Which Spin Liquid Is It?

Some results concerning the character and stability
of various spin liquid phases,

and

Some speculations concerning candidate spin-liquid
phases as the explanation of the peculiar
behavior of dmit and other complicated materials

Maissam Barkeshli, Hong Yao, and Steven A. Kivelson, arXiv:1208.3869

UIUC - 2012

Theory of Superconducting Phases with Stable Fermi Surfaces

E. Fradkin, E. Berg, C. Chen
J. Tranquada, C. Varma
M. Bakeshli, H. Yao,

Some experimental motivation

(I don't promise these effects are really intrinsic.)

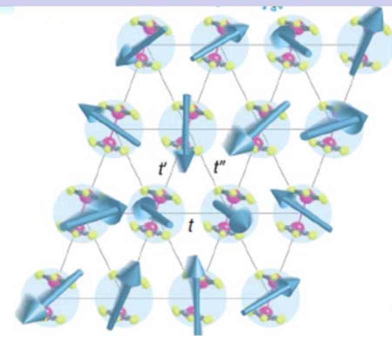
1) Disorder independent residual specific heat
in the superconducting state at low T
in "ultra-clean" YBCO: $C/T = \gamma \approx 2$mJ/mole K^2.

S. Riggs *et al*, Nat. Phys. 7, 332 (2011)

2) Fermi-liquid-like low temperature C/T and χ in
insulating $EtMe_3Sb[Pd($**dmit**$)_2]_2$
(and other organic salts), e.g.
$\gamma \approx 20$mJ/mole K^2 and $\kappa/T \approx 0.1$-.02 W/K^2m .

e.g. M. Yamashita *et al*, Nature Commun. (2011)
and Science **328**, 1246 (2010)

Candidate spin liquid material: $EtMe_3Sb[Pd(dmit)_2]_2$ = dmit

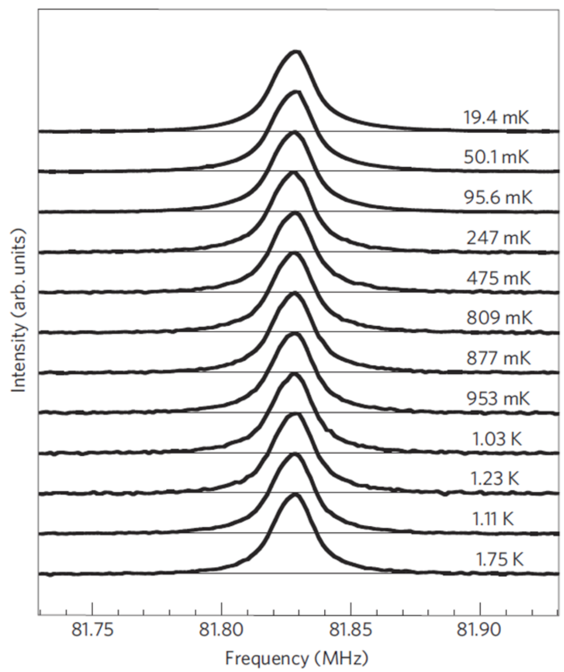


- A charge insulator
- S = 1/2 triangular lattice with J ~ 250 K

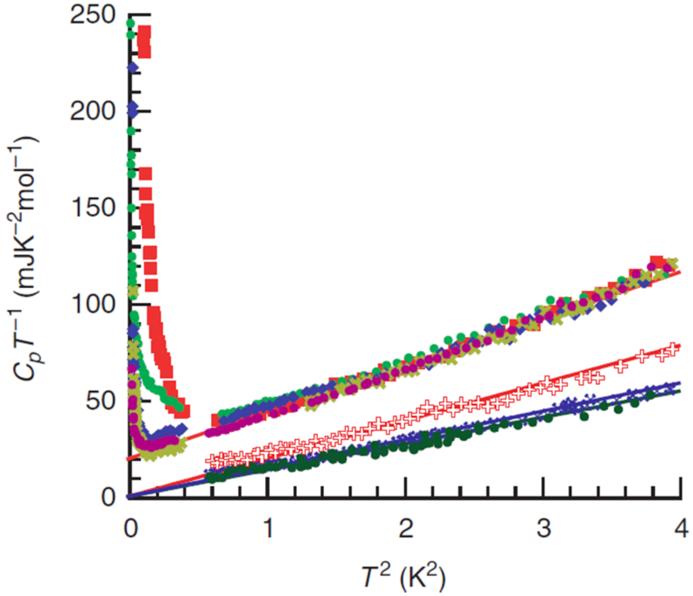
1) NMR: no magnetic order down to 20 mK.

2) As T→0, χ (susceptibility) → finite const.

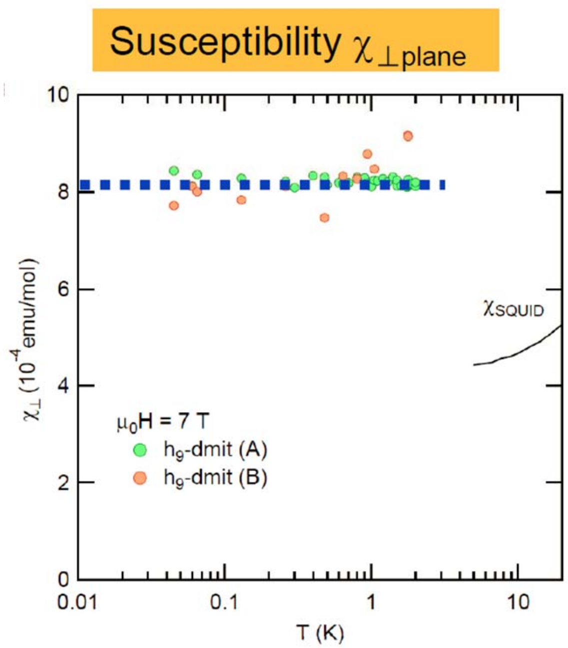
3) As T→0, γ = C/T → finite const.



Itou et al, Nat. Phys. 2010



S. Yamashita et al,
Nat. Comm. 2011



D. Watanabe et al,
Nat. Comm. 2012

All sorts of caveats are in order

There are all sorts of barriers to extrapolating the
thermodynamic data to low T.

NMR spectra are at least somewhat inhomogeneous.

Anderson insulator (non-interacting) is an insulator
with a constant $\rho(E_F)$.

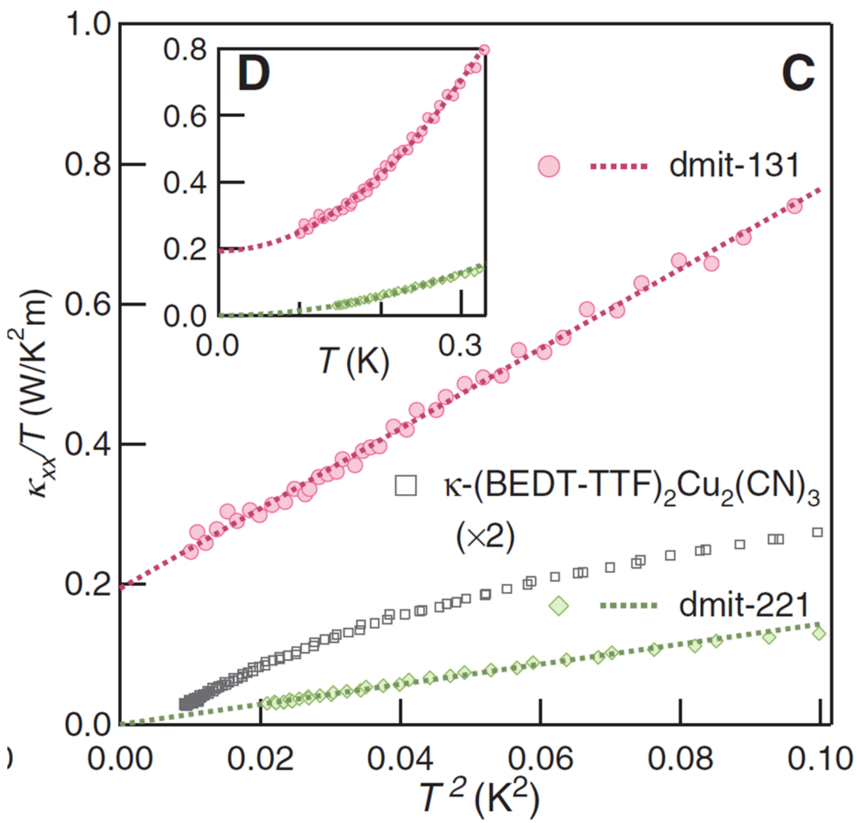
Interactions make complicated magnetic behavior
(e.g. random singlet phase)
which can be quite subtle.

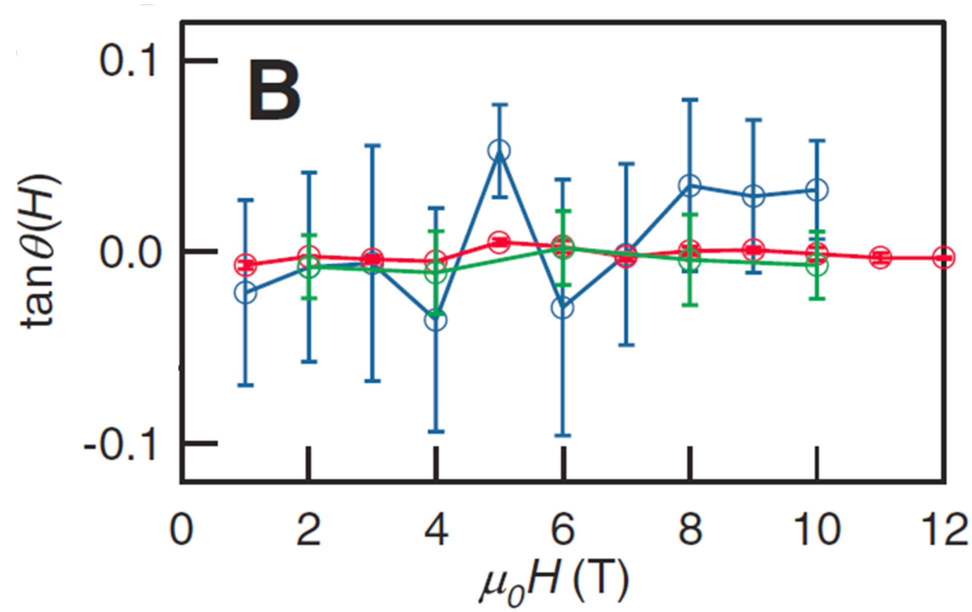
Even more subtle are possible quantum critical magnetic
phase (with weak disorder).

Candidate spin liquid material: dmit

- Thermal transport measurements:
  - As T→0, $\kappa_{xx}/T \to$ finite const.
  - Thermal Hall angle is almost zero, within error bar.

Infinite violation of WF!



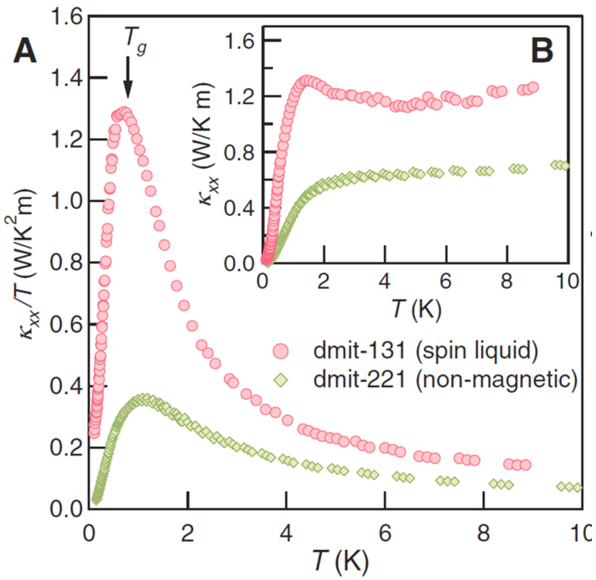


$$\tan[\theta(H)] = \kappa_{xy}/\kappa_{xx}$$

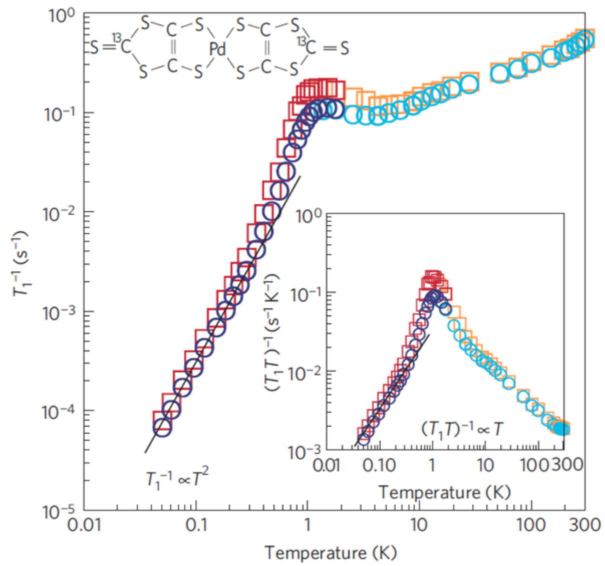
M. Yamashita et al, Science 2010

Candidate spin liquid material: dmit

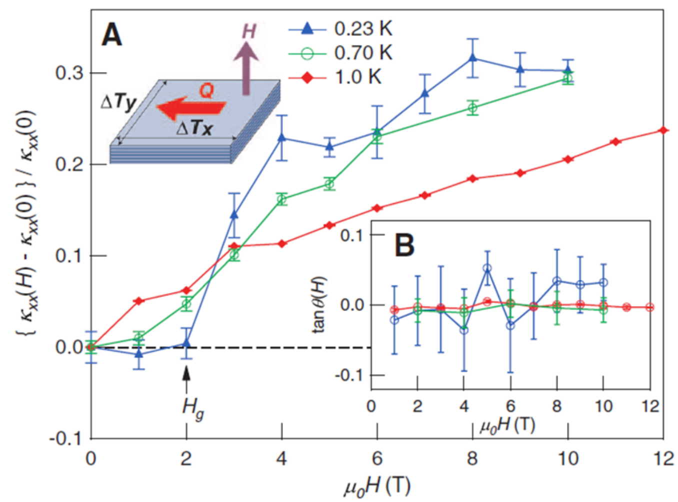
- Some anomalous features around 1K or 2T.



M. Yamashita et al, Science 2010



Itou et al, Nat. Phys. 2010



M. Yamashita et al, Science 2010

Phase transitions around 1K?

Which spin liquid is dmit?

Suppose we take as a working hypothesis

- The experiments are taken at face value.
    1) specific heat: $C \sim (k_B)^2 \rho\, T$, ρ = density of states.
    2) susceptibility: $\chi \sim \rho\, (\mu_B)^2$
    3) thermal conductivity: $\kappa_{xx} \sim T$
    4) thermal Hall angle ~ 0 within error bar

- Disorder does not play a fundamental role.
    1) from thermal conductivity, mean free path l 0.5
    2) from NMR relaxation measurement, stretch exponent ~ 1.

Is there a theoretically consistent candidate spin-liquid which is either stable or at most marginally unstable, which is consistent with these "facts?"

Theory of Superconducting Phases with Stable Fermi Surfaces

BdG Hamiltonian for a d-wave superconductor in the presence of additional broken symmetries.

$$H(\vec{k}) = \sum_a \Psi^\dagger \left[\tau_z \vec{v}_{Fa} \cdot (\vec{k} - \vec{q}_a) + \tau_x \vec{v}_{\Delta a} \cdot (\vec{k} - \vec{q}_a)\right] \Psi + H' + H''$$

$$\Psi(\vec{k}) = \langle \psi_{\vec{k},\uparrow}, \psi^\dagger_{-\vec{k},\downarrow} \rangle$$

Now, add a perturbation due to “competing” orders:

$$H' = \sum_a \Psi^\dagger \left[\phi_a \tau_z + \tilde{\phi}_a \tau_x\right] \Psi$$

$$H'' = \sum_a \Psi^\dagger \left[\Phi_a + \tilde{\Phi}_a \tau_y\right] \Psi + \ldots$$

H’’ breaks time reversal symmetry etc.

E. Berg *et al*, PRL **100**, 027003 (2008)

BdG Hamiltonian for a d-wave superconductor in the presence of additional broken symmetries.

Nodal quasiparticles persist in d-wave superconductor with weak coexisting -

Nematic order

Commensurate Charge Density Wave (*e.g.* stripe) order
(unless the ordering vector spans two nodes)

Commensurate Spin Density Wave (*e.g.* stripe) order

(unless the ordering vector spans two nodes)

D-density wave order; Neel order; etc.

E. Berg *et al*, PRL **100**, 027003 (2008)

Two (time reversal symmetry breaking) examples of d-wave superconductors with pseudo Fermi surfaces

1) Superconducting ferromagnet with spin-polarized Fermi pockets.

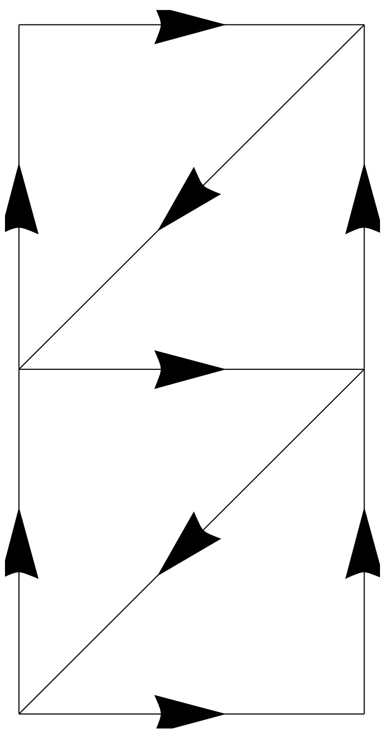
$$H'' = -h \sum_a \Psi^\dagger \Psi = -h \sum_a \left[\psi^\dagger_{\vec{k}\uparrow} \psi_{\vec{k}\uparrow} - \psi^\dagger_{-\vec{k}\downarrow} \psi_{-\vec{k}\downarrow} \right]$$

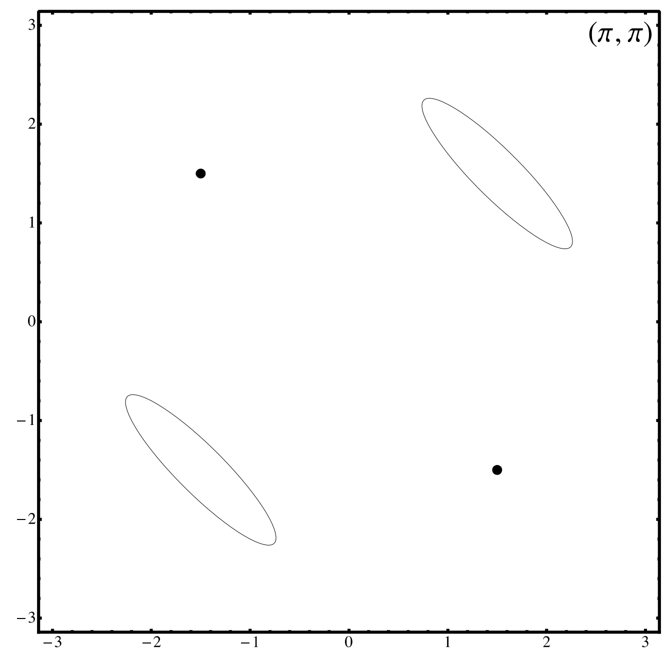
This “pseudo-Fermi surface” is marginally unstable (Cooper instability) to triplet pairing.

Broken time-reversal symmetry but preserves inversion symmetry

Two (time reversal symmetry breaking) examples of d-wave superconductors with pseudo Fermi surfaces

2) Superconductivity with coexisting "Varma" current loop order





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Two (time reversal symmetry breaking) examples of d-wave superconductors with pseudo Fermi surfaces

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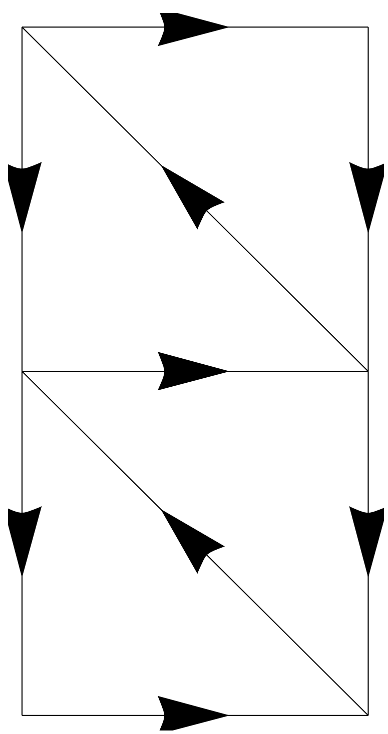


Breaks time-reversal and inversion.

Produces two pseudo-Fermi pockets that are perturbatively stable!

$$\epsilon_{\vec{k}} \neq \epsilon_{-\vec{k}}$$

E. Berg *et al*, PRL **100**, 027003 (2008)

A Second Class of Superconducting States with Pseudo-Fermi surfaces:

Pair-density-wave and its generalizations.

Generalized definition of a pair-density wave phase:

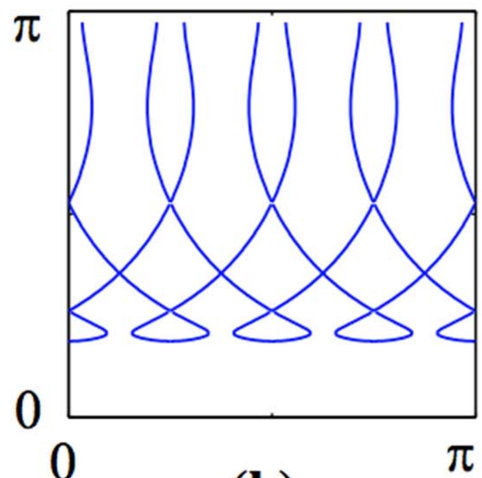
$$T_{\lambda/2}(\Delta_{PDW}) = - \Delta_{PDW}$$

therefore the spatial average of all components = 0

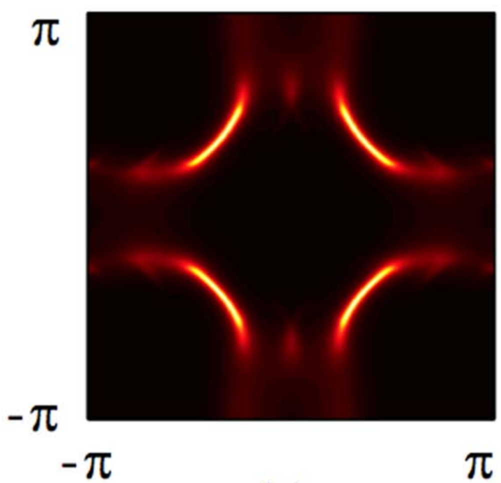
Issue of gauge invariance
and the proper definition
of Δ is technical and I can
discuss it later.

Himeda et al (2002)
E. Berg *et al*, PRL **100**, 027003 (2008)
S. Baruch and D. Orgad, PRB (2008)
M. Zelli, C. Kallin, and A.J. Berlinsky,
PRB (2011).
Mross and Senthil, PRL **108** (2012)

A Second Class of Superconducting States with Pseudo-Fermi surfaces: Pair-density-wave and its generalizations.



(b)



(e)

Structure of the Fermi surface for a period 8 d-wave-like pair-density wave.

S. Baruch and D. Orgad, PRB (2008)

A Second Class of Superconducting States with Pseudo-Fermi surfaces: Pair-density-wave and its generalizations.

Stability analysis with respect to uniform superconductivity:

$$\delta F = \cancel{\alpha \left[\Delta^* \Delta_{PDW} + c.c.\right]}$$

$$+u_1 |\Delta|^2 |\Delta_{PDW}|^2 + u_2 \left[\Delta^* \Delta_{PDW} + c.c.\right]^2 + \ldots$$

$$+v \left[\Delta_{4e}^* \Delta_{PDW}^2 + c.c.\right] + \ldots$$

Implies existence of uniform charge 4e superconducting order

Gentle melting of PDW can lead to uniform charge 4e superconductor presumably with a Fermi surface.

U(1) gauge symmetry broken to Z_4

(F.S. is, presumably, marginally unstable.)

E. Berg et al, PRB (2007)
E. Berg et al, Nature Phys (2009)

Issue of stability and fluctuations:

If $E(\mathbf{k}) \neq E(-\mathbf{k})$, quasi-particle FS is perturbatively stable.

If $E(\mathbf{k}) = E(-\mathbf{k})$, quasi-particle FS is marginally unstable.

Gauge field fluctuations and fluxoid excitions –

(There are differences depending on dimensionality, etc.
Vortex lines (in 3d) vs. point vortices (in 2d).)

Gapped by Higgs mechanism – breaks U(1) to Z_{2n}

Quantized vortices with $\phi = \pm\phi_0/2$ (Z_2) or $\phi = \pm\phi_0/4$ (Z_4)

(There are some differences in the spin-liquid realization.)

Now to spin liquids

Some scars from the spin liquid wars:

$$c_\sigma = f_\sigma c_1 c_2 b_1^\dagger b_2$$

Hafnian phase

Topological insulator

Bose condensate

331 state

Some scars from the spin liquid wars:

Ideally, we would like, for each candidate spin-liquid:

1) Controlled or exact solution of "engineered" model.

2) Effective field theory which permits perturbative stability analysis.

3) Some (presumably numerical) evidence that it exists in "non-engineered" models – preferably one that is physically realizable.

4) Of course, best of all, an experimental realization.

Fractionalization and emergent gauge theories:

Electron breaks into spinons (neutral) and holons (charged)
coupled to wildly fluctuating U(1) gauge field.

Insulating state = holon vacuum.

With luck, this leads to low energy theory of
fermionic spinons coupled to emergent U(1)
“statistical” gauge field.

Baskaran and Anderson; Fradkin and SAK;
Laughlin; Wen, Wilczek and Zee;
Sachdev and Read; Weigmann and Polyakov;
Nayak, Balents, and Fisher; Senthil and Fisher
Lee and Wen; etc.

Fractionalization and emergent gauge theories:

With luck, this leads to low energy theory of
fermionic spinons coupled to emergent U(1)
"statistical" gauge field.

The low energy physics now looks like that of an interacting
spinon metal but with $\alpha_{eff} \sim 1$.

What are the stable
(or at most marginally unstable)
phases that can occur?

Fractionalization and emergent gauge theories:

The low energy physics now looks like that of an interacting spinon metal but with $\alpha_{eff} \sim 1$.

In d=2, gauge fluctuations make the spinon Fermi liquid highly unstable.

This could lead to a non-Fermi liquid (with non-Fermi liquid power laws).

(This is not a viable “candidate” according to the stated criteria.)

Fractionalization and emergent gauge theories:

The low energy physics now looks like that of an interacting spinon metal but with $\alpha_{eff} \sim 1$.

This could lead to a uniform spinon-paired superconducting state = "fully gapped Z_2 spin liquid."

Note that the Anderson-Higgs phenomenon quenches the gauge field fluctuations.

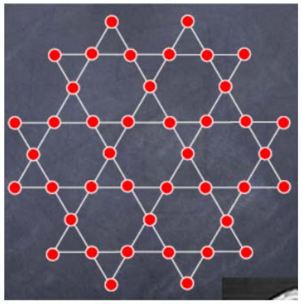
The existence of such a phase in exactly solvable models has been well established, starting with the quantum dimer model on the triangular lattice (Moesner &Sondhi)

The existence of such a phase in "natural" models has become increasingly convincing from numerics, for instance in the J_1-J_2 model on the square lattice - Figuerido *et al* 1989, Jiang, Balents and Yao (2012) …

Search for quantum spin liquids: numeric studies

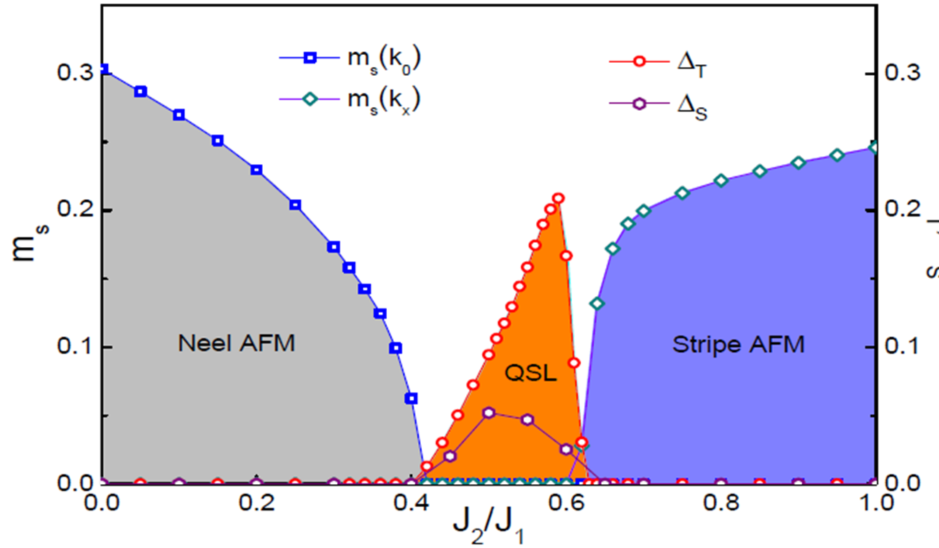
- DMRG studies of the Kagome Heisenberg antiferromagnet: A short-range RVB-like state.

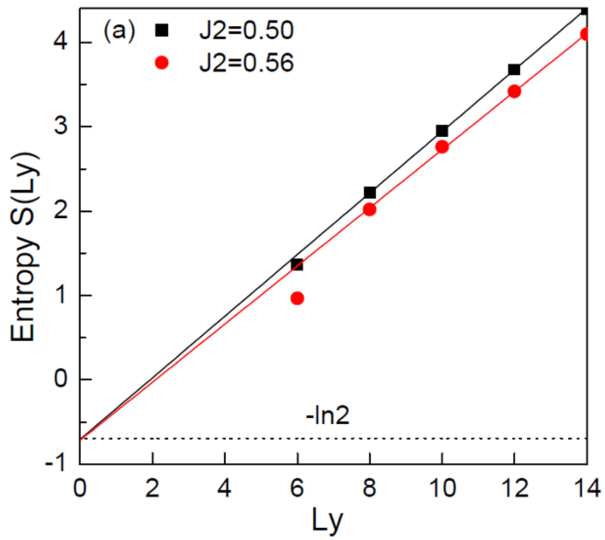
    S. Yan, Huse & White, Science 2011;
    H-C. Jiang, Z.-Y. Weng, D. N. Sheng, PRL 2009;
    Dependrock, McCulloch, and Schollwoeck PRL 2012;



- The J_1-J_2 AF Heisenberg model on the square lattice

    - DMRG (H.-C. Jiang, HY, and Balents, PRB 2012)





    - Tensor network state (L. Wang, Z.C. Gu, X.G. Wen, and Verstraete, 2011)

(This is not a viable "candidate" according to the stated criteria.)

Fractionalization and emergent gauge theories:

The low energy physics now looks like that of an interacting spinon metal but with $\alpha_{eff} \sim 1$.

This could lead to a uniform, nodal spinon-paired superconducting state = "nodal Z_2 spin liquid."

The existence of such a phase in exactly solvable models has been well established, starting with the Kitaev model on a honeycomb lattice.

(This is not a viable "candidate" according to the stated criteria.)

Fractionalization and emergent gauge theories:

The low energy physics now looks like that of an interacting spinon metal but with $\alpha_{eff} \sim 1$.

This could lead to a uniform, spinon-paired superconducting state with broken time-reversal and reflection symmetry (like the loop ordered state).

The existence of such a phase in exactly solvable models has been established by us in a version of the Gamma Matrix Model.

This phase is stable, has a spinon pseudo-Fermi surface, but it spontaneously breaks time-reversal symmetry and requires a finite T phase transition at which the broken symmetries occur!

(This is a viable “candidate” according to the stated criteria.)

Fractionalization and emergent gauge theories:

The low energy physics now looks like that of an interacting spinon metal but with $\alpha_{eff} \sim 1$.

This could lead to a pair-density wave analogue (called “Amperian pairing”).

This phase has a spinon pseudo-Fermi surface which is only marginally unstable, and with no spontaneously broken symmetries.

(This is not a viable “candidate” according to the stated criteria.)

S-S. Lee, P.A. Lee, and T. Senthil, PRL (2007)

Fractionalization and emergent gauge theories:

The low energy physics now looks like that of an interacting spinon metal but with $\alpha_{eff} \sim 1$.

This could lead to a weakly melted version of the spinon-pair-density-wave, which would result in a uniform Z_4 spin liquid.

This phase has a spinon pseudo-Fermi surface which is only marginally unstable, and with no spontaneously broken symmetries.

It has half quantum fluxoids

Berg, Fradkin, and SAK   Radzihovsky and Vishwanath

Salient features of the putative spin-liquid insulator, $EtMe_3Sb[Pd(dmit)_2]_2$

It is an insulator with one electron per unit cell.

There is no magnetic ordering to the lowest T, although J ~ 250K.

C/T approaches 20mJ/mole K^2 as T tends to zero (with caveats).

The Wilson ratio, χ T/C, is approximately 1.

The thermal conductivity, $\kappa/T \approx 0.1$-.02 W/K^2m

There is a peak in C at around 1 K which could signify a rounded phase transition, but also might not.

The role of disorder is …

Otherwise, it is an excellent candidate for a spin-liquid with a pseudo-Fermi surface.

Bottom line:

1) The only stable candidate is a spin-liquid version of a current loop ordered unconventional superconductor:

   This must involve a finite T phase transition to a state of broken time-reversal and inversion symmetry.

   This has a simple field theoretic description and is the exact solution of an engineered model

   We have no evidence that it occurs in any physically reasonable “natural” model.

Comment about vortices in Z_2 spin liquid

In quantum dimer model (as in Z_2 lattice gauge theory)
the vortex (vison) does not break time-reversal symmetry –
it is a singlet excitation.

In an Abelian-Higgs model, the vortex and the anti-vortex are
related by time-reversal symmetry, but are distinct!

However, if lattice effects are taken into account, they
mix via quantum tunnelling –
splitting between the symmetric (time reversal even
and antisymmetric (time reversal odd) superposition
is exponentially small in strongly Type II limi.

SAK, PRB 1989.

Bottom line:

1) The only stable candidate is a spin-liquid version of a current loop ordered unconventional superconductor:

2) There are several viable marginally unstable candidates:

a) Spin liquid version of PDW. (Could be singlet or triplet.)

Probably, this breaks spatial symmetries and so requires a finite T transition.

b) Could be a melted version of this – *i.e.* a Z_4 spin liquid.

No phase transition required.

Thank you for listening

References:

M. Barkeshli, H. Yao, and SAK, “Gapless spin liquids: stability and possible experimental relevance,” arxiv:1208.3869 (2012).

SAK and C. M. Varma, “Fermi pockets in a d-wave superconductor with coexisting loop-current order,” arxiv:1208.6498 (2012).

E. Berg, E. Fradkin, and S. A. Kivelson, “Charge-4e superconductivity from pair-density wave order in certain high-temperature superconductors,” Nature Physics **5**, 830 (2009).