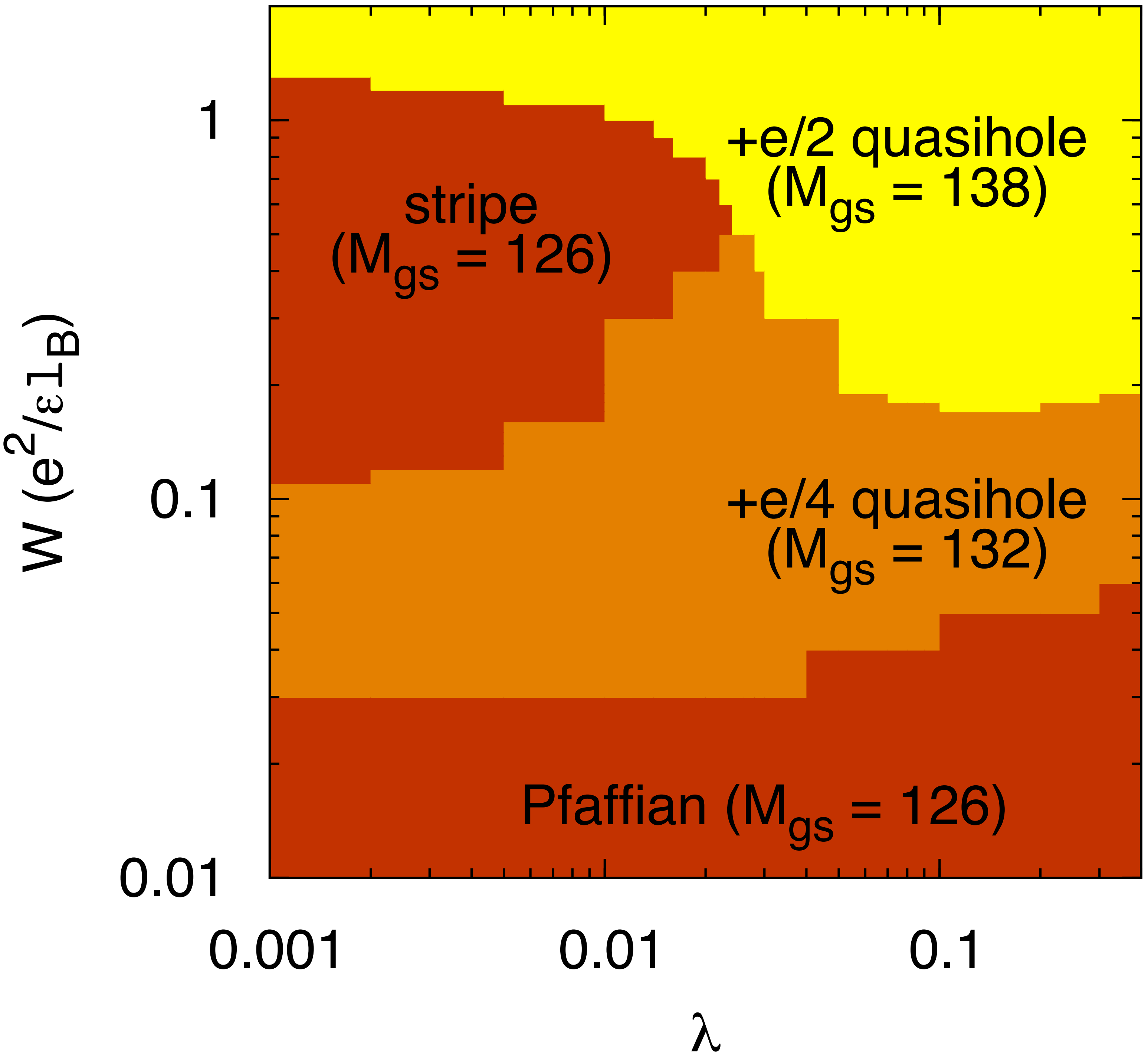
$$H_{probe} = W \sum_m \exp\{-\frac{m^2}{2\sigma^2}\} c_m^\dagger c_m$$

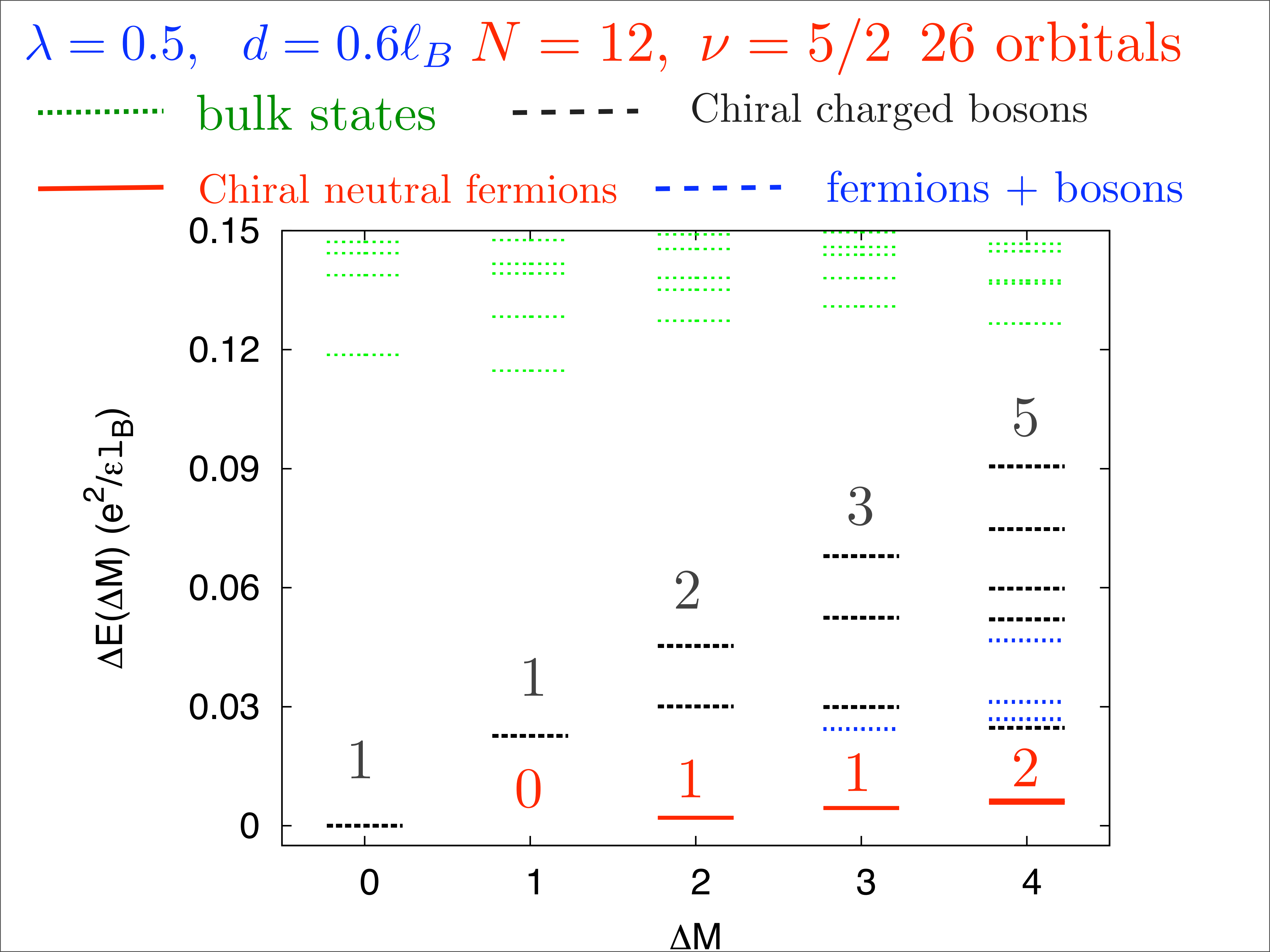
To nucleate and localize $e/4$ quasi-particles

12 electrons in 22 orbitals



12 electrons in 22 orbitals



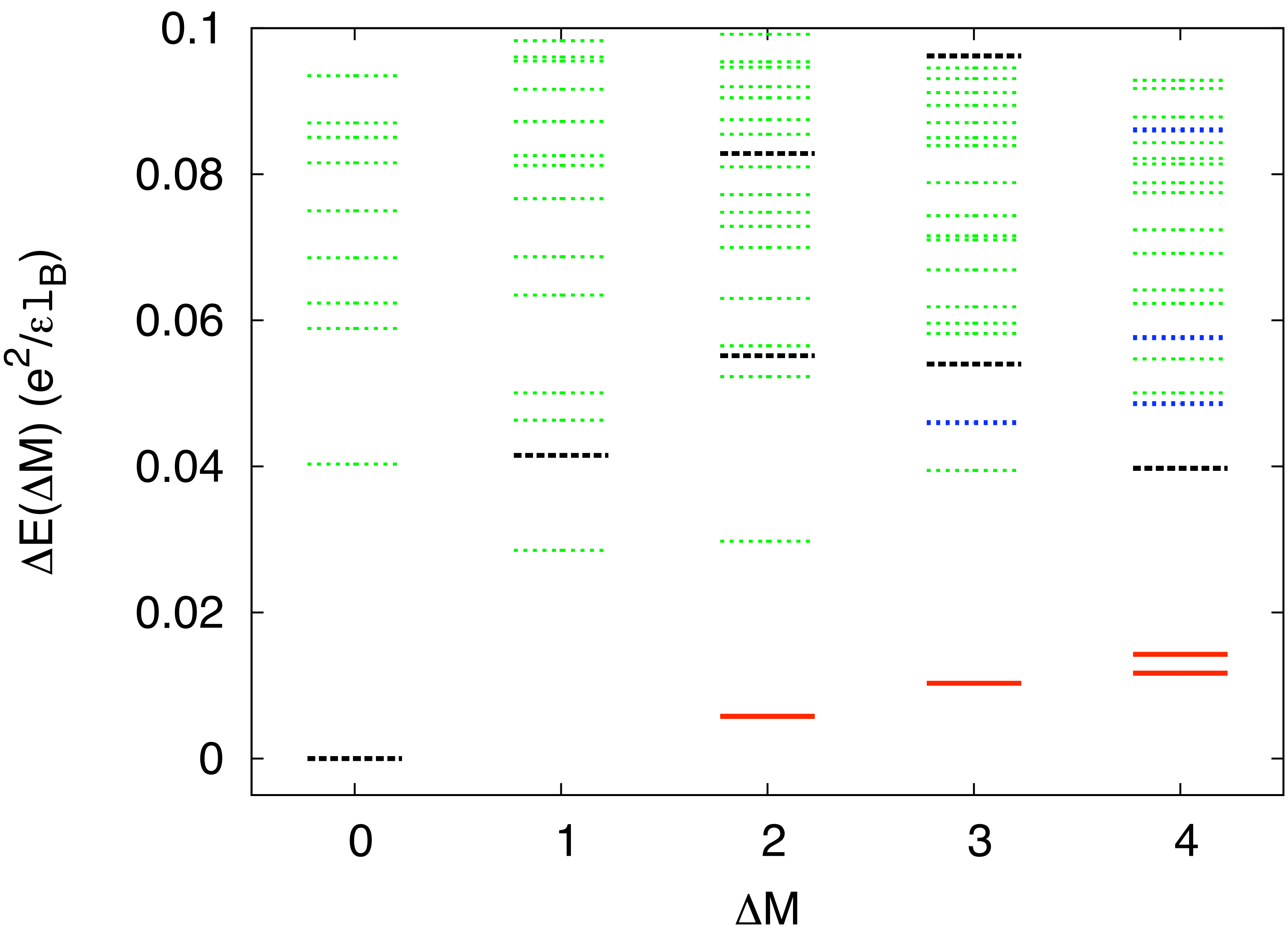


$\lambda = 0.5, \quad d = 0.6\ell_B$ $N = 12, \ \nu = 5/2$ 26 orbitals
bulk states
Chiral charged bosons
Chiral neutral fermions
fermions + bosons
0.15
0.12
0.09
0.06
0.03
0
ΔE(ΔM) (e²/ε1_B)
0
1
2
3
4
ΔM
1
1
0
2
1
3
1
5
2

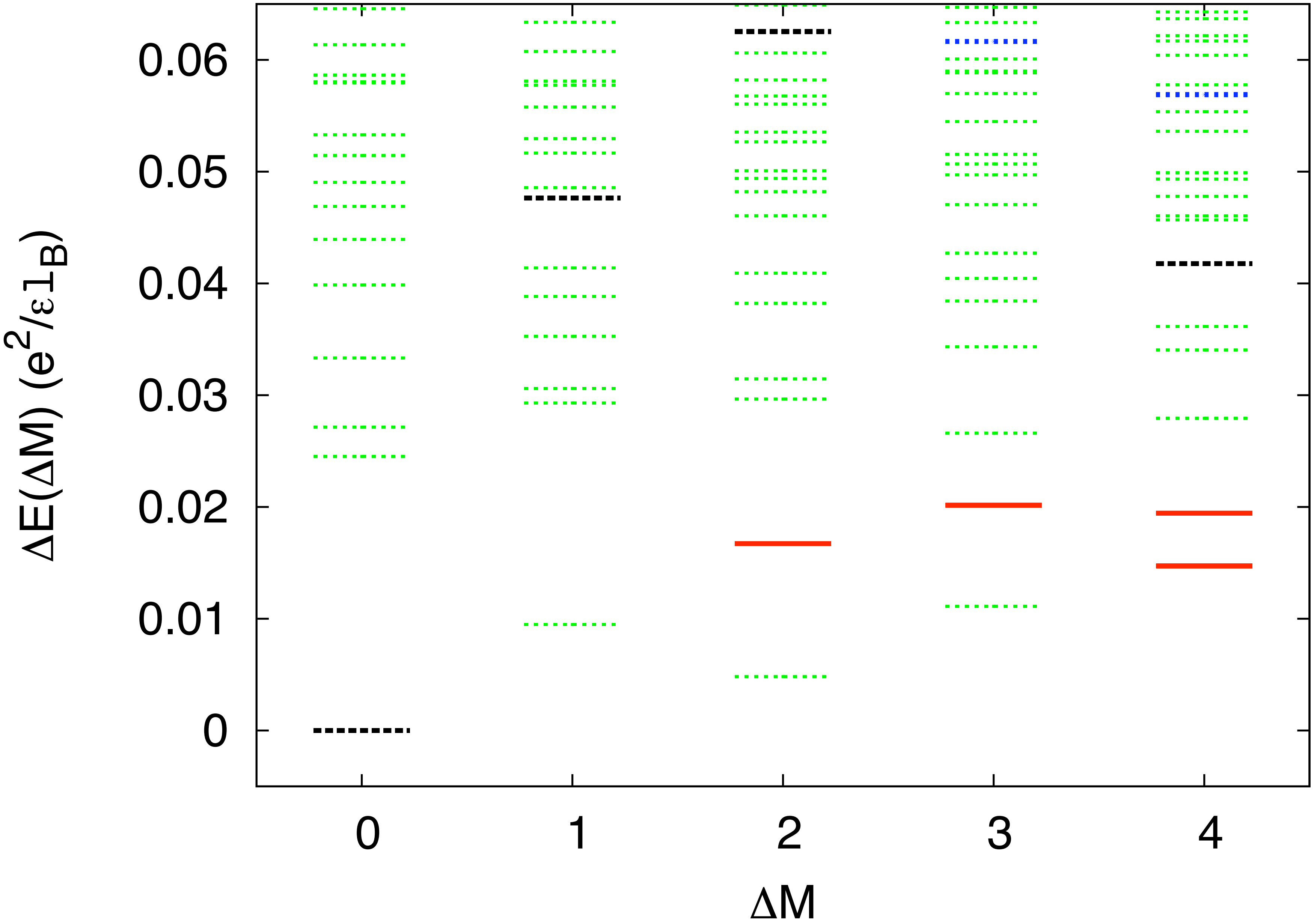
$\lambda = 0.1, \quad d = 0.6\ell_B$ $N = 12, \; \nu = 5/2$ 26 orbitals

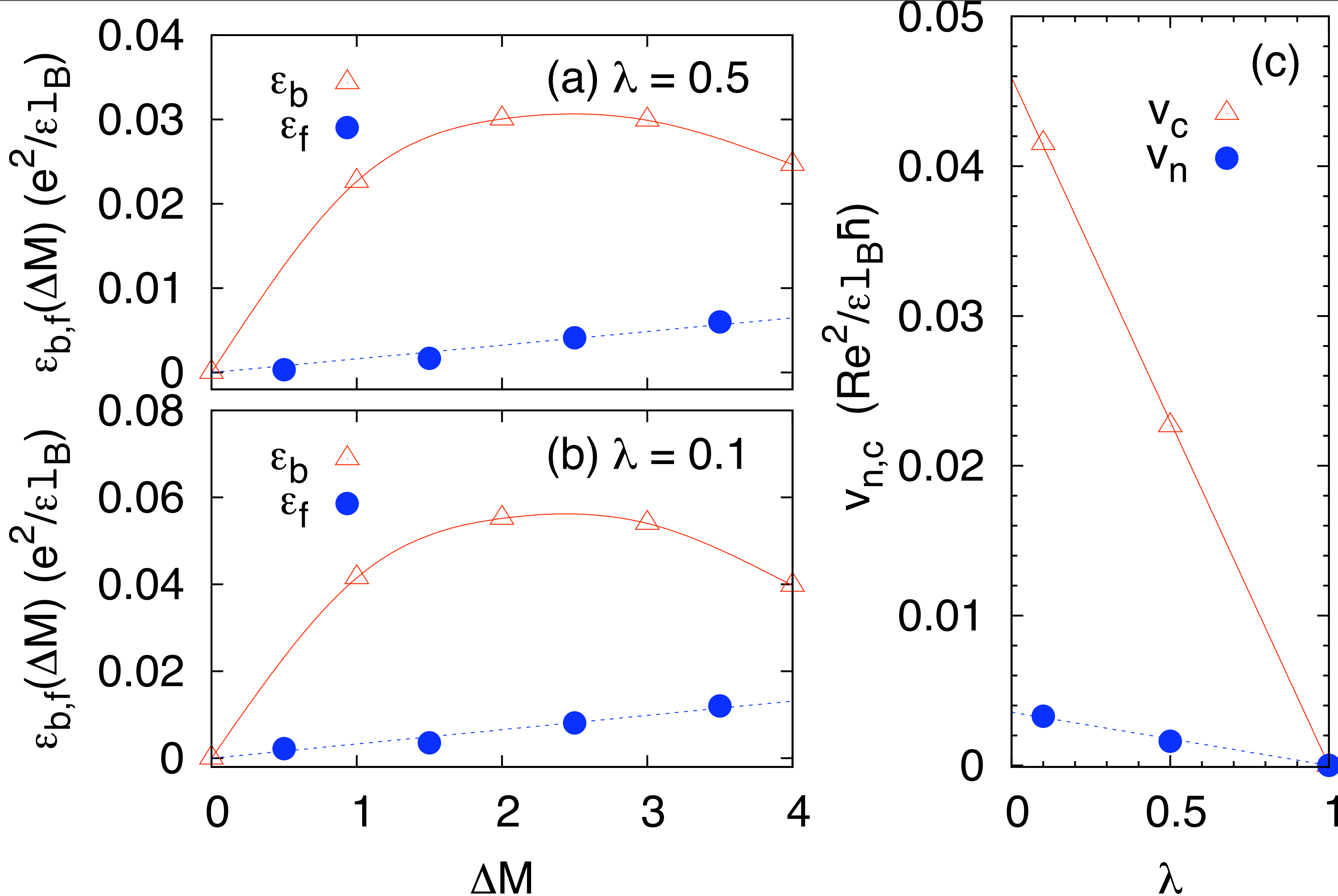
bulk states — Chiral charged bosons

Chiral neutral fermions — fermions + bosons



Pure Coulomb $\lambda = 0$





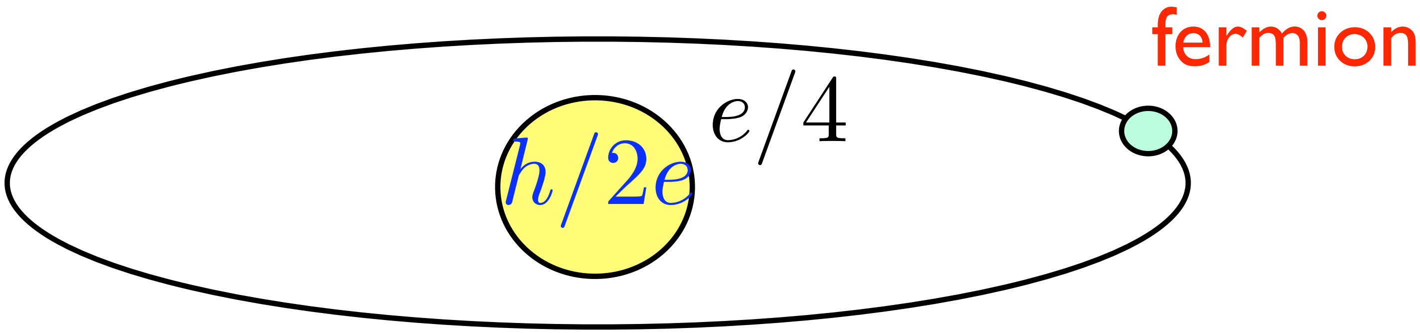
$$\Delta M = \sum_{l_B} n_B(l_B) l_B + \sum_{l_F} n_F(l_F) l_F$$

$$\Delta E = \sum_{l_B} n_B(l_B)\epsilon_B(l_B) + \sum_{l_F} n_F(l_F)\epsilon_F(l_F)$$

Non-interacting bosons and fermions

Non-Abelian statistics Odd number of quasi-holes in the bulk

Milovanovic and Read (1996), Wen (1993)



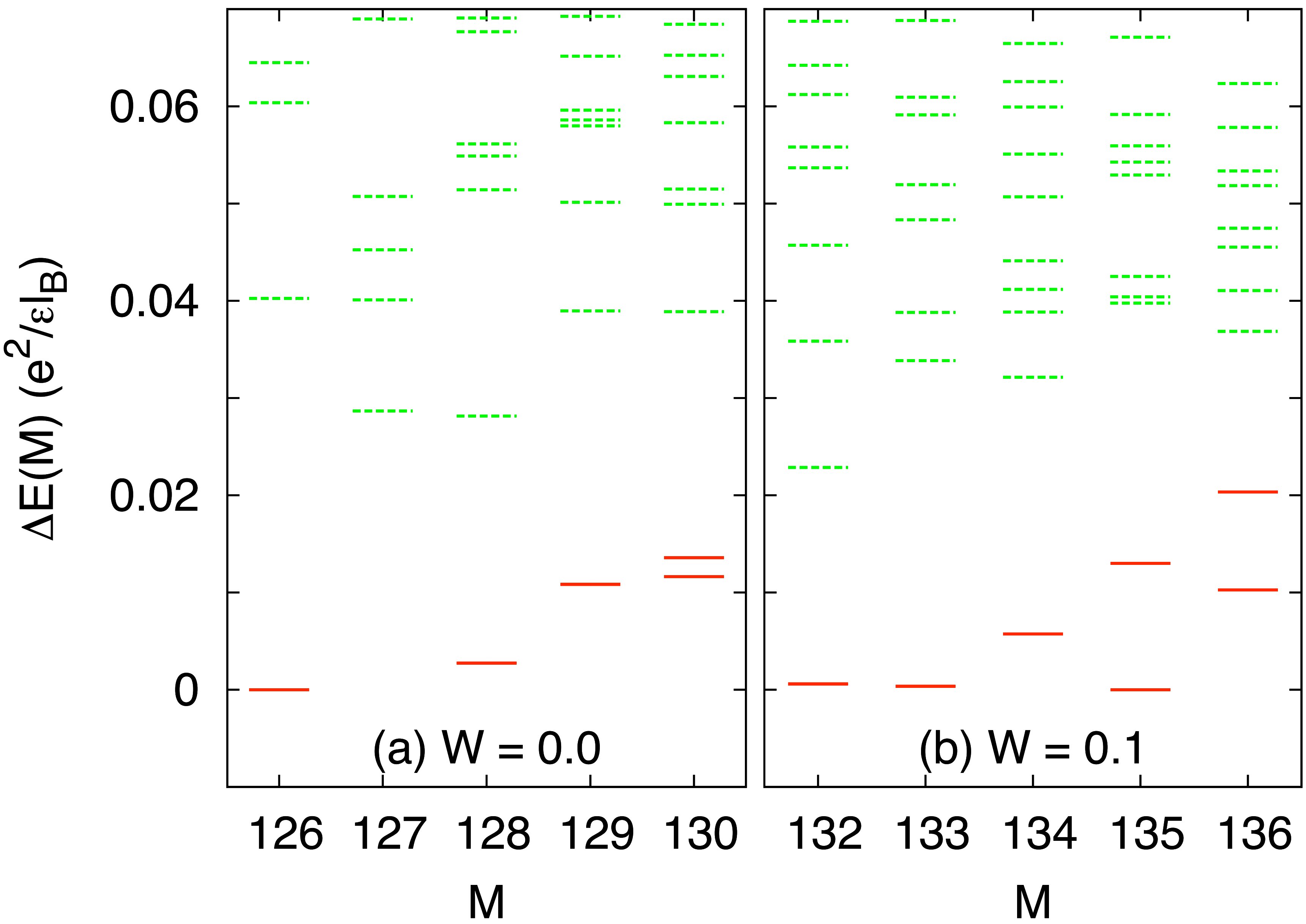
Majorana fermion accumulates a phase of π since a full flux will yield a phase of 2π (twisted sector)

This changes the spectrum of the Majorana fermions

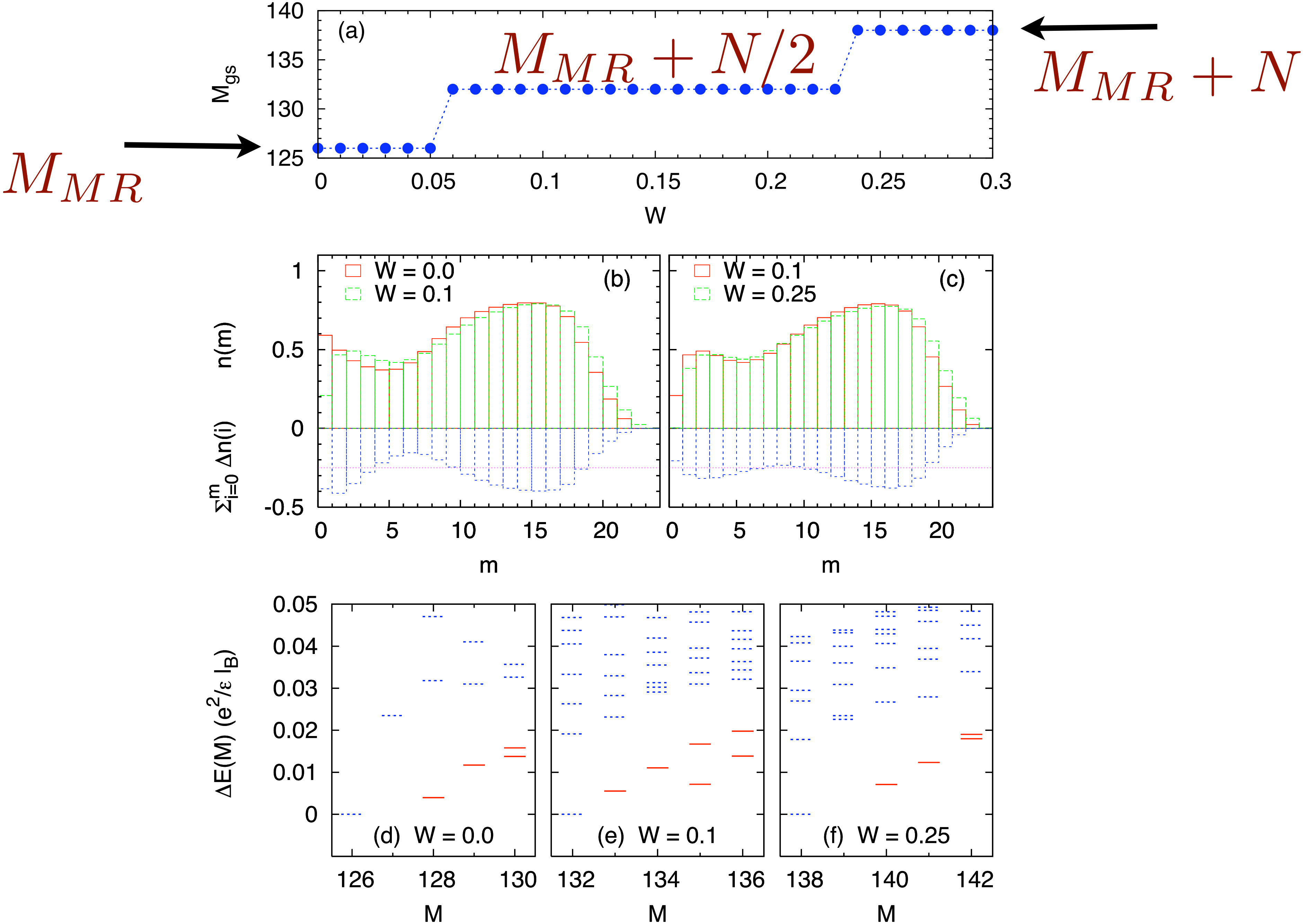
from: ΔM: 1(0), 2(1),3(1),4(2) to ΔM: 1(1),2(1),3(2),4(2)

$\lambda = 0.1 \ d = 0.7\ell_B \ N = 12$ in 24 orbitals, $\nu = 5/2$

from: ΔM: 1(0), 2(1),3(1),4(2) to ΔM: 1(1),2(1),3(2),4(2)



$\lambda = 0.5 \quad d = 0.5\ell_B \quad N = 12$ in 24 orbitals



from: ΔM: 1(0), 2(1),3(1),4(2) to ΔM: 1(1),2(1),3(2),4(2)

Discussion

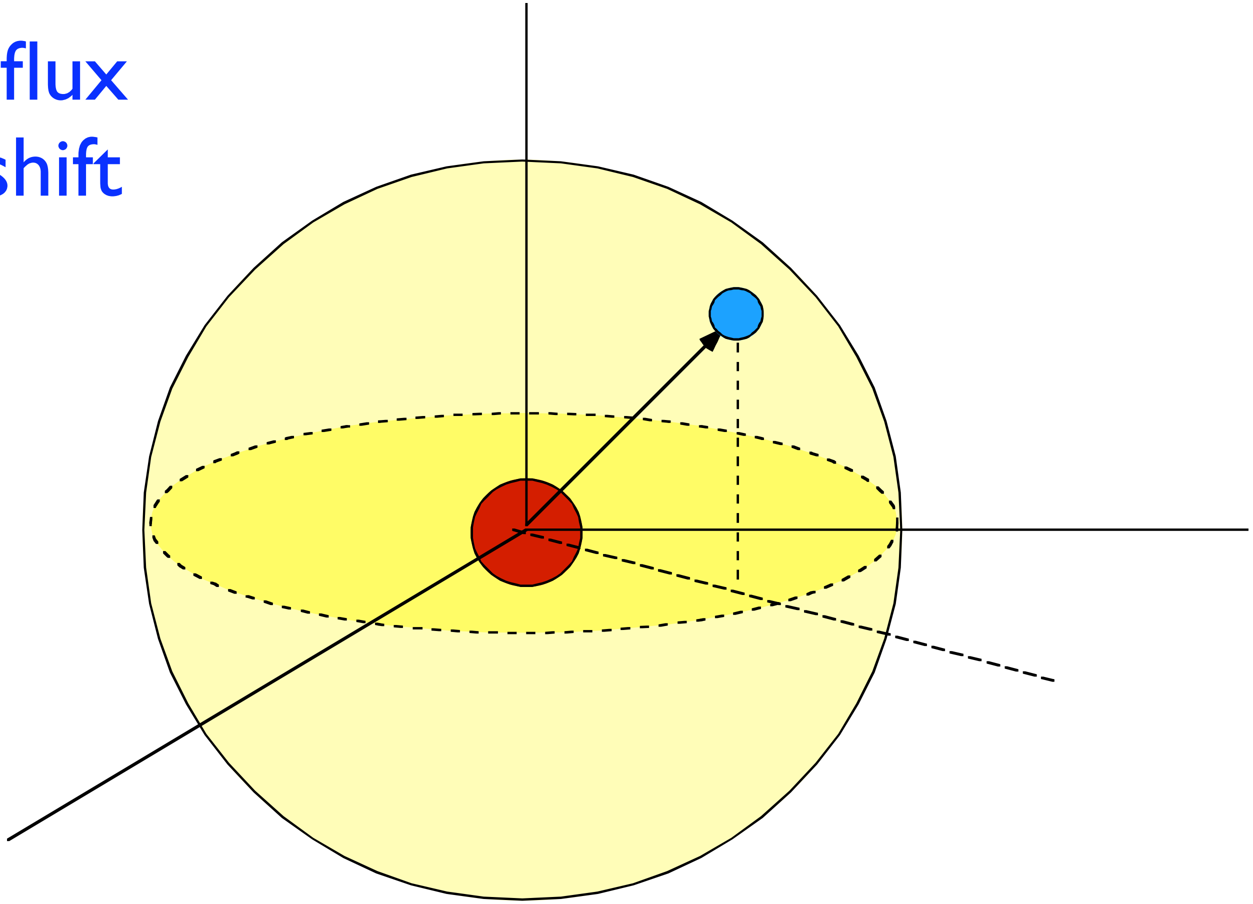
- Extrapolating to the Coulomb potential yields v_c=0.046R and v_n=0.0036R in units of $e^2/\epsilon\ell_B\hbar$
- In Ga-As systems, we have v_c~5×10^6 cm/s and v_n~4×10^5 cm/s
- We can estimate the decoherence length to be about 4 *u*m (using the result of Bisharat and Nayak)

Studies of the Moore-Read state on the sphere

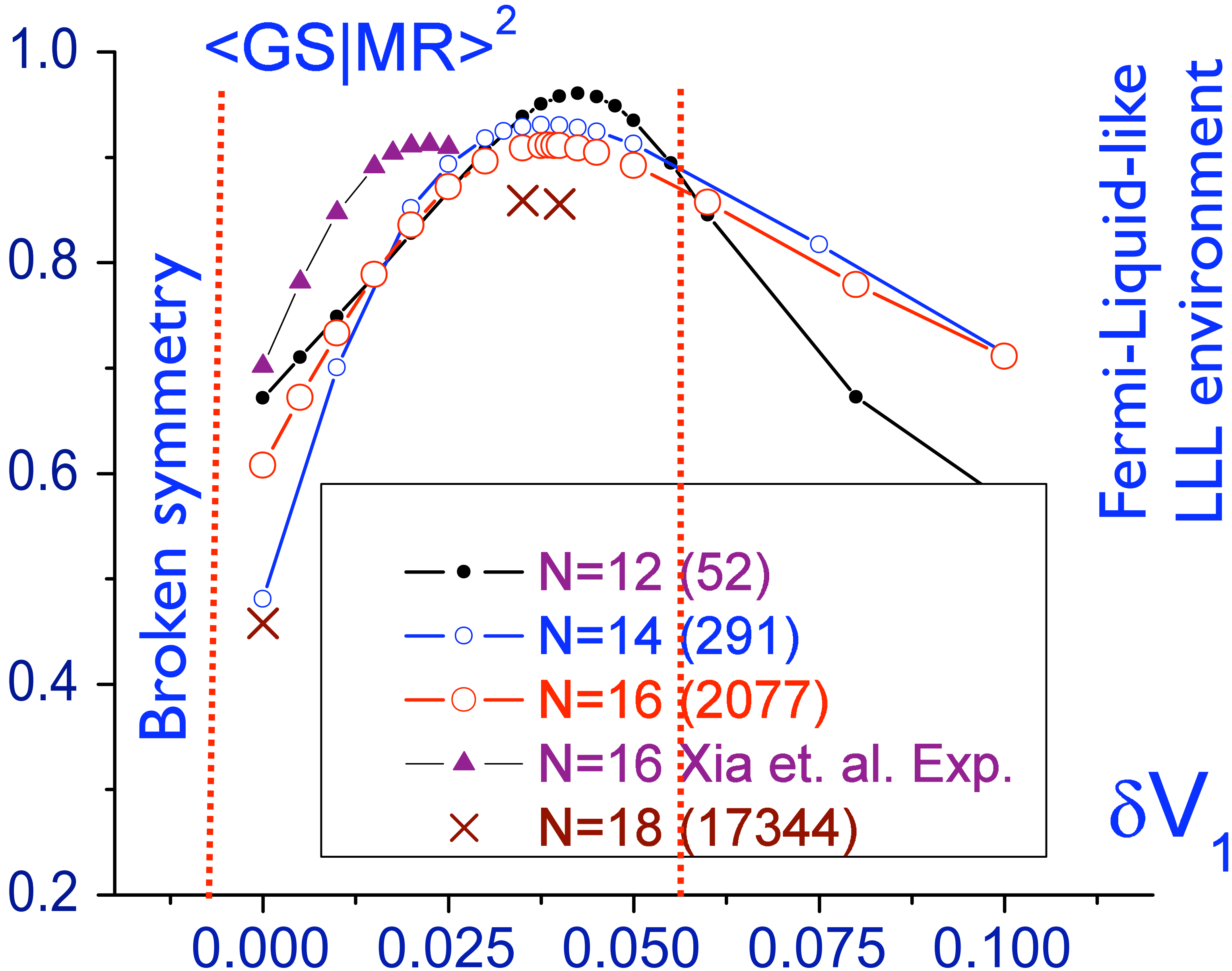
On the sphere charge-flux relations have a finite shift

$$N_\phi = \nu^{-1} N - \underline{m}$$

Haldane 1983



The ground state at 2N-3 flux



PBC Ground State Quantum Numbers Degeneracy and Topological Order

- There are $\bar{N}^2$ $(\nu = \frac{N}{N_\phi} = \frac{p\bar{N}}{q\bar{N}} = \frac{p}{q})$ 2-D conserved wave-vectors ***K*** forming the many-body BZ (Haldane 1986)

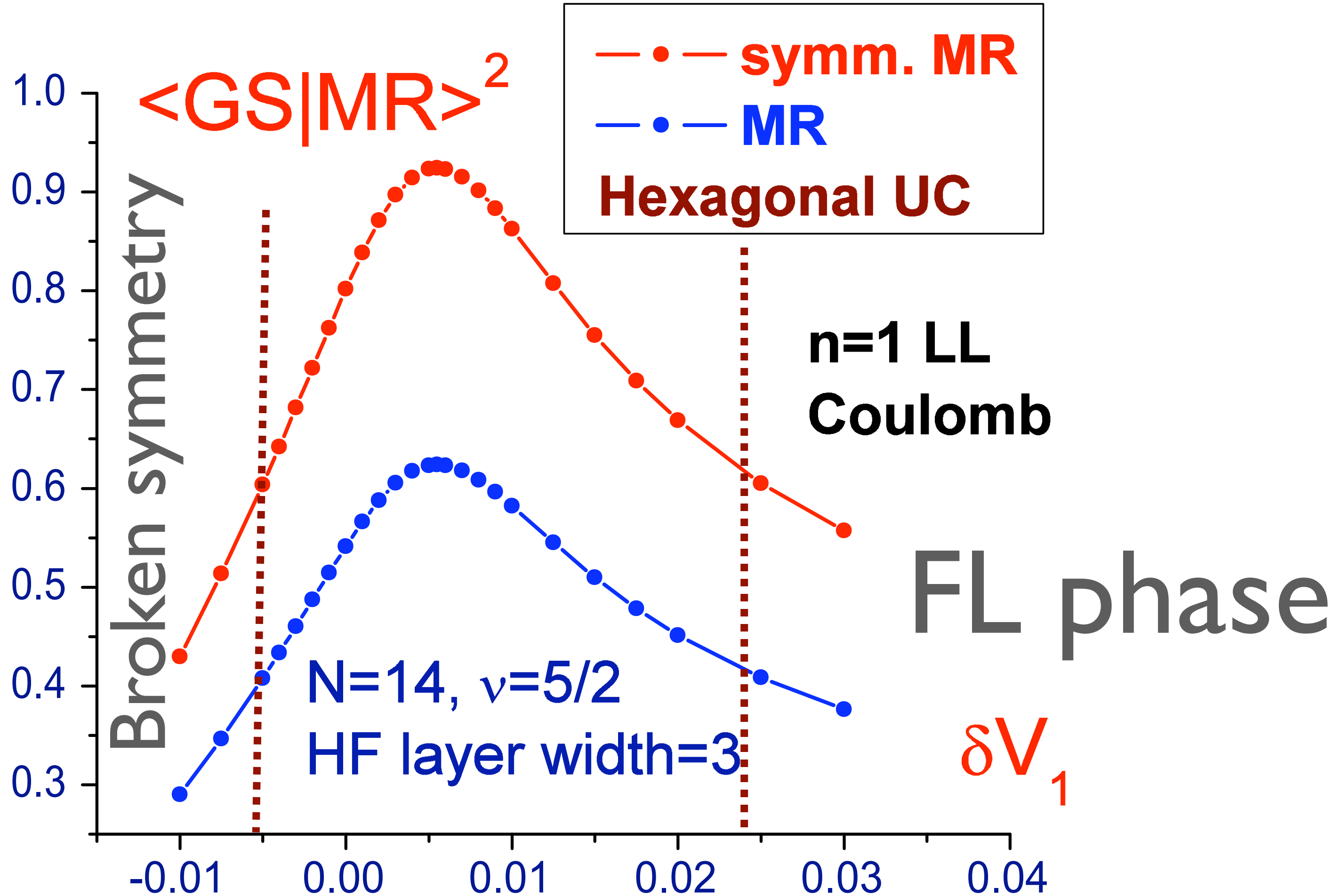
- For Incompressible Hall states $\vec{K} = \vec{G}/2$

  G is a reciprocal vector $\vec{K}_{1/3} = \vec{K}_{2/5} = 0$

$$\vec{K}_{5/2} \neq 0$$

- Degeneracy of Abelian Hall states is just q

  while for 5/2 it is 3q=6 same as MR state!

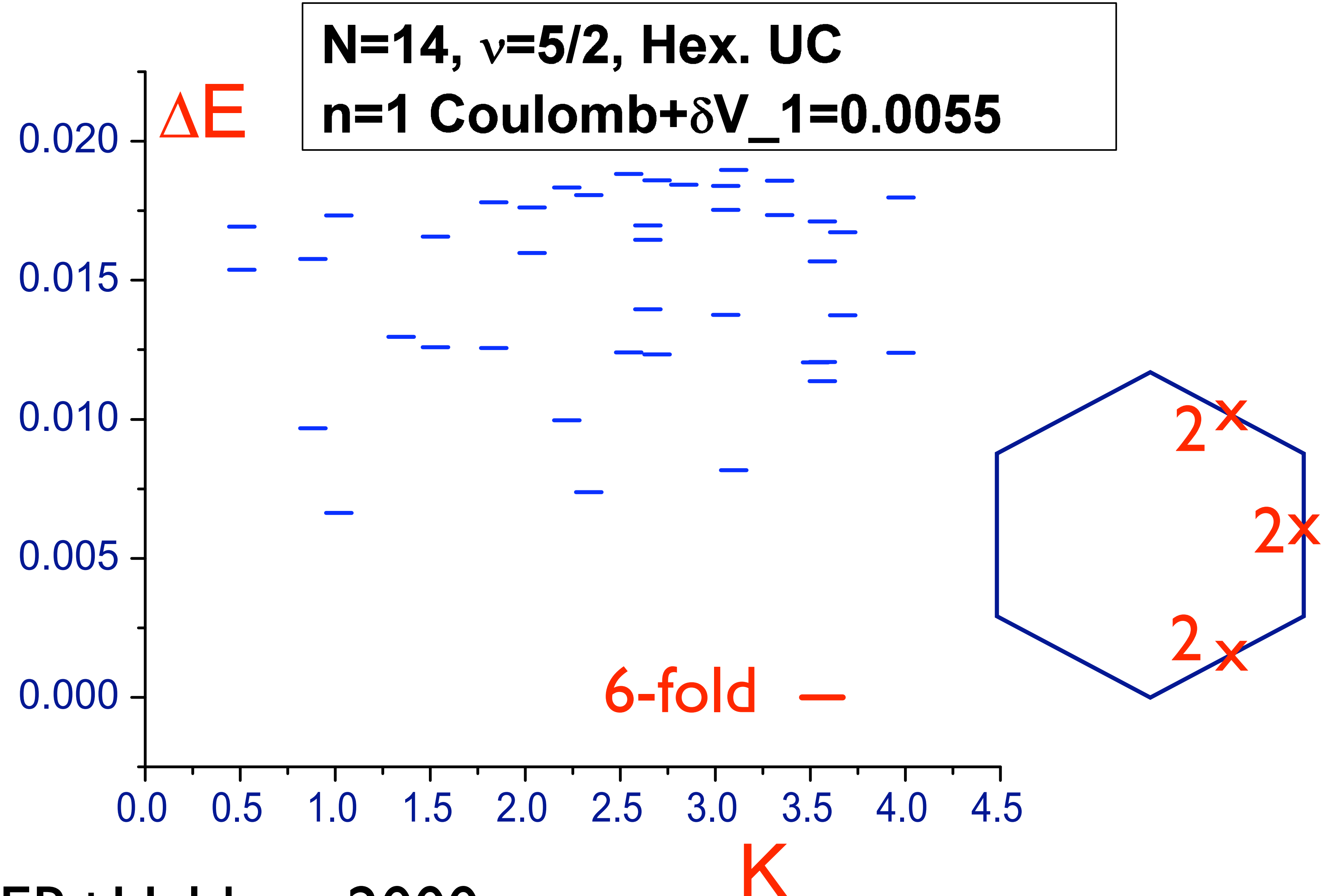


$$\Psi_{MR} = \alpha\psi_+ + \beta\psi_-$$

$$|\alpha|^2 + |\beta|^2 = 1 \quad |<\psi_\pm|\psi_\pm>|^2 = 1$$

$\psi's$ have opposite parities under PH conjugation

The spectrum shows a clear gap



ER+Haldane 2000

Rapidly Rotating Ultra-Cold Atoms with N. Read and N. Cooper

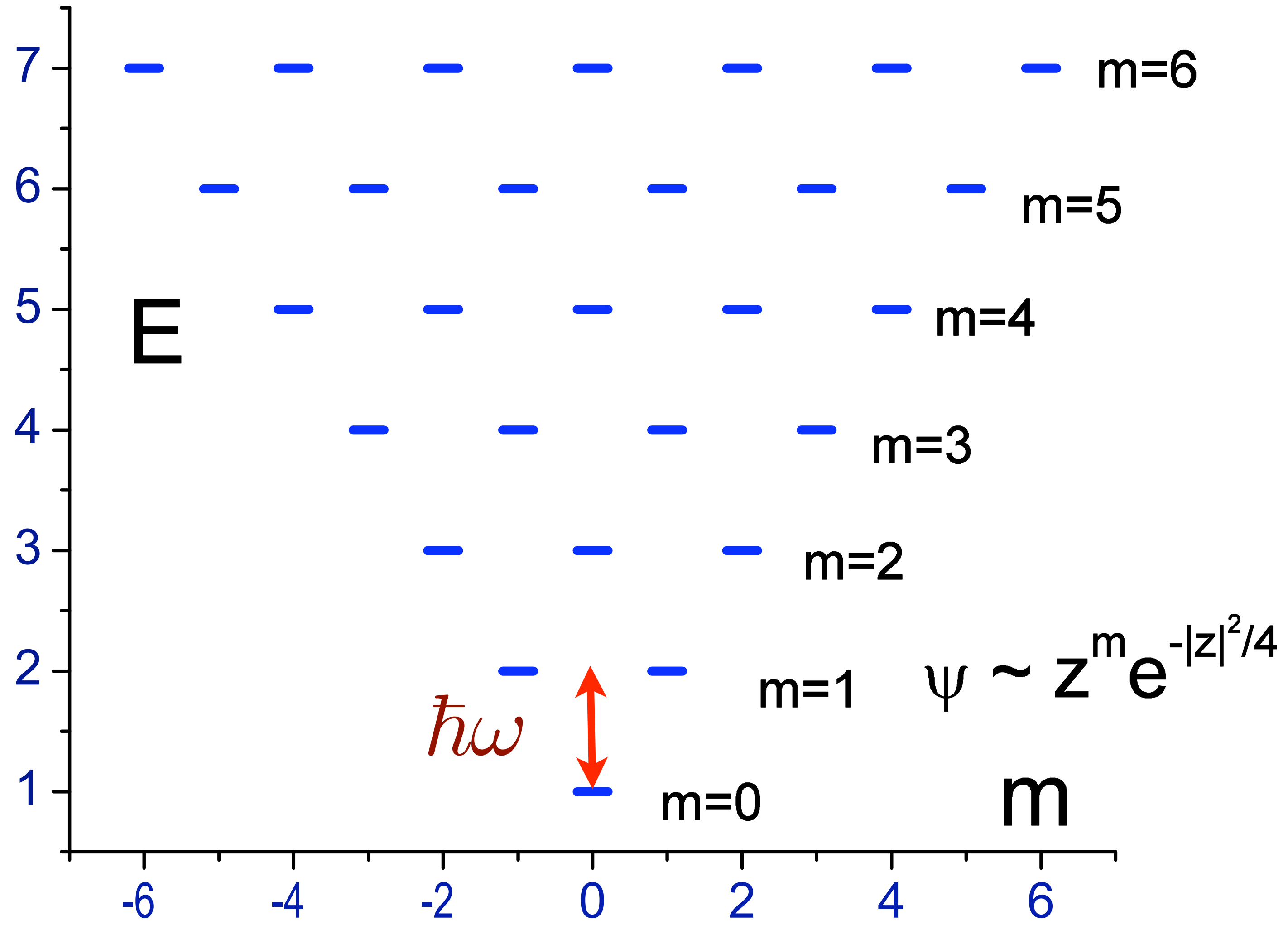
$$H = -\frac{\hbar^2}{2m}\sum_i^N \nabla_i^2 + \frac{1}{2}m\omega^2\sum_i^N r_i^2 + g\sum_{i<j}^N \delta(\mathbf{r}_i - \mathbf{r}_j)$$

$$g = \frac{4\pi\hbar^2 a_s}{m}$$

a_s is the S-wave scattering length

$$H_{\rm rot} = H - \mathbf{\Omega}\cdot\mathbf{L}_{\rm tot}$$

2-d harmonic oscillator
E vs. angular momentum m



- At ultra-cold temperatures the dominant scattering is s-wave which can be modeled by a contact pseudo-potentials

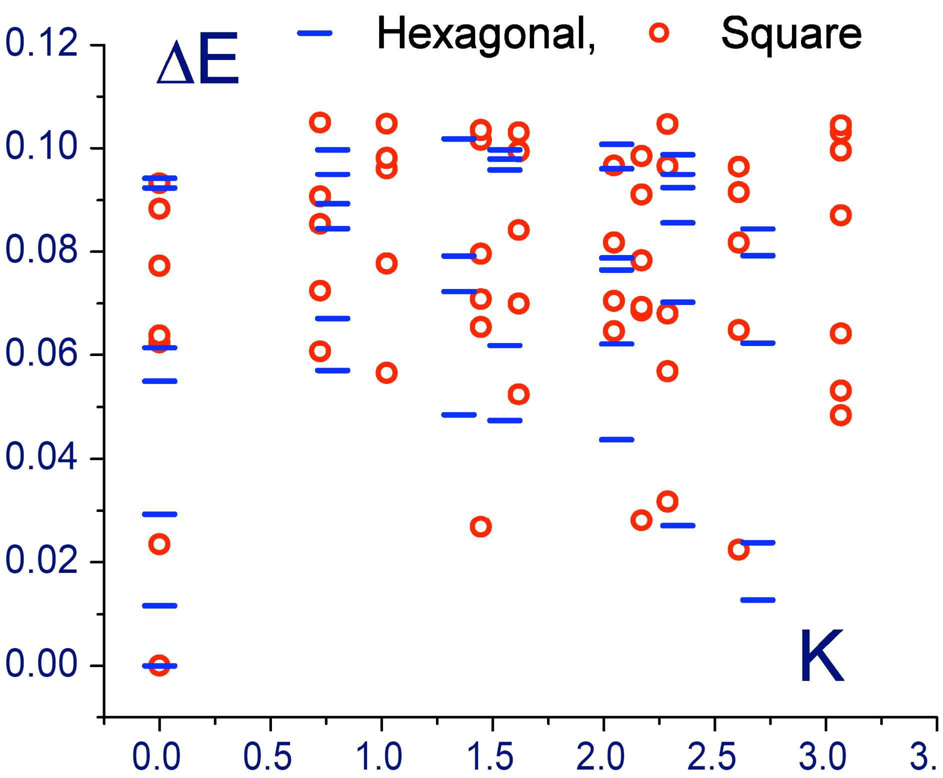
- Enter lowest Landau level if gn<< Landau level spacing, where n is the density (dilute limit), Wilkin, Gun and Smith (1998), Wilkin and Gun (2000), the counterpart of the filling factor is: N/N_V where N_V is the number of vortices.

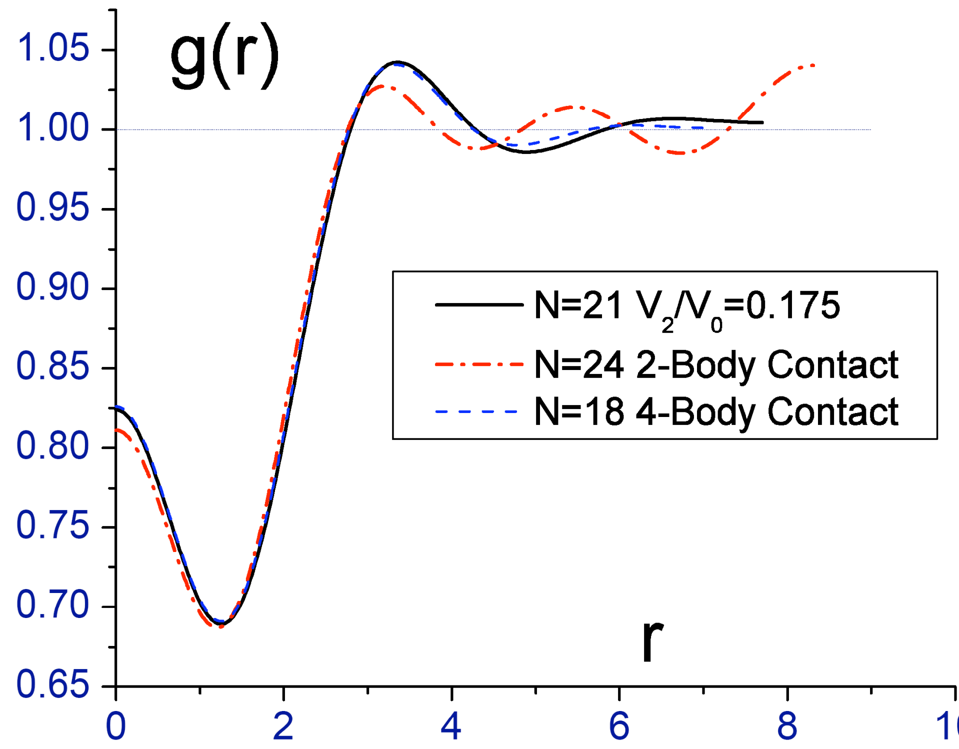
- With rapid rotations crystal of vortices (already observed) are expected to melt (for fillings <6)to quantum Hall fluid phases Cooper, Wilkin and Gun (2001).

Dipolar Forces

- Griesmier et. al. (2005) have succeeded in condensing chromium (with a permanent dipole moment).
- We will assume dipoles point along the rotation axis: $V(r) \sim C_d / r^3$
- We can parametrize the dipolar interactions with pseudo-potentials V_m m=0,2,... (bosons)
- The contact potential is just V_0 (includes contact and dipolar parts) which can be tuned by means of Feshbach resonance.
- Quantify the dipole to contact interaction by the ratio V_2/V_0

Pure Contact Potential



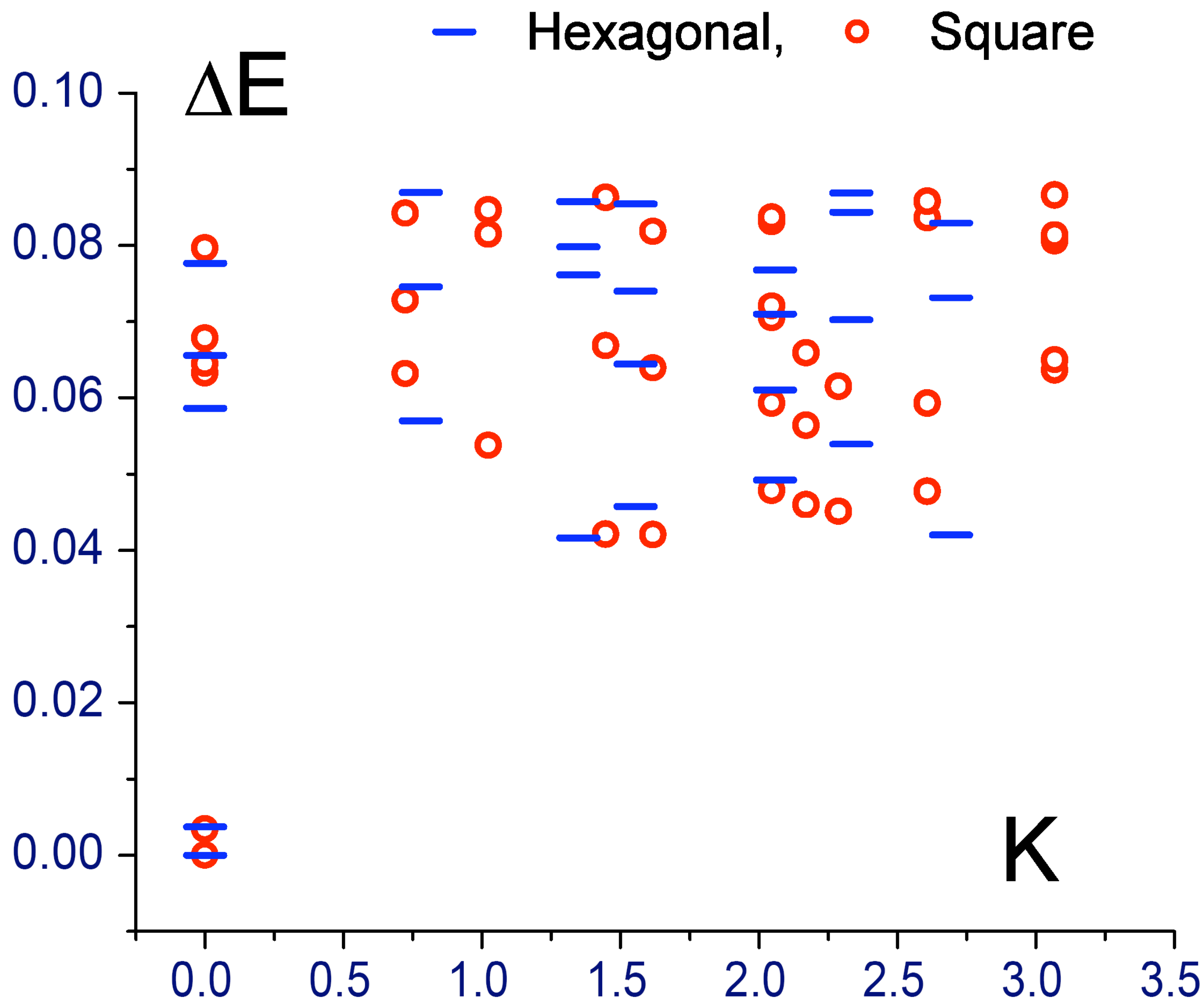


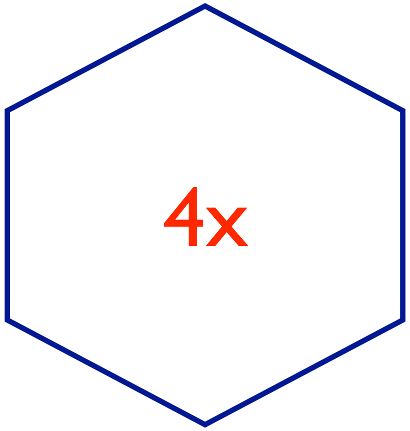
$N = 18,\ \nu = 3/2,\ \text{PBC},$

$\nu = 3/2,\ \text{Sphere}$

$\alpha = 0$

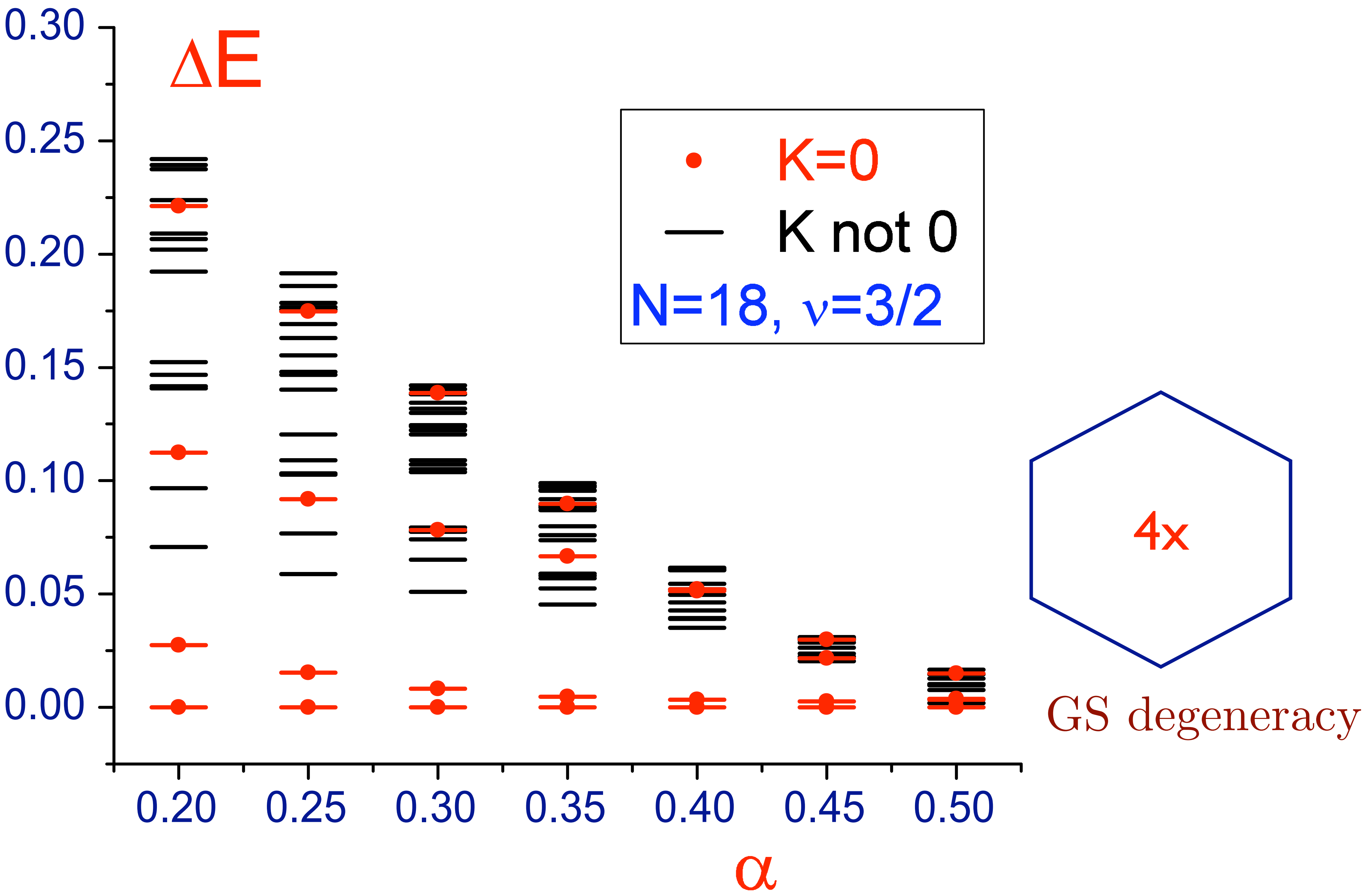
Spectrum plotted vs. K



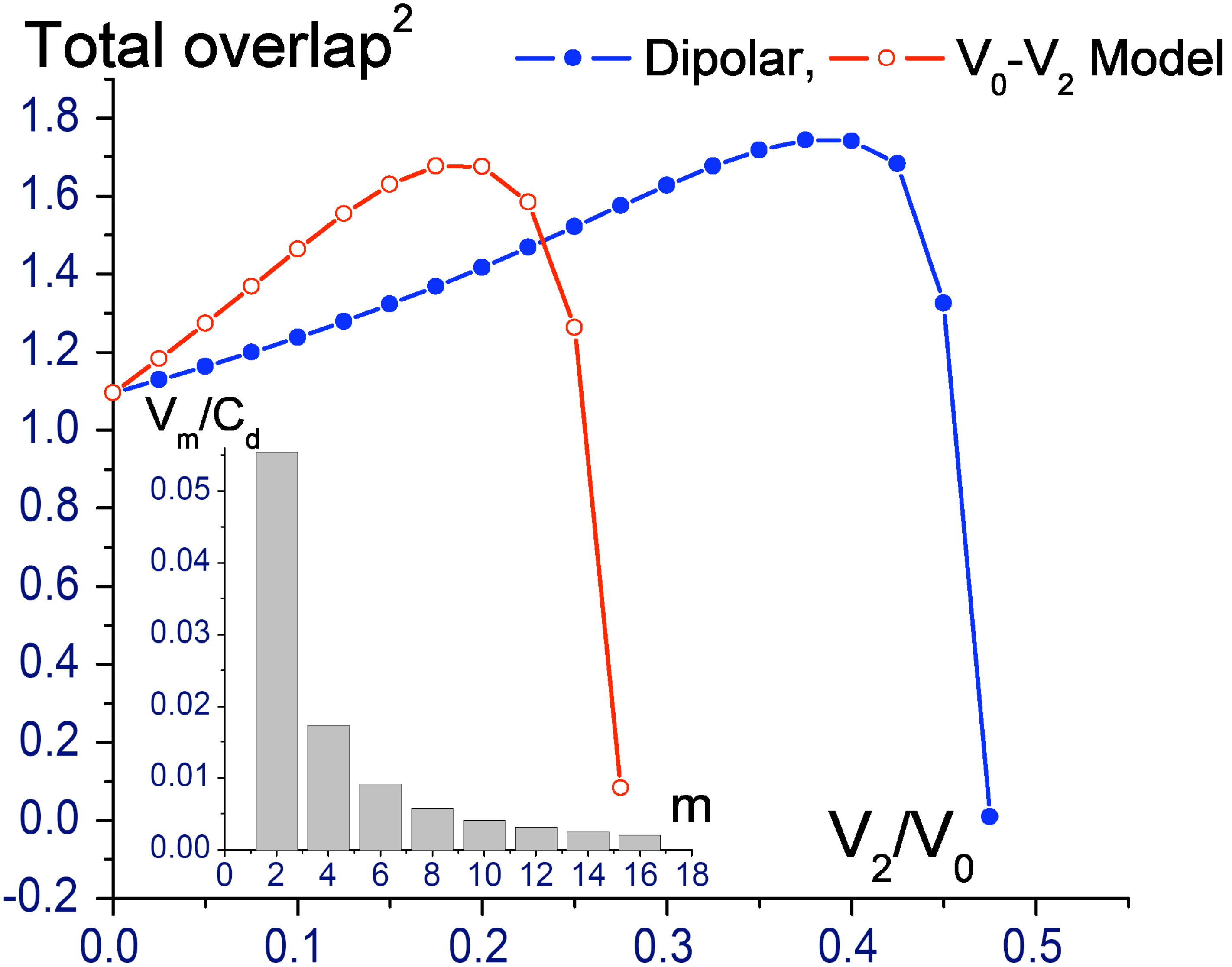


k=3 RR state GS degeneracy

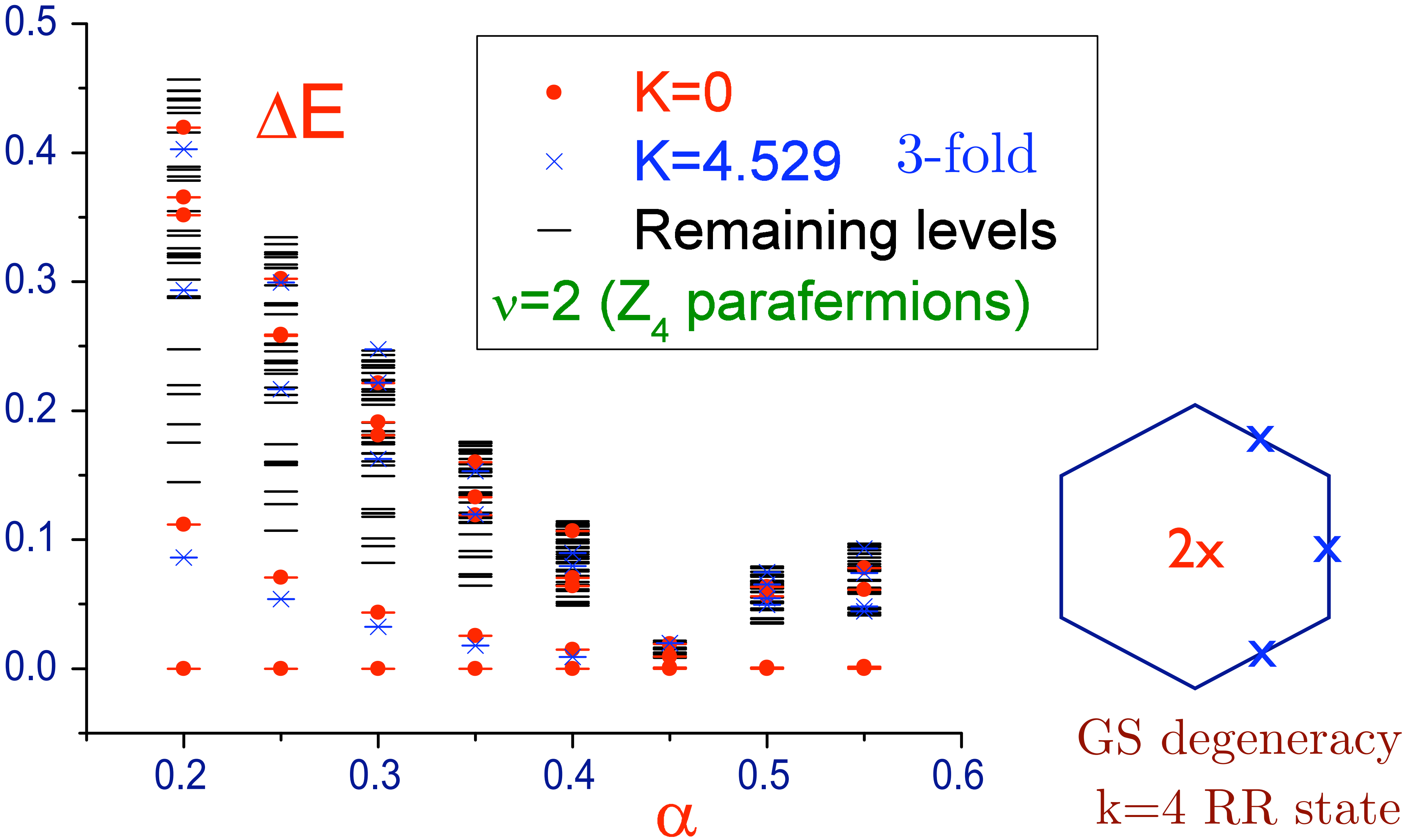
$$N = 18, \nu = 3/2, \ \mathrm{PBC}, \ \alpha = \frac{V_2}{V_0} = 0.38$$



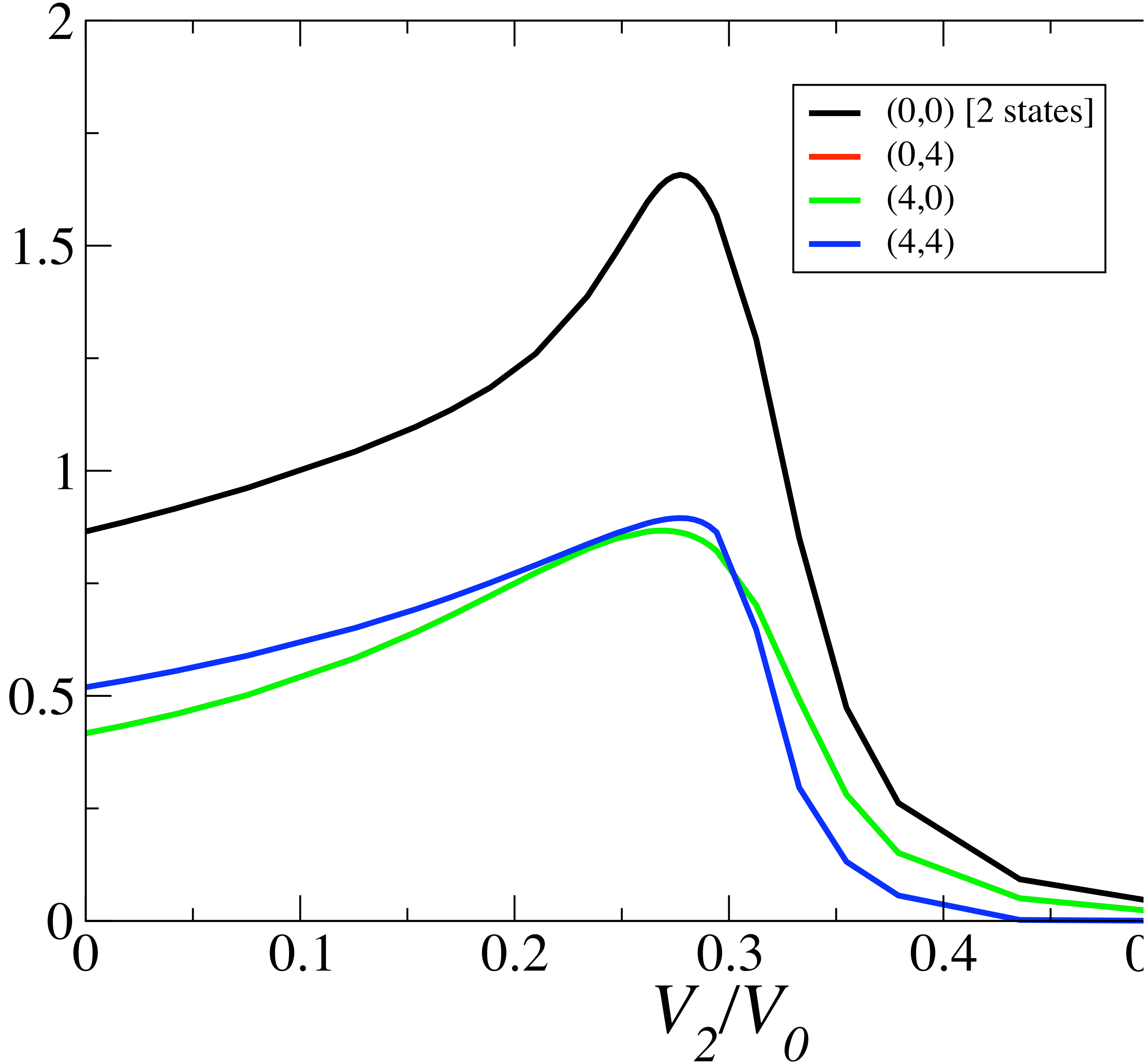
ΔE
0.30
0.25
0.20
0.15
0.10
0.05
0.00
K=0
K not 0
N=18, ν=3/2
4x
GS degeneracy
0.20
0.25
0.30
0.35
0.40
0.45
0.50
α



The Hilbert space dimension not reduced by point symmetries is 242,000



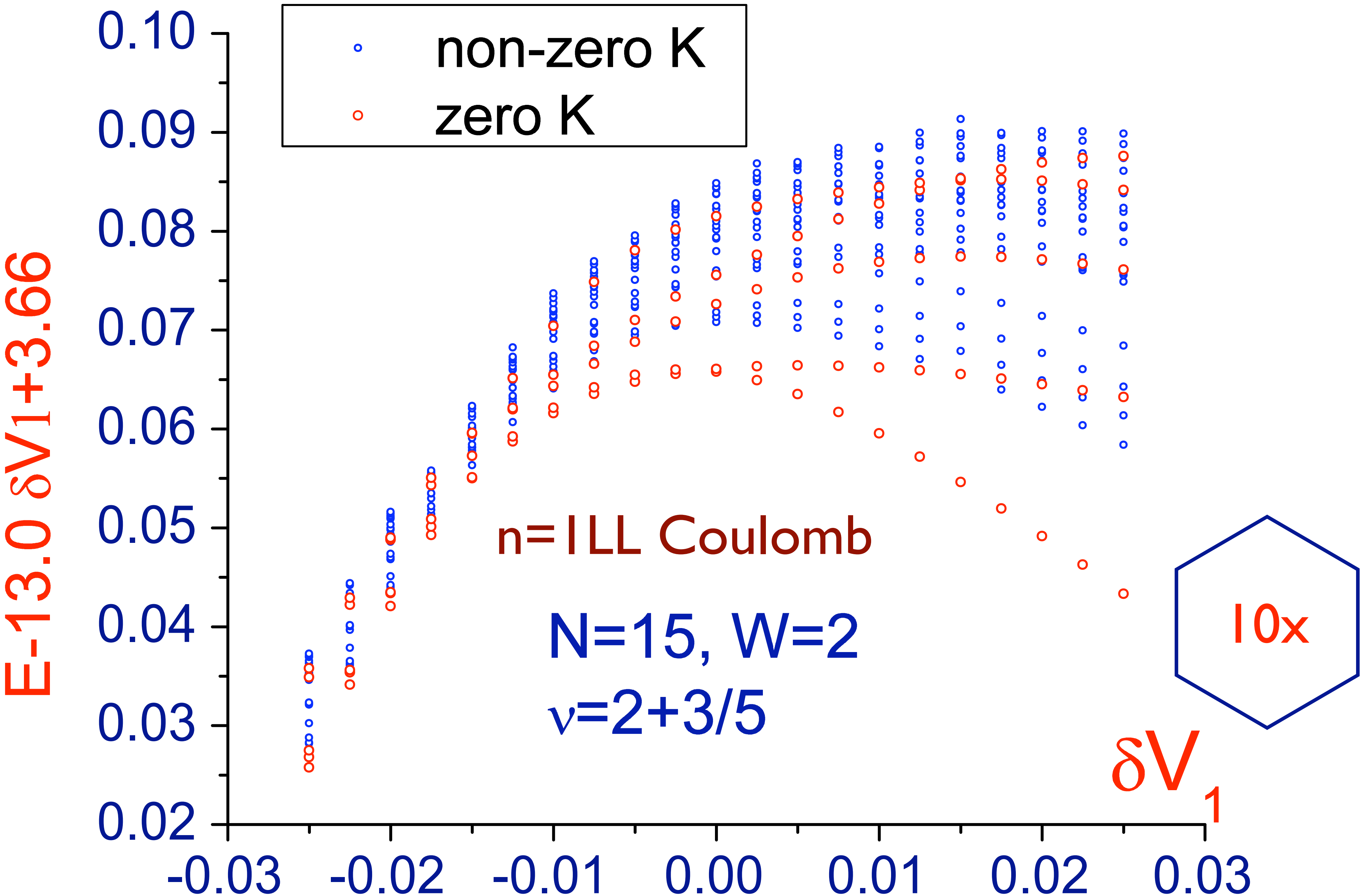
0.5
0.4
0.3
0.2
0.1
0.0
0.2
0.3
0.4
0.5
0.6
α
ΔE
K=0
K=4.529
3-fold
Remaining levels
ν=2 (Z4 parafermions)
2x
GS degeneracy
k=4 RR state



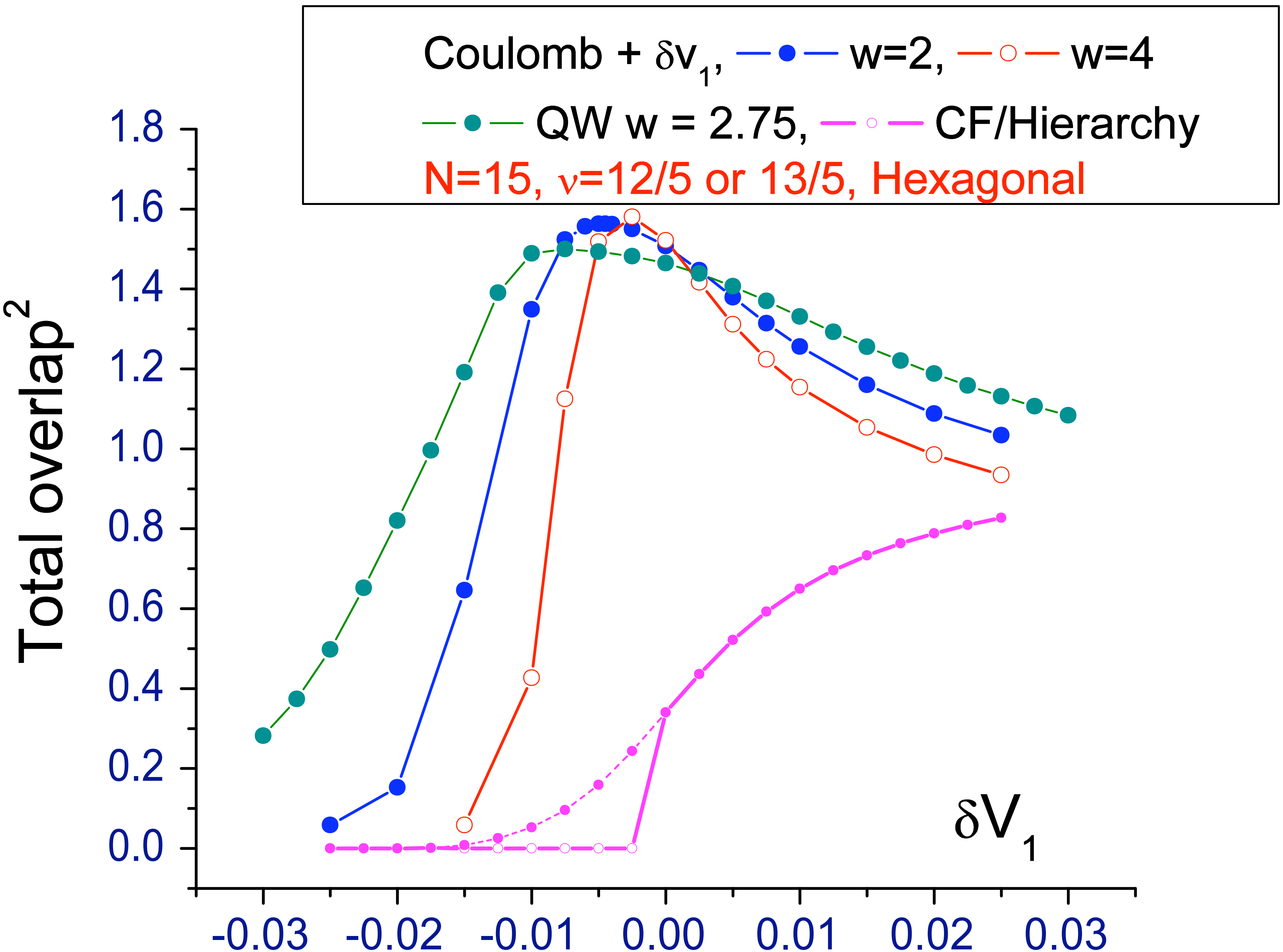
2
1.5
1
0.5
0
0
0.1
0.2
0.3
0.4
(0,0) [2 states]
(0,4)
(4,0)
(4,4)
V_2/V_0

Electrons at 12/5 or 13/5 Landau level filling

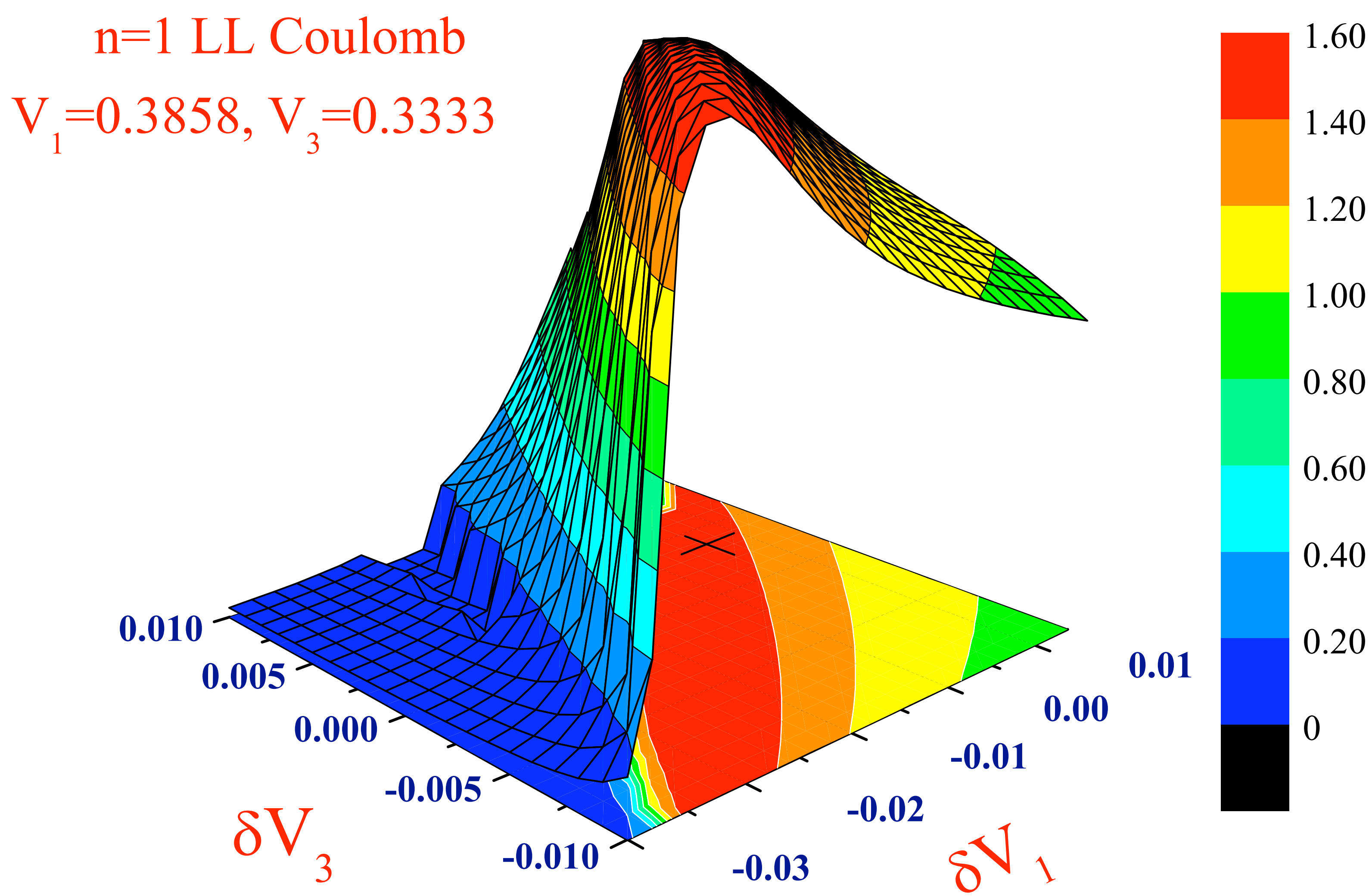
With N. Read

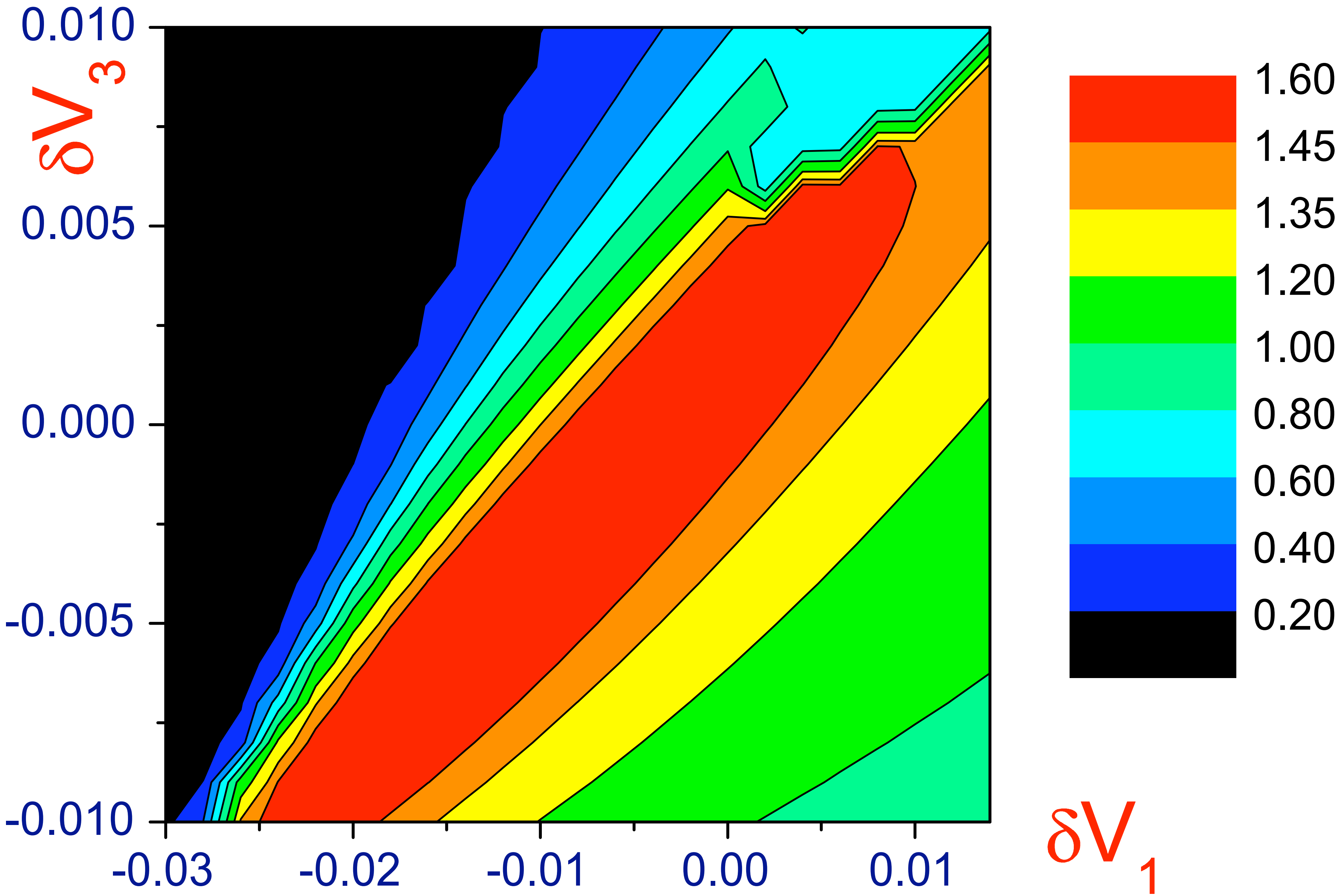


non-zero K
zero K
n=1LL Coulomb
N=15, W=2
ν=2+3/5
10x
E-13.0 δV1+3.66
δV1
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03

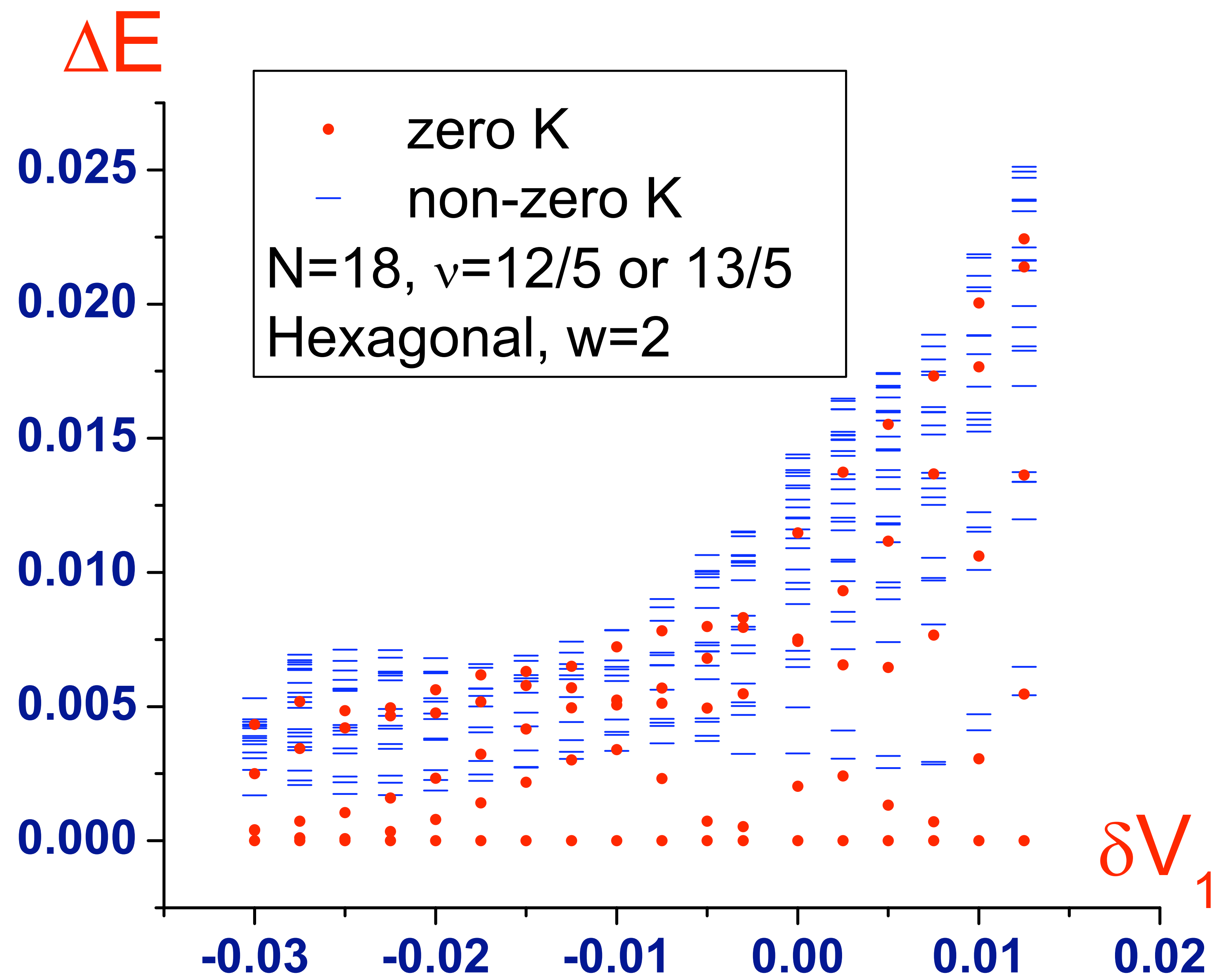


Coulomb + δv1, w=2, w=4
QW w = 2.75, CF/Hierarchy
N=15, ν=12/5 or 13/5, Hexagonal
Total overlap2
δV1
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.03
-0.02
-0.01
0.00
0.01
0.02
0.03

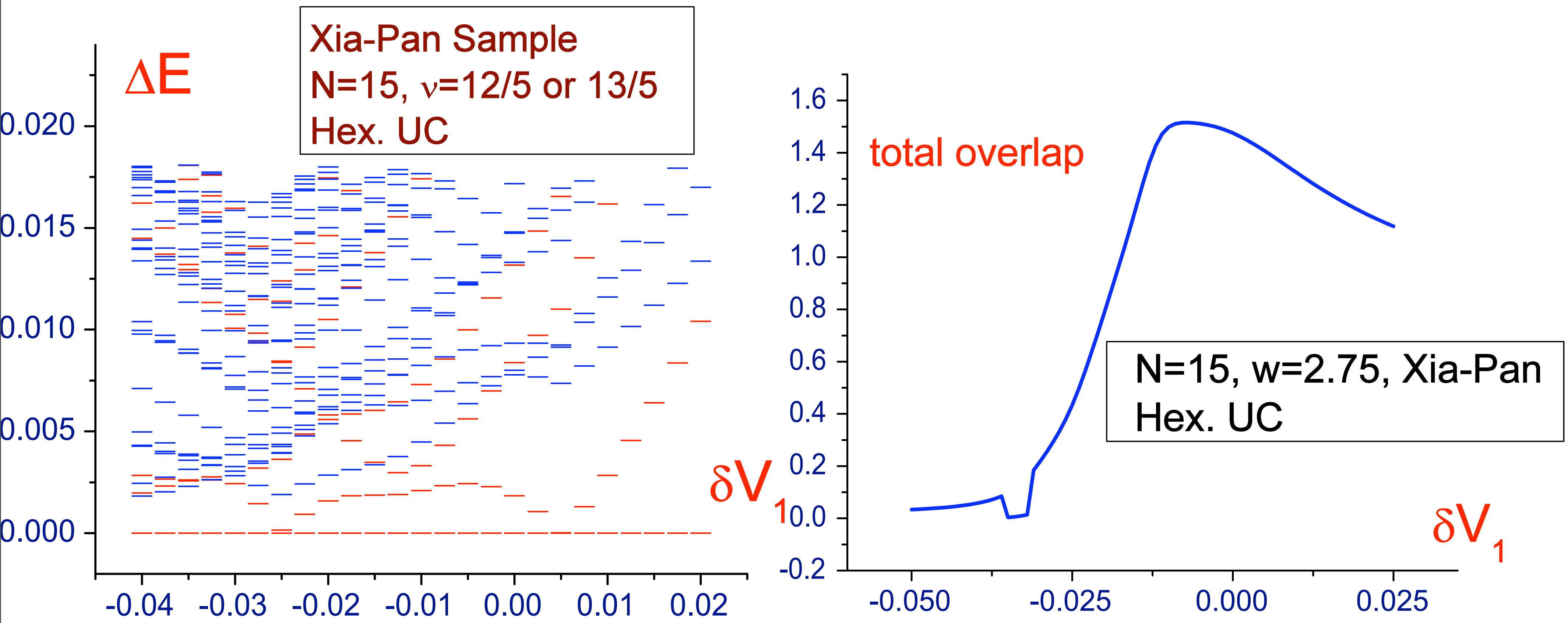

n=1 LL Coulomb
V_1=0.3858, V_3=0.3333
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0
0.010
0.005
0.000
-0.005
-0.010
δV_3
0.01
0.00
-0.01
-0.02
-0.03
δV_1



0.010
0.005
0.000
-0.005
-0.010
δV3
-0.03
-0.02
-0.01
0.00
0.01
δV1
1.60
1.45
1.35
1.20
1.00
0.80
0.60
0.40
0.20



ΔE
zero K
non-zero K
N=18, ν=12/5 or 13/5
Hexagonal, w=2
0.025
0.020
0.015
0.010
0.005
0.000
-0.03
-0.02
-0.01
0.00
0.01
0.02
δV1



ΔE
Xia-Pan Sample
N=15, ν=12/5 or 13/5
Hex. UC
0.020
0.015
0.010
0.005
0.000
-0.04
-0.03
-0.02
-0.01
0.00
0.01
0.02
δV1
total overlap
N=15, w=2.75, Xia-Pan
Hex. UC
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.050
-0.025
0.000
0.025
δV1