Non-Abelian adiabatic transport of conformal block trial wavefunctions

N. Read

Topological (gapped) phases in 2+1 relate to modular tensor categories.

Conformal blocks as trial wavefunctions give MTC <u>same</u> as their RCFT or else gapless.

Application in case of non-unitary RCFT.

Gapped (topological) phase

→ top. qu. field th (TQFT)

or tensor category

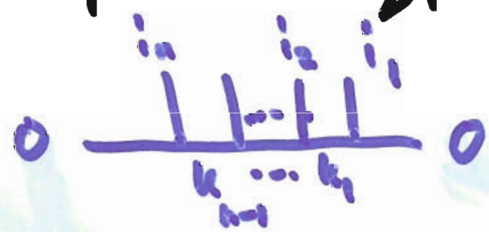
Moore + Seiberg
Witten
Reshitikhin
+ Turaev
1988-90

Finite (?) set of qptcle types i

Fusion $\phi_i \times \phi_j = \sum_k N_{ij}^{k} \phi_k$

N_{ij}^{k} = integers ≥ 0

→ degeneracy of n well-separated qptcles



If all type i

$$\dim = (N_i^{\,n})_{00}, \quad N_i = \text{matrix}, \quad (N_i)_{jk} = N_{ij}^{k}$$

$$\sim (\text{largest eval of } N_i)^n \quad \text{as } n \to \infty$$

i^* = dual of i, $\phi_i \times \phi_{i^*} = 1 + \ldots$

Operations on these spaces

↑ "time"

Braiding $c_{V_i,V_j} =$ 

→ braid gp rep"
↓
YB eq

Creation/Destruction

$i_{V_i} =$ 

, $e_{V_i} =$ 

(note order)

Axiom: 

$=$ 

$= id$

Twist: 2π rotation. Thicken to ribbon:

$\theta_i =$  $=$  $=$ 

\+ further conditions

Apply to general objects V also

Refs:
Kassel
Bakalov + Kirillov
Turaev

Quantum trace and quantum dimension:

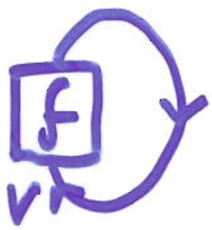
Want  but  not yet defined

Use 

so  =  $= e'_{V_i}$

Also  =  $= i'_{V_i}$

Define $f: V \to V$

$$q\mathrm{tr}\, f = $$  (not only for simple V_i)

Nice property:

$$q\mathrm{tr}\, f \otimes g = q\mathrm{tr}\, f \cdot q\mathrm{tr}\, g$$

Simples = qparticle type i, define

$$q\dim V_i = $$  $= d_i$

Also

$$\tilde{S}_{ij} = $$ . <u>Modular</u> TC if $\tilde{S}$ is invertible.

Hermitian structure Turaev

For any map f, have antilinear map $f \to \bar{f}$ s.t.

$$\bar{\bar{f}} = f$$
$$\overline{id_V} = id_V$$

$$\overline{c_{V,W}} = c_{V,W}^{-1}$$
$$\overline{\theta_V} = \theta_V^{-1}$$

$$\bar{i}_V = e_V c_{V,V*} (\theta_V \otimes id_{V*}) = e_V'$$
$$\bar{e}_V = i_V'$$

(Then $S = \tilde{S}/D$ is unitary as matrix.)
Also d_i real.

"Unitary" if positive definite

$$q\mathrm{tr}_V \bar{f} f \geq 0 \quad \text{all } f$$

This captures $\boxed{\text{positivity of QM}}$ in 2+1.

In particular

$$d_i = q\mathrm{tr}_1 \, e_{V_i}' \, i_{V_i} = q\mathrm{tr}_1 \, \bar{i}_{V_i} \, i_{V_i} \geq 0$$

and actually $d_i \geq 1$.

$$d_i = \| i_V^{i*} \|^2 > 0$$

In MTC, N_i is non-negative (entries)

$\to$ Perron-Frobenius Thm
+ Verlinde formula
lead to

$\boxed{\text{if also unitary, then } d_i \text{ is largest eval of } N_i}$

So deg for n qp's type $i \sim d_i^n$
in <u>unitary</u> case.

If not unitary, $d_i < 0$ allowed,
seems to occur in every case.

In non-unitary rational CFT ($\exists h_i < 0 \Rightarrow$ non-unit)
as 1+1 QFT
existence of $h_i < 0 \Rightarrow$ some $d_j < 0$.

What is relation between "unitarity" in 2D RCFT and in 3D TQFT?

Conformal blocks as trial wavefunctions

Moore + N. R.
1991

Conf. blocks are holomorphic fns Ψ_a in "hole" coords z_j, qp coords w_k (all type i)

From RCFT, arise as (unnormalized) corr fns

$$\langle \Phi_\psi(z_1,\bar{z}_1) \cdots \Phi_\psi(z_N,\bar{z}_N)\, \Phi_\tau(w_1,\bar{w}_1) \cdots \Phi_\tau(w_n,\bar{w}_n) \rangle$$

$$= \sum_a \left| \Psi_a(z_1 \ldots z_N, w_1 \ldots w_n) \right|^2$$

- Single valued (symm/antisymm) in z_i's
- multivalued in w_k's, s.t.

$$\Psi_a \to \sum_b \Psi_b M_{ba}$$

under exchange of w_k's by analytic cont^n

(monodromy) M ind. of z's, w's.

- braiding! Indeed, all of MTC structure

Charge sector, background charge, ...

$\Psi_a(z_1 .. w_1 ..)$ are wfns of states $|w_1 .. w_n\rangle$

Inner product $\langle \Psi_a | \Psi_a \rangle = \int \prod_{j=1}^{N} d^2 z_j \, |\Psi_a(z_1 .. w_1 ..)|^2$

Must compute exchange, twist by <u>adiabatic transport</u>

Berry: result (<u>holonomy</u>)

$$\boxed{B = M \, P e^{i \oint A \cdot dw}}$$

$$A_{w,ab} = i \langle \Psi_a(w) | \frac{\partial \Psi_b}{\partial w}(w) \rangle$$

$$A_{\bar{w}} = (A_w)^{\dagger}$$

P = path ordering

assuming $|\Psi_a\rangle$ <u>orthonormal</u>.

If $|\Psi_a(w)\rangle$ also holomorphic in w, then

$A = 0$ and

$$\boxed{B = M}$$

"holonomy equals monodromy"

as desired M+R 91

– read off M from CFT

As Ψ_a holomorphic in w_k, issue is orthonormality (up to w-ind factors)

$$\langle \Psi_a(w_1 \ldots) | \Psi_b(w_1 \ldots) \rangle = Z_{ab}(w_1 \ldots w_n)$$

Want $Z_{ab} \sim \delta_{ab}$ (w-ind coeff)

for well-separated w_k.

Go grand-canonical, then

$$\sum_a Z_{aa} = \left\langle e^{\lambda \int d^2 z \, \Phi_\psi(z, \bar{z})} \prod_k \Phi_\sigma(w_k, \bar{w}_k) \right\rangle_{CFT}$$

– perturbed CFT.

Problem in <u>2D</u> field th / stat mech.

What is long distance behavior of correlator under the pert?

– either 1) massive

2) massless

3) other?

1) Massive 2D phase

Expect correlations of typical operators to approach <u>constants</u>, exp'ly small corrections

But that is $\sum_a Z_{aa}$.

Z_{ab} cannot all be nonzero because of monodromy:

$$Z_{ab} \to \sum_{cd} M^\dagger_{ac} Z_{cd} M_{db}$$

So $Z = M^\dagger Z M$

M are <u>unitary</u> rep of braid gp

If rep is irrep, Schur's lemma $\Rightarrow Z \propto \mathbb{1}$

Works even in general case to give

$$B = M$$

(Other parts of Z_{ab} are mutually non-local, can't all have exp^{ns} in same phase.) Like σ, μ in Ising $T \neq T_c$.

2) Massless 2D phase

$$Z_{ab} \sim \frac{1}{|w_1 - w_2|}\left(1 + O\left(\frac{1}{|w_1 - w_2|^3}\right)\right)$$

- get power corrections to holonomy, no good in gapped phase (and other problems)

Such a case presumably a gapless phase (in 2+1 sense).

Can handle spin similarly

Conclusion:

Either adiabatic transport agrees with monodromy (w. exp'y small corrections), or not a gapped phase at all.

In particular,

use of any RCFT with negative qu. dims cannot give a gapped phase

- ie all known non-unitary examples