Universal Sound Attenuation in Amorphous Solids at Low-Temperatures

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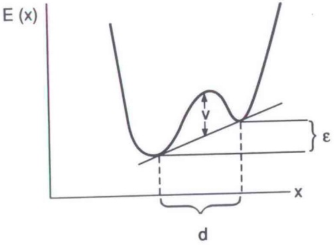
Outline

The Tunneling Two-Level Model (TTLS)

- Objects with two metastable configurations

$$H = \begin{pmatrix} \epsilon & -\Delta_0 \\ -\Delta_0 & -\epsilon \end{pmatrix} + e \begin{pmatrix} \gamma & 0 \\ 0 & -\gamma \end{pmatrix}$$



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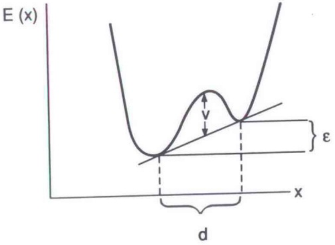

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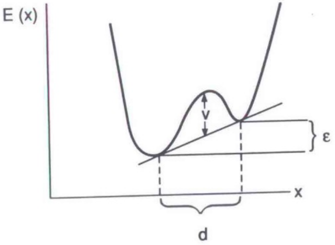

- $\Delta_0 \sim \Omega e^{-\xi}, \; \xi = d\sqrt{2M_0 V}$
- Distribution of TTLS parameters: $P(E, \Delta_0) = \frac{\bar{P}}{\Delta_0}$

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- Non-linear acoustic properties (saturation, echoes, ...)

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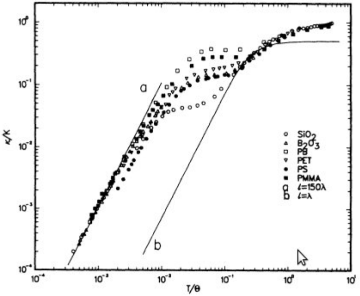

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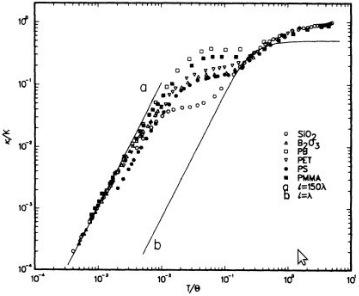

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- Full Non-phonon Hamiltonian:

$$\hat{H}_{many\ \ stuff} = \sum_{s}^{N}\hat{H}_0^{(s)} + \sum_{s<s'}^{N}\sum_{ijkl}^{3}\Lambda_{ijkl}\hat{T}_{ij}^{(s)}\hat{T}_{ij}^{(s')}.$$

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- For N coupled blocks, solve the ultrasonic absorption

$$Q^{-1}_{ijkl}(\omega) \sim \sum_n \hat{T}^{ij}_{0n} \hat{T}^{kl}_{n0} \delta(E_n - E_0 - \omega)$$

$T^{ij}_{mn} \equiv \langle n | T^{ij} | m \rangle$, $\hat{T} \equiv \hat{T}_1 + \hat{T}_2 + \hat{T}_3 \dots$ Many-body non-phonon stress.
$E_n \equiv E_{n_1} + E_{n_2} + E_{n_3} \dots$ Many-body levels.

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- $\bar{Q}^{-1} / \bar{Q}_0^{-1} = U_0 / U.$

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- $\hat{H} = \hat{H}_0 + \hat{V}$. No correlations between $\hat{T}$'s

$$\mathrm{Tr}\hat{H}^2 - \mathrm{Tr}\hat{H}_0^2 = \mathrm{Tr}\hat{V}^2$$

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  Quantum Many-Body System $\implies$ Dominating Powers are Large.
- $N_s^2(U^2 - U_0^2) = K \bar{Q}^{-2} U^2 N_s^2$, regardless of c_k, c_{0k}.
  Now we know U/U_0.

Combine Argument-1 and Argument-2

- $\bar{Q}^{-1}$ can be related to $\bar{Q}_0^{-1}$:

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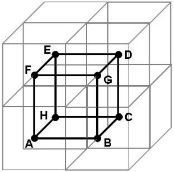

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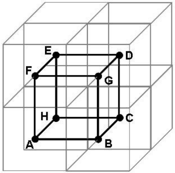

- We know how to iterate blocks to superblocks: The rest is RG.

Combine two Argument-1 and Argument-2

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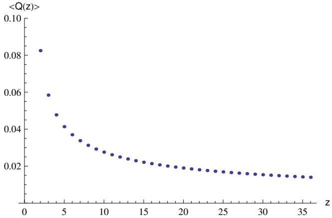

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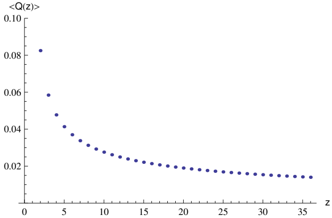

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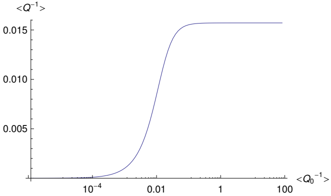
- Universal Q or universal "sample size"?

Microscopic $\bar{Q}_0$ dependence

- Dependence on microscopic parameters:



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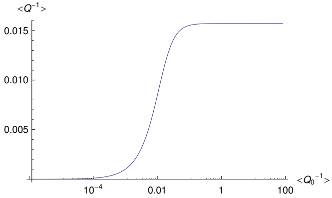

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- We need Q^{-1} in the ω ~MHz-GHz range. What is the ω dependence?

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- Then $Q^{-1}(\omega = 1MHz) \sim 2 \times 10^{-4}$ ✓

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$$\chi_{0t,l} = \frac{\partial^2}{\partial e_{ij}^2} \sum_{a<b}^{N} \langle \phi(r_{ab}) \rangle \bigg|_{e_{ij}=0} \quad (1)$$

$$= \sum_{a<b}^{N} \left[\frac{\partial |r_{ab}|}{\partial e_{ij}^2} \frac{\partial \phi(r_{ab})}{\partial r_{ab}} + \frac{\partial^2 \phi(r_{ab})}{\partial r_{ab}^2} \left(\frac{\partial r_{ab}}{\partial e_{ij}} \right)^2 \right]_{e_{ij}=0} \quad (2)$$

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- $\chi_{0l} = \frac{1}{L^3} \int \frac{\phi''(r)}{r^2} r^2 dr \int r_x^4 d\Omega.$
- $\frac{c_t}{c_l} = \sqrt{\frac{\chi_{0t}}{\chi_{0l}}} = \frac{1}{\sqrt{3}}$ ✓

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- No assumptions regarding microscopic nature: Arbitrary "stuff" with arbitrary number of levels.

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- T-dependence of $Q^{-1}(\omega, T)$, $K(T)$, and $\Delta c(T)/c$ fit better than TTLS.

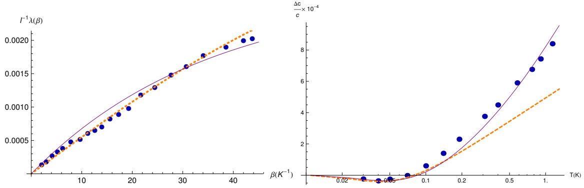

Figure: Q vs $\beta = 1/T$ (left) and $\Delta c/c$ vs T (right). Dashed line: TLS (without ad-hoc fit functions). Solid line: Present Theory. Dots: Golding&Graebner (1976)

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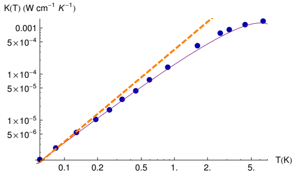

Figure: $K(T)$ vs T. Dashed line: TTLS prediction. Solid line: Present Theory. Dots: Zeller and Pohl (1972)

Acknowledgements

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