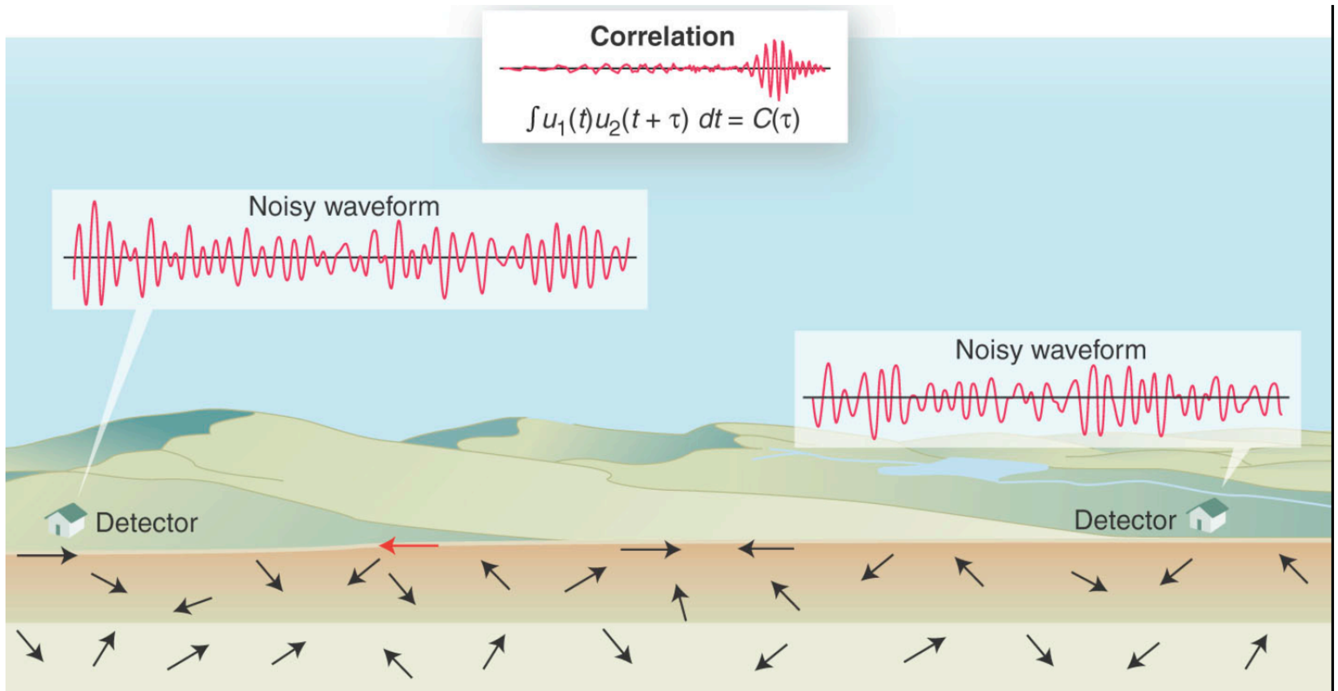
Seismic Noise Correlations

- RL Weaver, U Illinois, Physics



Karinworkshop
May 2011

Over the last several years, Seismology

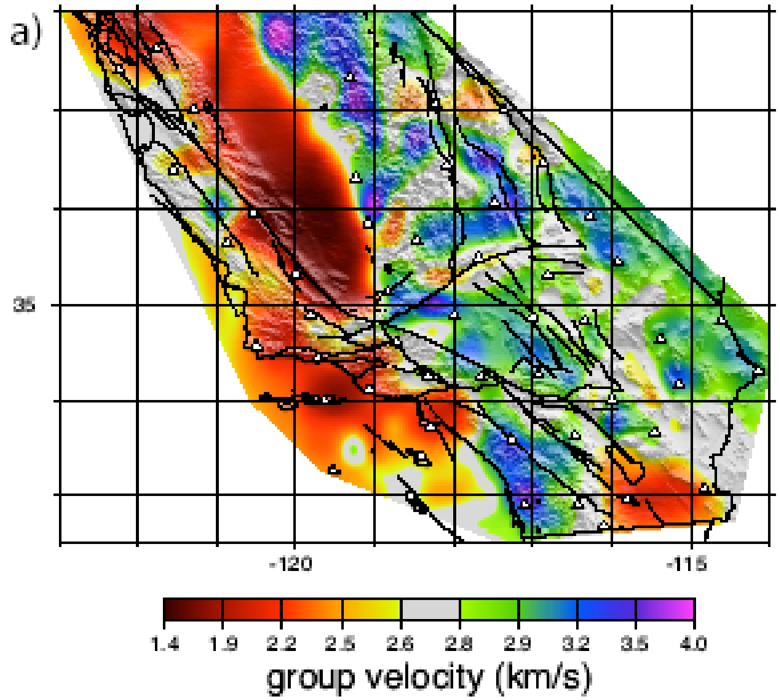
has focused growing attention on
Ambient Seismic Noise
and its Correlations.

Citation count on one of the seminal papers:

2011	20	6.7114 %	to date
2010	75	25.1678 %	
2009	60	20.1342 %	
2008	57	19.1275 %	
2007	40	13.4228 %	
2006	37	12.4161 %	
2005	9	3.0201 %	

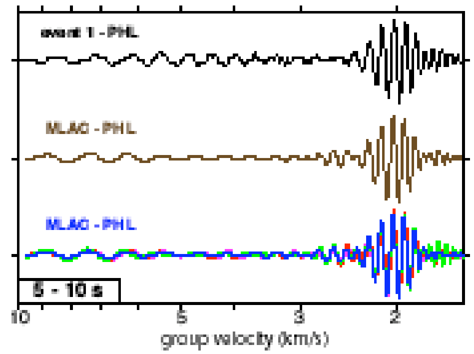
High-resolution surface-wave tomography from ambient seismic noise
NM Shapiro *et al* *SCIENCE* **307** 1615-1618 MAR 2005

The reason is (in part) due to the striking maps of seismic velocity that noise reveals . . .



A map of Surface-Wave Velocity in California

Obtained from correlating seismic noise



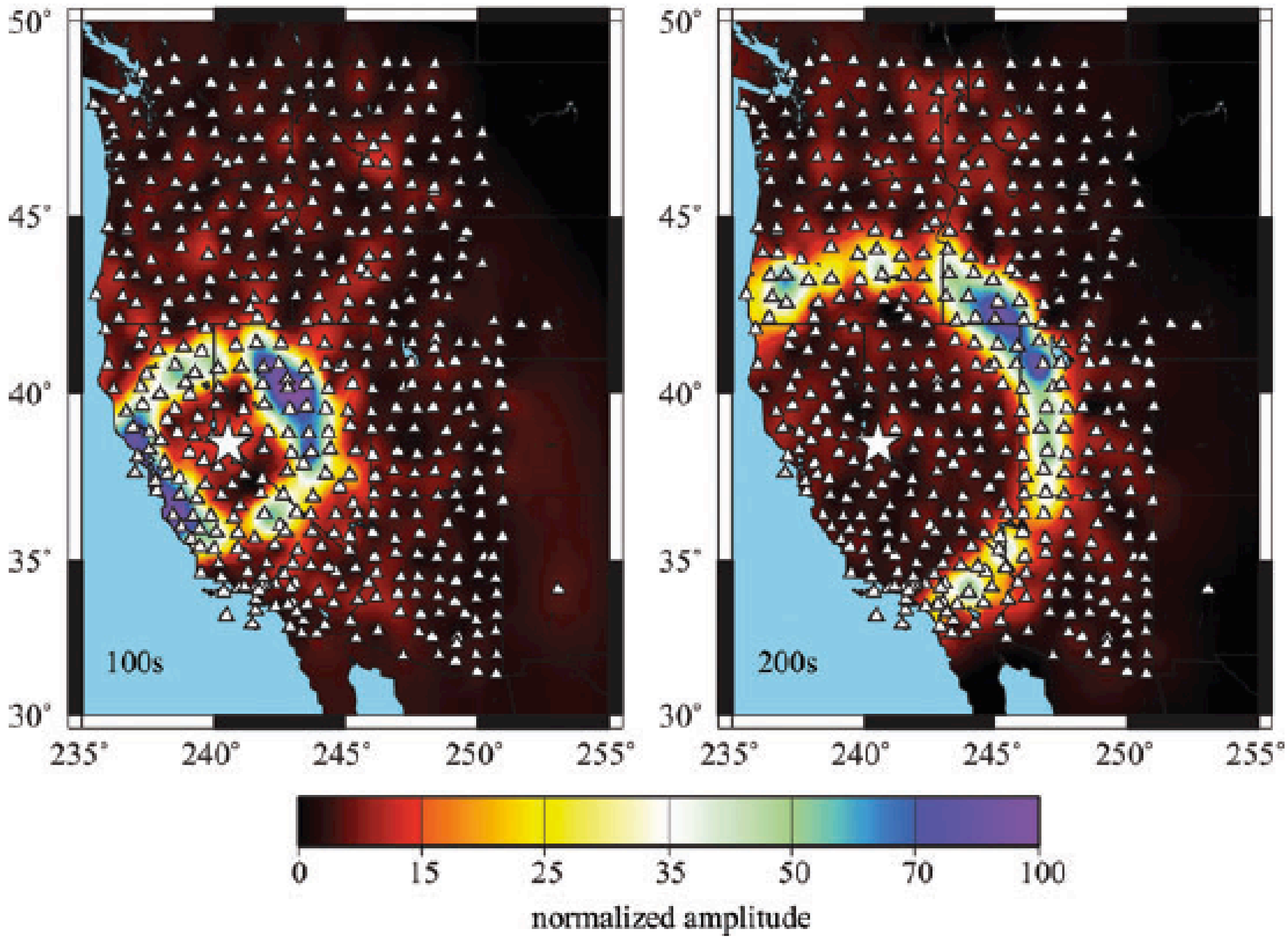
earthquake

1 year of correlations

4 one-month correlations

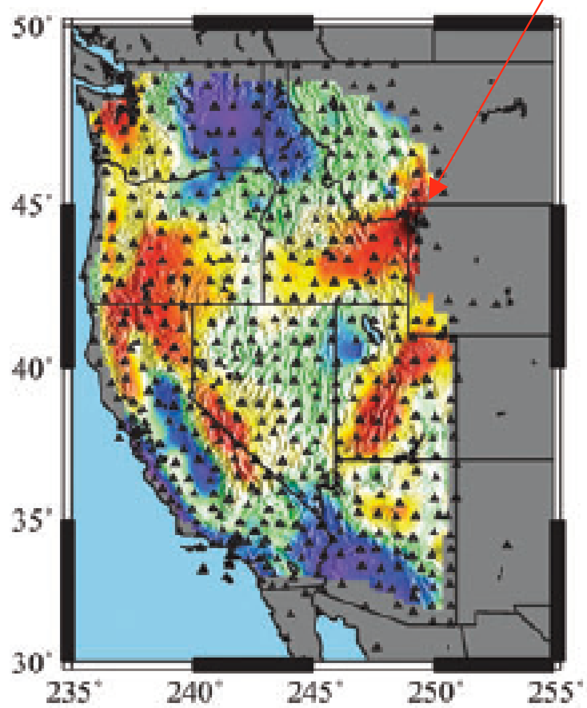
Frequencies ~0.02 < f < 1 Hz; 3km < λ <150 km

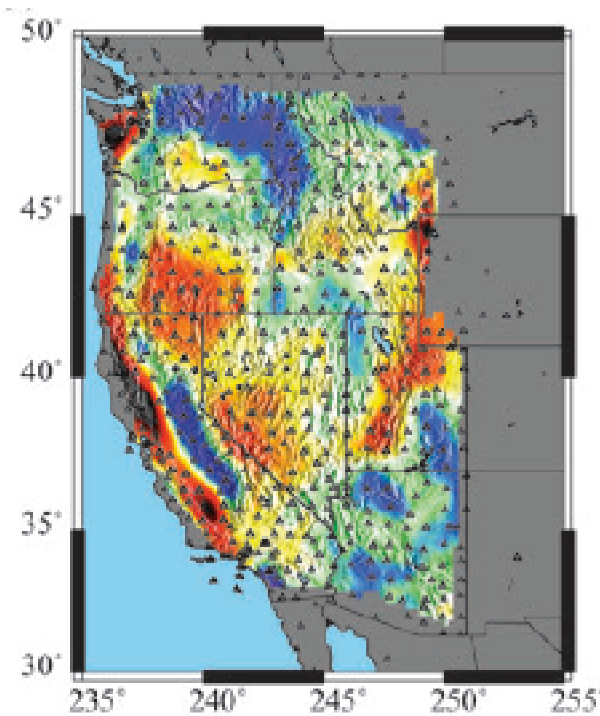
Lin and Ritzwoller and Snieder (2009) Geophys J Int
3 years of data on a bigger array



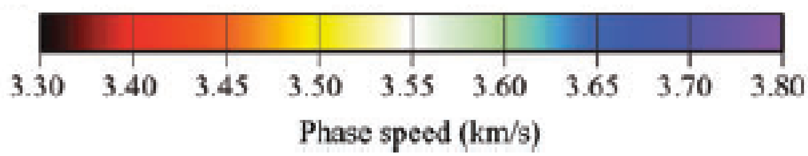
Tomographically generated maps of wave speed

Hot spot in Yellowstone

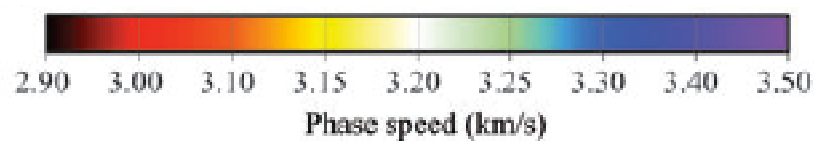




24 sec

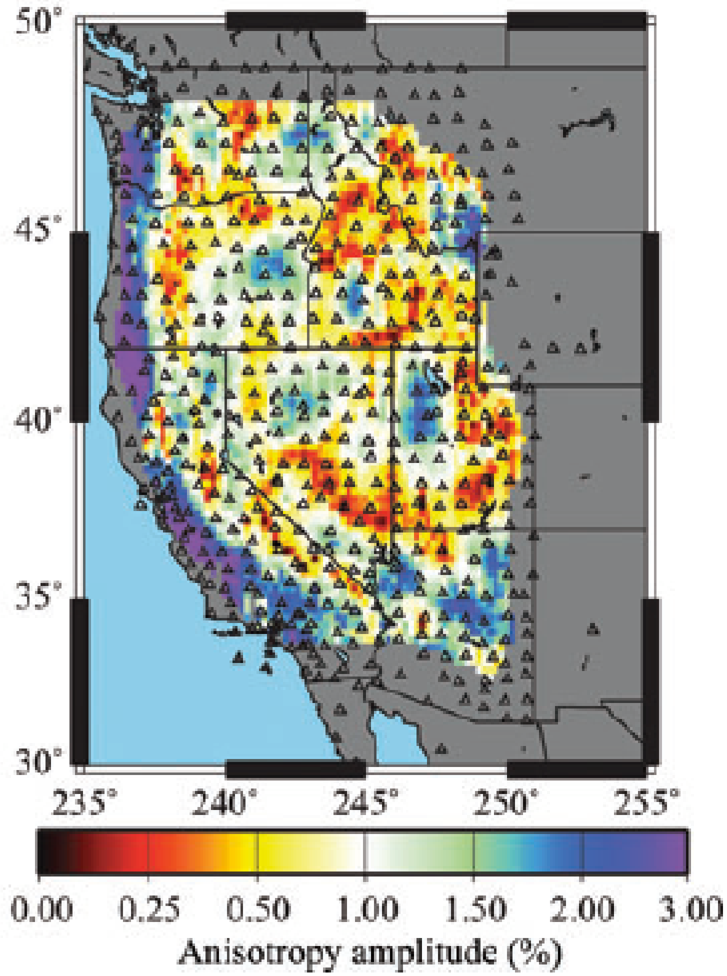


12 sec



Different properties at different frequencies
i.e, different depths

They even resolve ~ 1% anisotropies in wave speed



Main Assertion of Theory:

$$G(\vec{x},\vec{y};\tau) \sim \frac{\partial}{\partial \tau} < \psi(\vec{x},t)\psi(\vec{y},t+\tau) > ?$$

Correlation of a diffuse field $\psi(\vec{x},t)$

gives the Green's function

(+ sundry fine print and qualifications)

Where did this assertion come from?

Why should we believe it?

How *much* should we believe it?

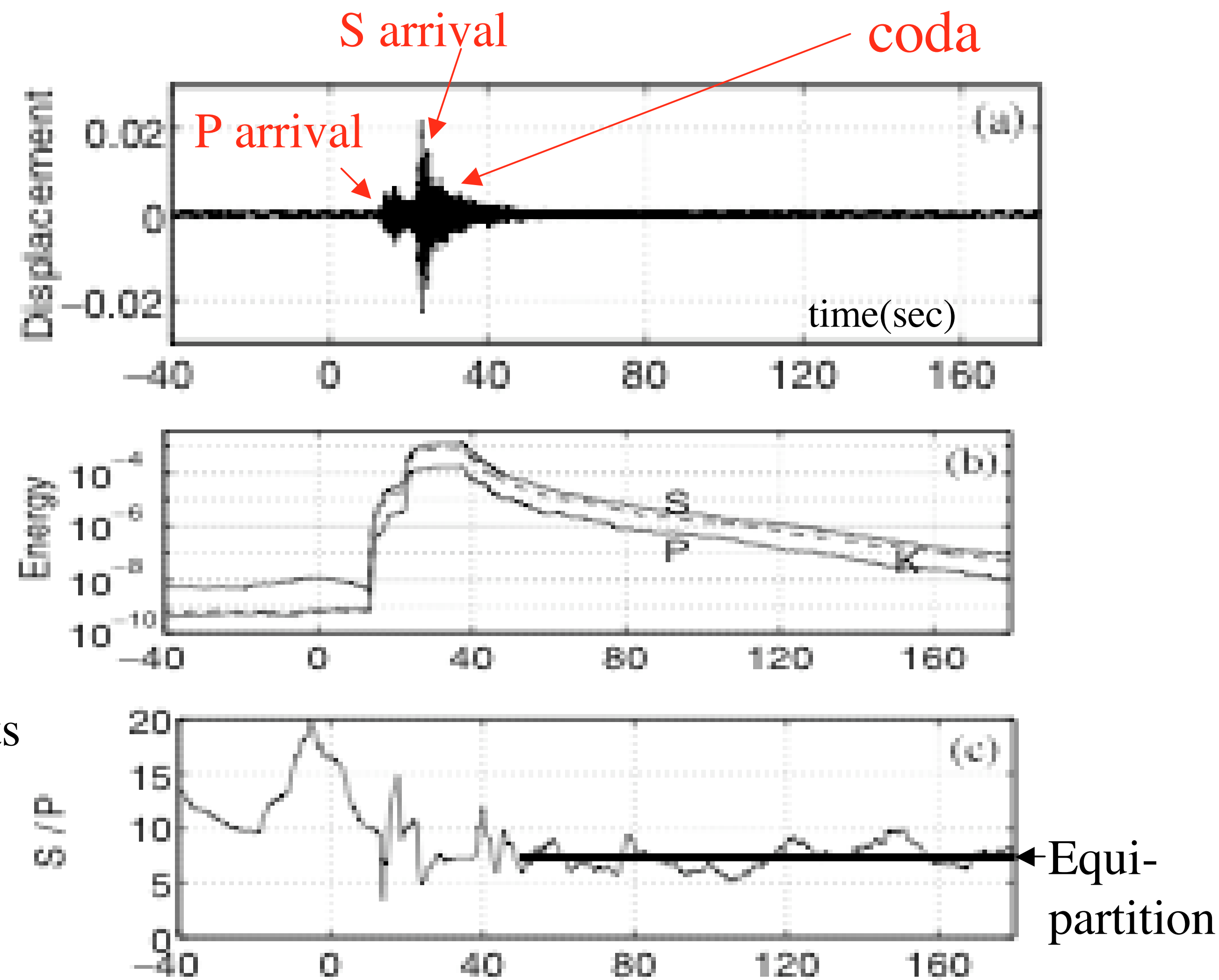
History of the approach . . .

Conversations with a seismologist, at a 1999 workshop, about the seismic coda - which appeared to be equipartitioned

An earthquake record

Ray arrivals are followed by low amplitude noise, or "*coda*"

The coda appears to achieve a steady state ratio of its energy contents For example, its shear-to-dilational energies: S/P



Phys Rev Lett **86** 3447-50 (2001)

History of the approach continued. . .

I then pointed out

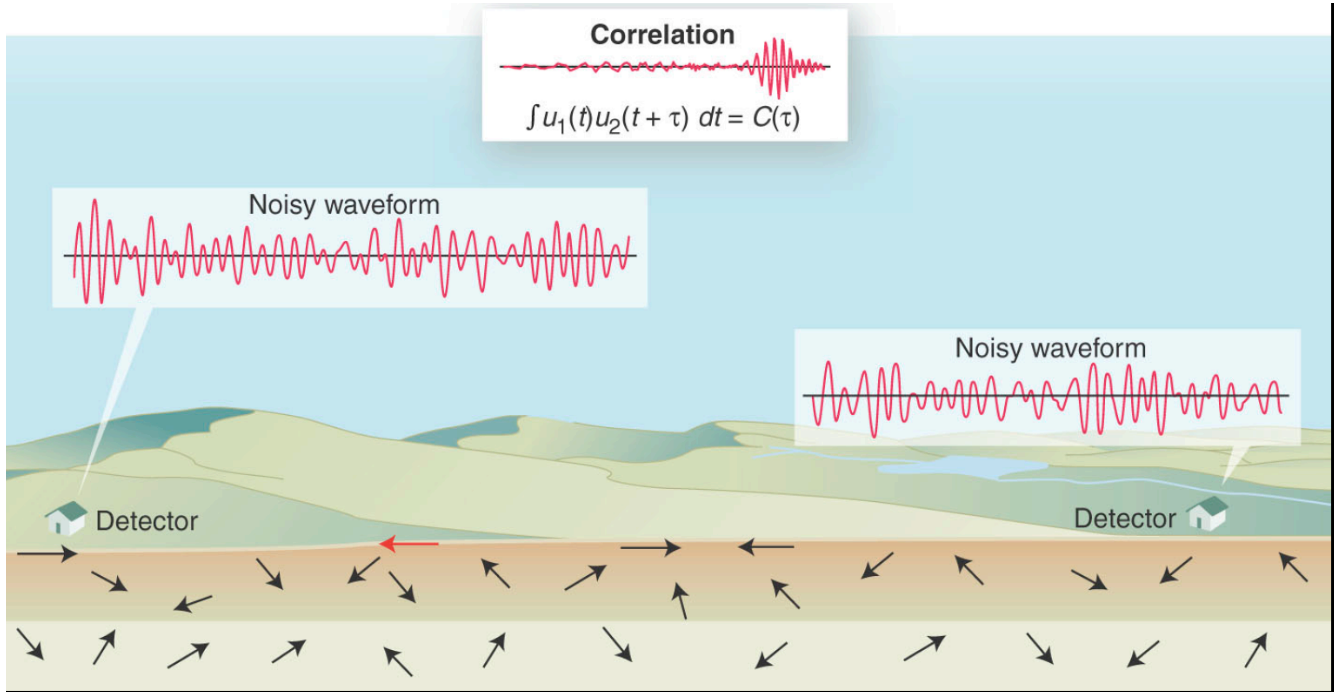
"if a wave field (e.g. seismic coda) is multiply scattered to the point of being equipartitioned, the field's correlations should be Green's function,

And we could recover lots of information without using a controlled source"

Geophysicist: "useful, if true"

Physicist: "Nonsense, can't possibly be true"

Hand-waving plausibility argument . . .



that there could well be a signature, an "arrival,"
at the correct travel time
- due to those few rays that happen to be going the right way
  But G exactly?
  And where's the proof?
  And won't other ray directions obscure the effect?

Standard Proofs . .

- For a thermally diffuse field
  - modal picture
  - fluctuation-dissipation theorem
- For a conventional acoustic diffuse field
  - modal picture (sensible only for closed systems)
  - plane wave picture (sensible only for homogeneous systems)
- Systems with uniformly distributed incoherent sources everywhere
- Heterogenous loss-free region without sources,
  but *insonified* by an external diffuse field

But what about imperfectly diffuse fields?

- There is an asymptotic ~validity to assertion

The simplest proof involves a common definition of a fully diffuse field, from room acoustics or physics of thermal phonons:
in terms of the normal mode expansion for the field in a finite body

$$\phi(x,t) = \Re \Sigma_{n=1}^{\infty} a_n\, u_n(x) \exp\{i\omega_n t\}$$

$$< a_n a_m^* > = \delta_{nm} F(\omega_n) / \omega_n^2$$ "equipartition"

n.b: this follows from maximum entropy where F ~ energy per mode ($k_B T$)

$$C \equiv < \phi\,(x,\, t)\;\; \phi\,(y,\, t+\tau) > \;=\; \frac{1}{2} \Re\, \Sigma_{n=1}^{\infty} F(\omega_n)\, u_n(x)\, u_n(y) \exp\{-i\omega_n \tau\} / \omega_n^2$$

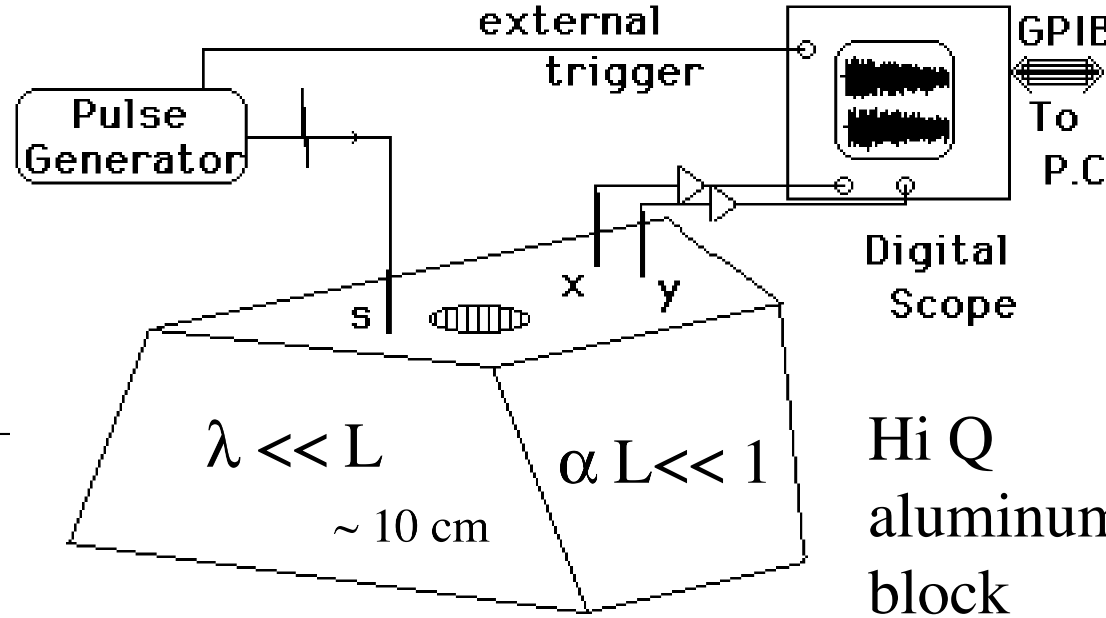
Compare with G . . .

$$G_{xy}(\tau) = \Sigma_{n=1}^{\infty} u_n(x)\, u_n(y)\, \frac{\sin \omega_n \tau}{\omega_n} \qquad [\text{ for } \tau > 0 \text{ , } 0 \text{ otherwise}]$$

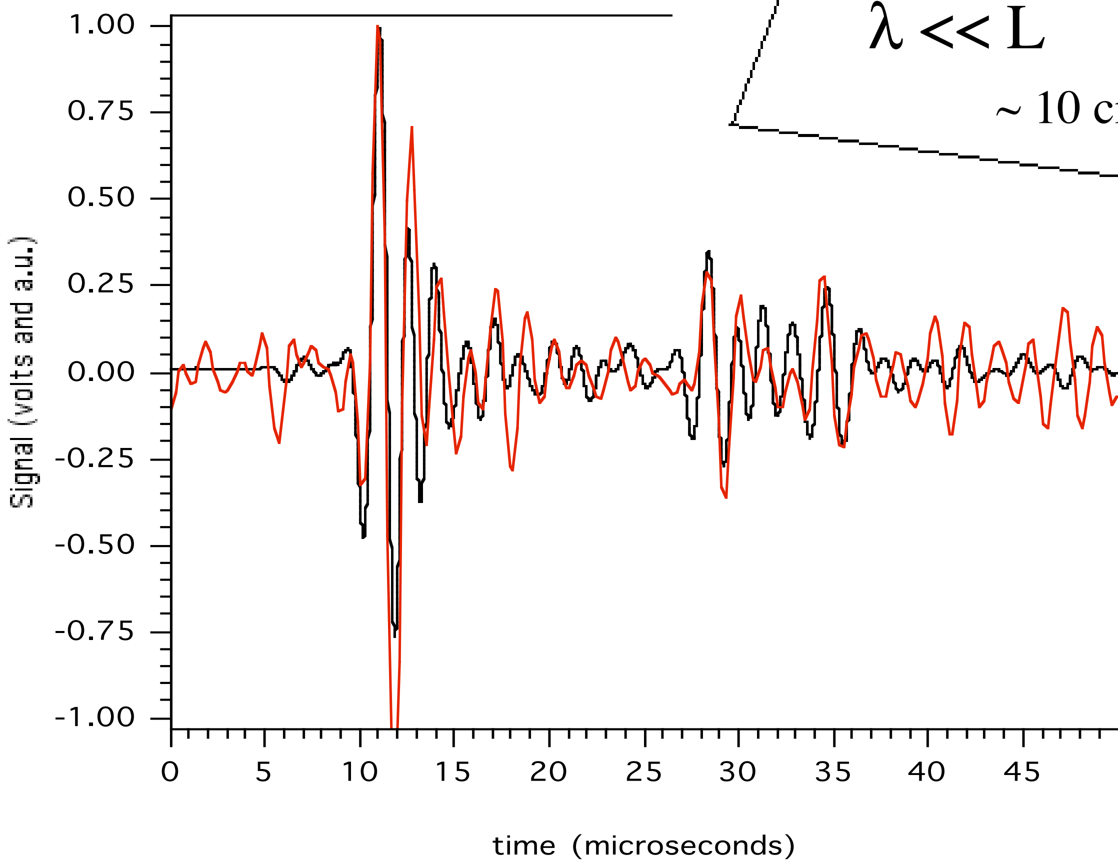
So, $\partial C/\partial \tau = G - G^{\text{time reversed}}$, i.e, $G - G^*$ or $\text{Im}\, G$ if *F* is constant

Verification?

e.g. . . .



Hi Q aluminum block



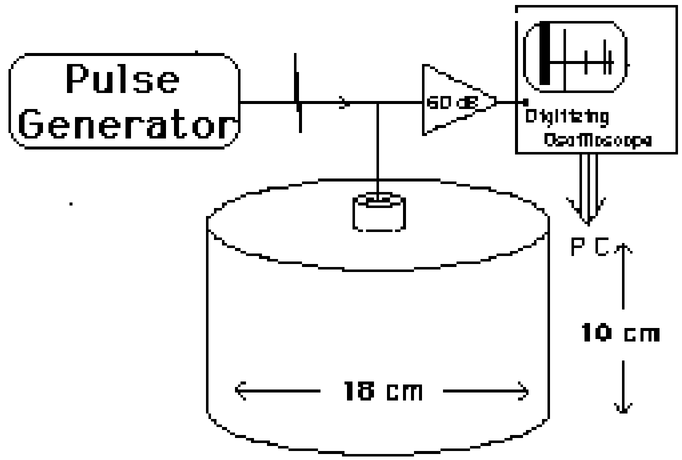
direct pitch/catch signal

correlation

J Acoust Soc Am. **110**, (2001)

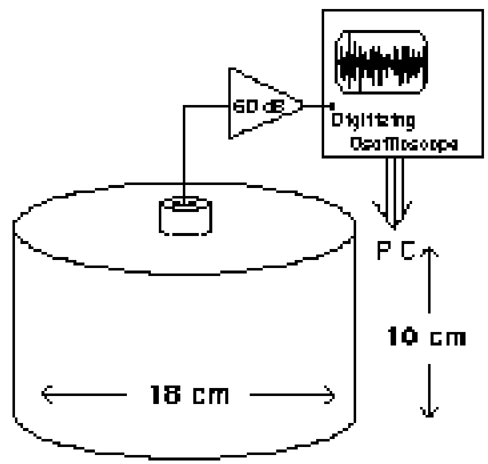
Comparison of a
Direct Pulse-Echo
Signal,

(conventional ultrasonics)



and

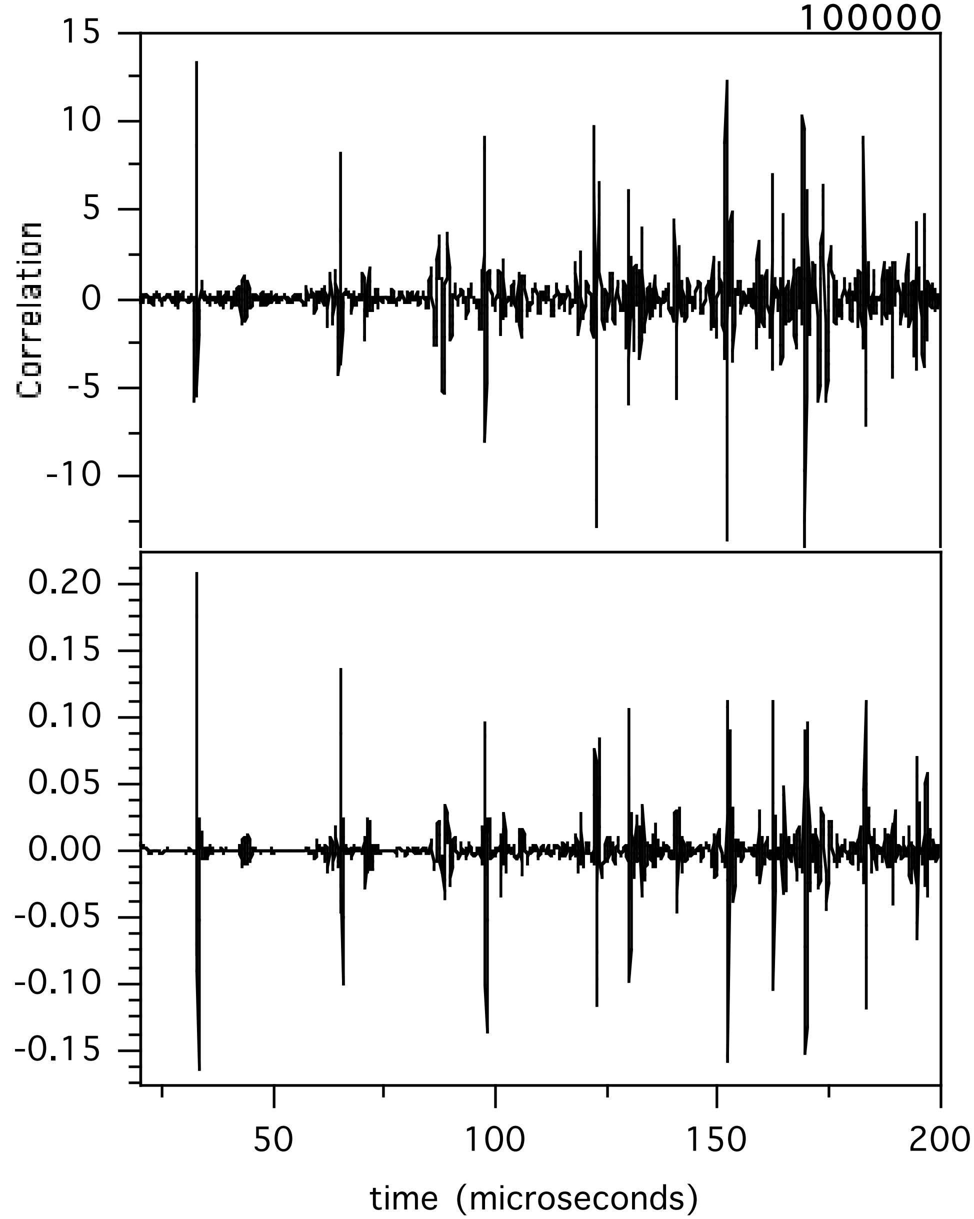
Thermal Noise
Correlation



Phys Rev Lett **87** 134301 (2001)

Correlation
Of
Thermal noise

rms $u \sim 3$ fm/$\sqrt{}$MHz



After
Capturing
320 seconds
Of data

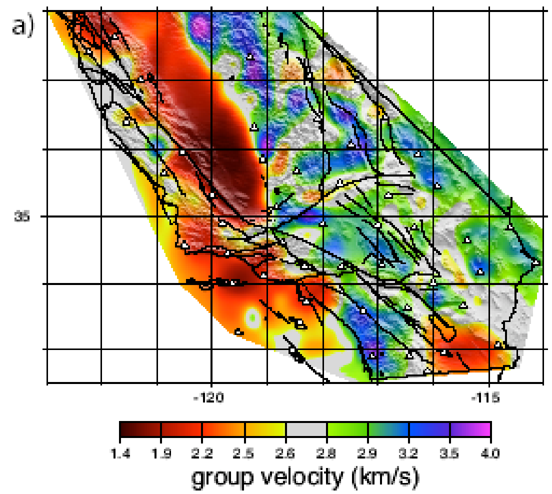
(and taking 2.5
hours to do so)

Direct
Pulse-Echo
Signal

But proofs that require full diffusivity
and/or finite bodies and closed acoustic systems,
May not be relevant for practice.

Ambient seismic noise(*), for example, is
NOT fully diffuse
It has preferred directions (sources in ocean storms)

Nevertheless, these
maps are impressive

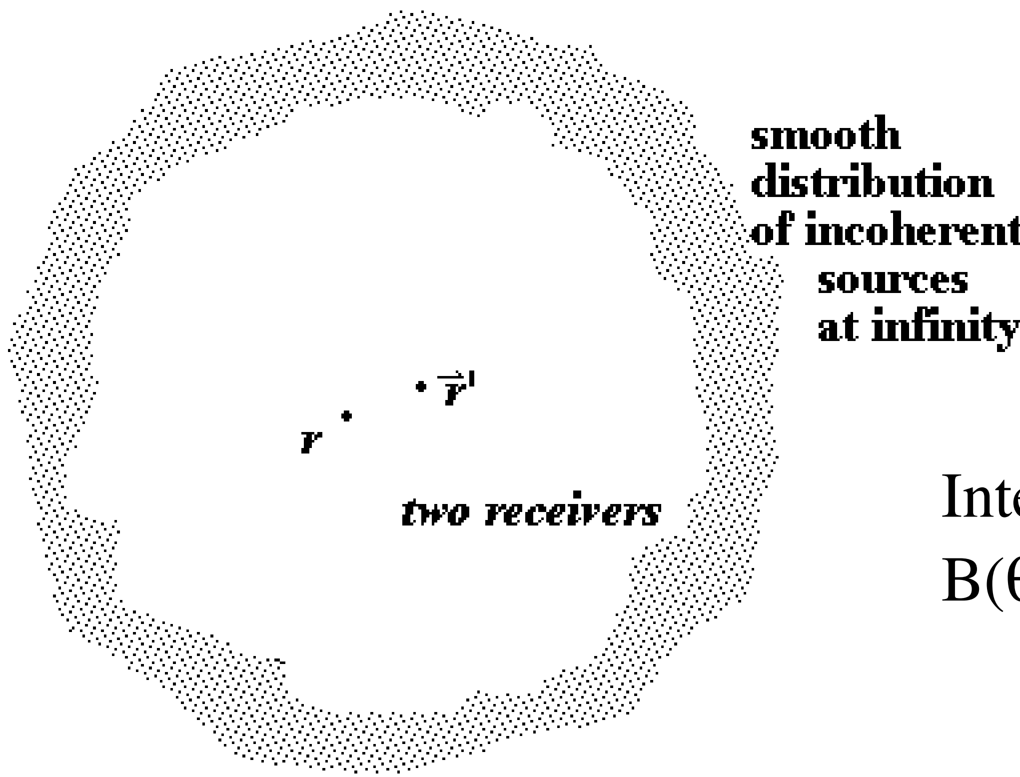


Why does it work?

*Late *coda* appears
fully diffuse, but
there isn't enough of it.

What if an incident field does not have isotropic intensity?
What it it is not equipartitioned?

Consider a homogeneous medium with incoherent sources at infinity



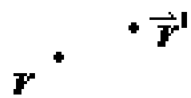
Intensity distribution
B(θ)

The field in the vicinity of the origin is a superposition of plane waves

$$\psi(\vec{r},t) = \int A(\theta) \exp(-ik\hat{\theta} \bullet \vec{r} + i\omega t)\, d\theta \qquad (\text{2-d})$$

with $\quad < A >= 0; \qquad < A(\theta)A^*(\theta') >= B(\theta)\delta(\theta - \theta')$

i.e, an incident plane wave intensity B(θ)



Which implies that the field-field correlation is

$$< \psi(\vec{r},t)\psi(\vec{r}\,',t') >= \int B(\theta) \exp(-i\omega\hat{\theta} \bullet (\vec{r} - \vec{r}\,')/c + i\omega(t - t'))\, d\theta$$

exact

$$C = \langle \psi(\vec{r},t)\psi(\vec{r}\,',t')\rangle = \int B(\theta)\exp(-i\omega\hat{\theta}\bullet(\vec{r}-\vec{r}\,')/c + i\omega(t-t'))\,d\theta$$

wavelet S(t) related to power spectrum of noise

$$= \frac{-1}{4\pi}\sqrt{\tfrac{2\pi}{\omega x}}\int_0^{+\infty} d\omega i\exp(i\omega(t-x/c))\tilde{S}(\omega-\omega_o) \quad \times$$

$$\{B(0)e^{i\pi/4} + B''(0)\tfrac{1}{2\omega x}e^{3i\pi/4} - B(0)\tfrac{i}{8\omega x}e^{5i\pi/4}..) + c.c.$$

Leading term

first correction

Permits us to show that the apparent arrival time is delayed relative to $|r\text{-}r'|/c$ by a fractional amount $B''(0)/2k^2|r\text{-}r'|^2B(0)$

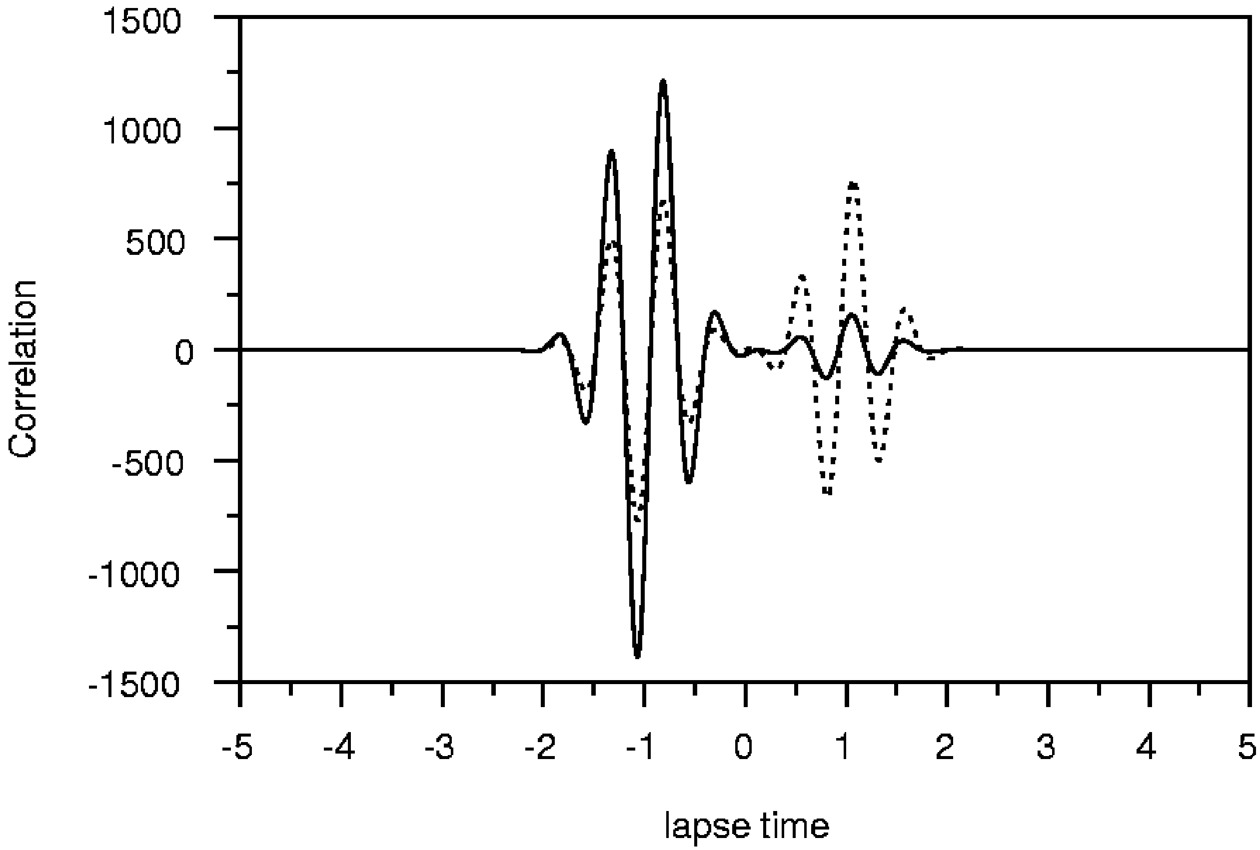
⇒The effect of non-isotropic B or arrival time is small in practice

⇒Hence the high quality of the maps of seismic velocity

even though the ambient seismic noise is not equipartitioned

Comparison of Correlation waveform (solid line)
and time-symmetrized G (dashed line)

For case of non-trivial ponderosity $B(\theta) = 1 - 0.8 \cos \theta$



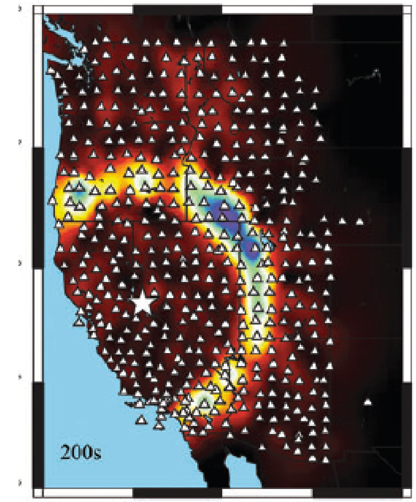
Froment et al 2009

Note: a) assertion fails, $G \neq dC/d\tau$

b) large differences in positive and negative time amplitudes

c) there are *tiny* shifts of apparent arrival time, as predicted

In sum, the method works well for arrival times, hence the good maps



The method is well suited to seismology because

Controlled sources are highly inconvenient, (earthquakes and nuclear explosions)

Recent advent of large arrays of long-period seismic stations and world-side access to their time records

For many years seismologists would record seismic time-records, ignore the noise, and examine the earthquakes

Now they throw out the earthquakes and keep the noise.

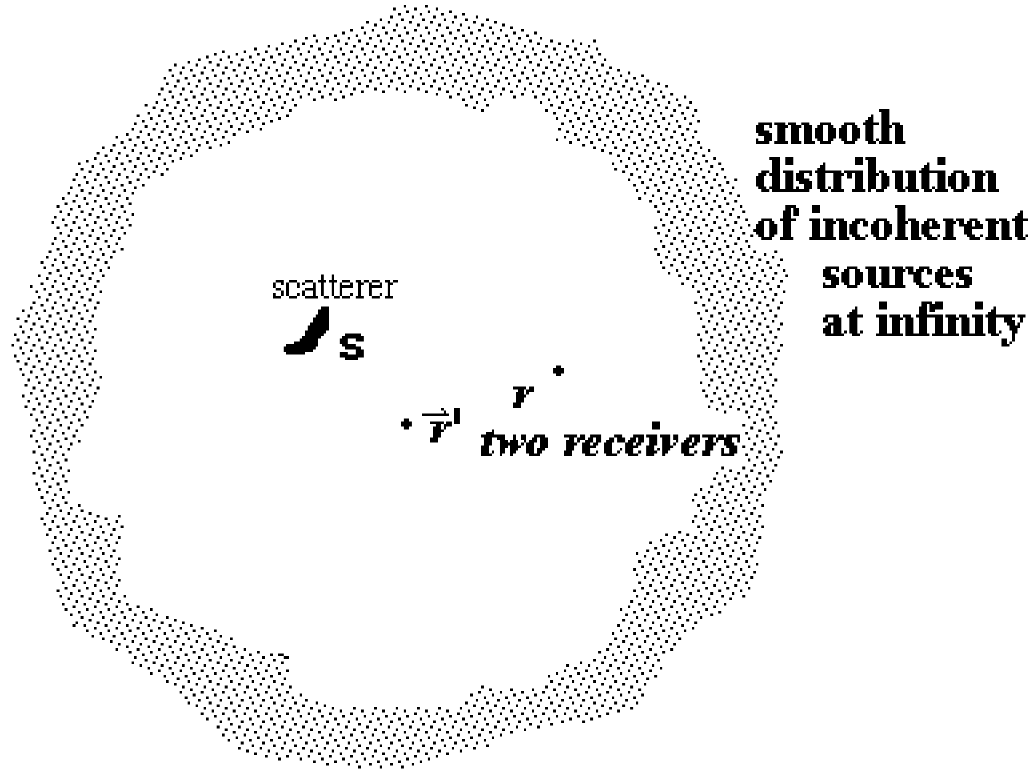
Other consequences of imperfectly partitioned ambient noise:

Spurious features in the correlations due to scatterers

Amplitude information is hard to interpret

Spurious arrivals..

Intensity distribution
$B(\theta)$



Correlations in the presence of a scatterer will show

a direct arrival at $\tau = |\mathbf{r}-\mathbf{r}'|/c$
an indirect arrival at $\tau = |\mathbf{r}-\mathbf{s}|/c + |\mathbf{r}'-\mathbf{s}|/c$

} parts of G

and

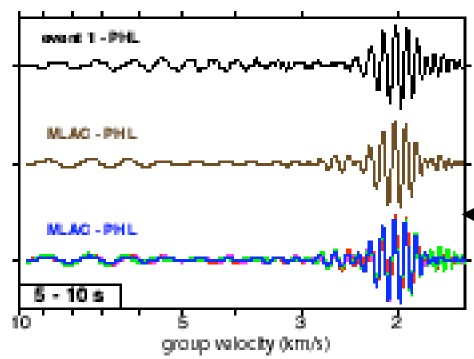
a spurious arrival at $\tau = |\mathbf{r}-\mathbf{s}|/c - |\mathbf{r}'-\mathbf{s}|/c$

Disappears if field is equipartitioned

Amplitude information?

The technique has been used very successfully in seismology to recover seismic velocities, with high spatial resolution. But . . .

Arrival time is evident



Arrival amplitude?
Is this meaningful?

If we really had *G*, we'd be able to infer attenuation also.

Issues include the unknown field intensity in the direction between the detectors

Ray amplitudes X depend on

attenuation α

"on-strike" intensity B

$$X_{i\to j} = B_i(\hat{n}_{i\to j})\sqrt{2\pi / \omega_o \mid \vec{r}_i - \vec{r}_j \mid}\, \exp(-\int_{\vec{r}_i}^{\vec{r}_j} \alpha(\vec{r})\, d\ell)$$

If field is not equipartitioned, then B varies. But how?

Noise intensity B varies in space like an RTE?

$$\hat{n}\cdot\vec{\nabla}B(\vec{r},\hat{n}) + 2\alpha(\vec{r})B(\vec{r},\hat{n}) = P(\vec{r},\hat{n}) + \oint B(\vec{r},\hat{n}')p(\vec{r},\hat{n},\hat{n}')d\hat{n}'$$

↑ sources

↑ Scattering into direction n

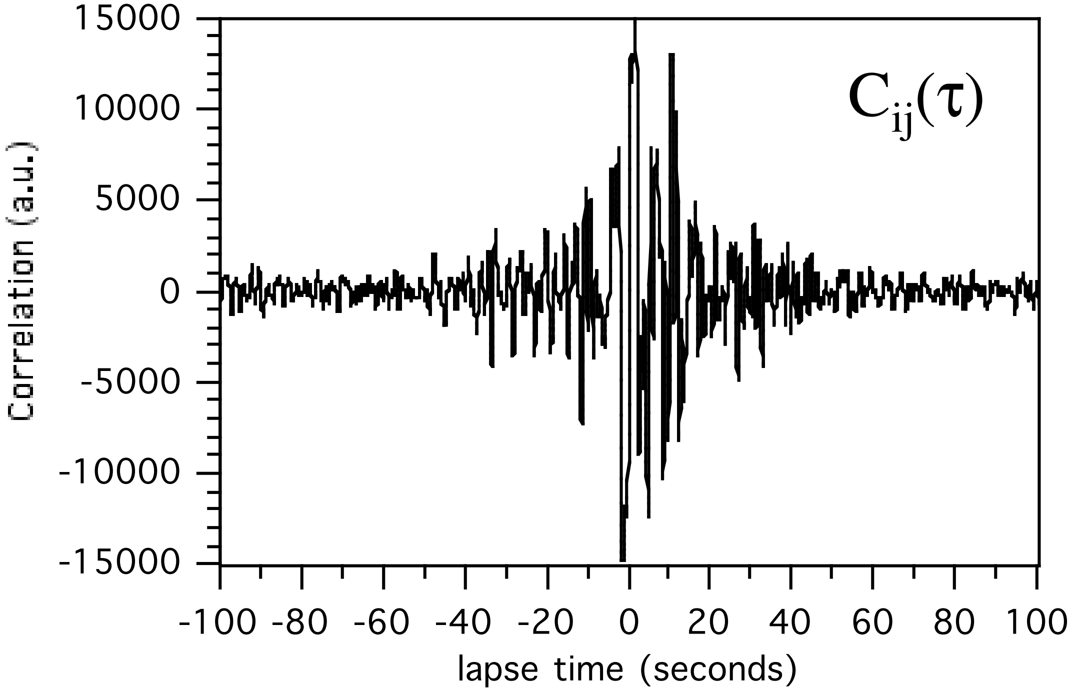
Another application

Detecting changes in a medium . . .

Brenguier et al *Science*: **321**. 1478 - 1481(2008)

correlated ocean-generated seismic noise on a daily basis from an array of seismometers in Parkfield Ca.

Typical Daily Correlation between two of the stations:



Very hard to interpret.

The correlations are about 80% converged.

No clear "arrivals."

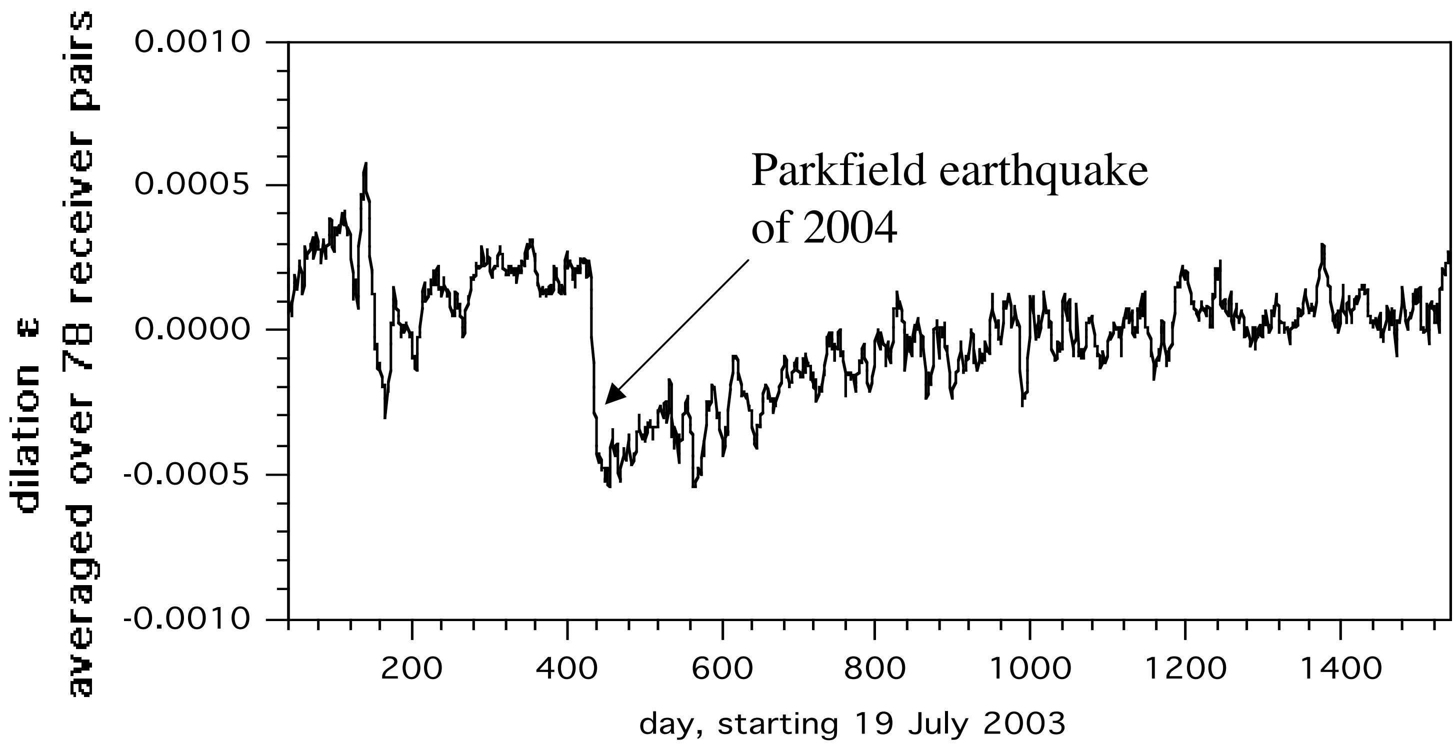
This is G??

They then constructed dilation correlation coefficients

$X(\varepsilon)$ (a measure of relative stretch)

(This is a 4th order statistic on the seismic field)

Between the C_{ij} on different dates and the (year-long) average C_{ij}



Method has been used to predict volcano eruptions

In Sum . .

It has been about 12 years now, and the topic is still growing, still hot,
especially in seismology

Applications in

High resolution seismic velocity maps
Maps of attenuation too?
Monitor changes in a medium
Maps of scattering ?

We still need better understanding
of the effects of imperfectly diffuse fields.