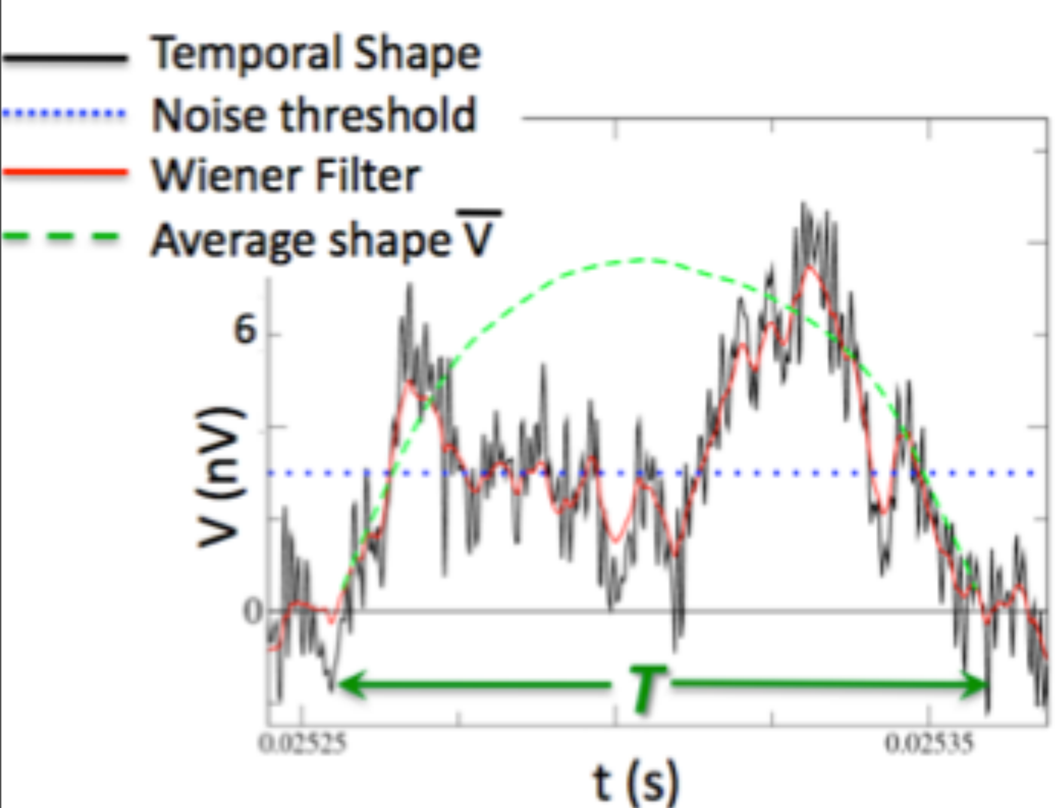
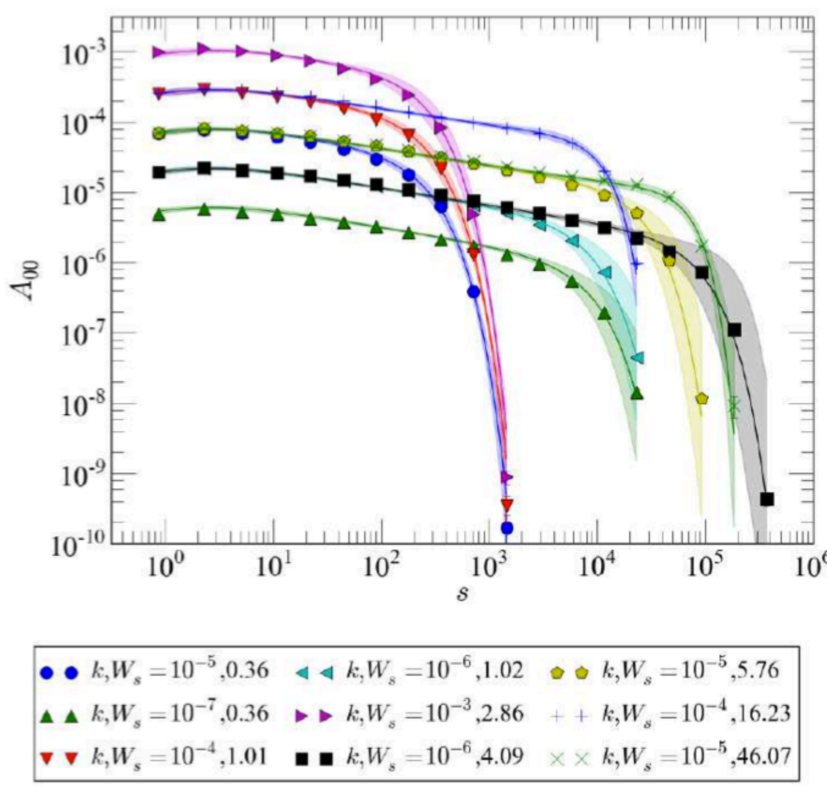
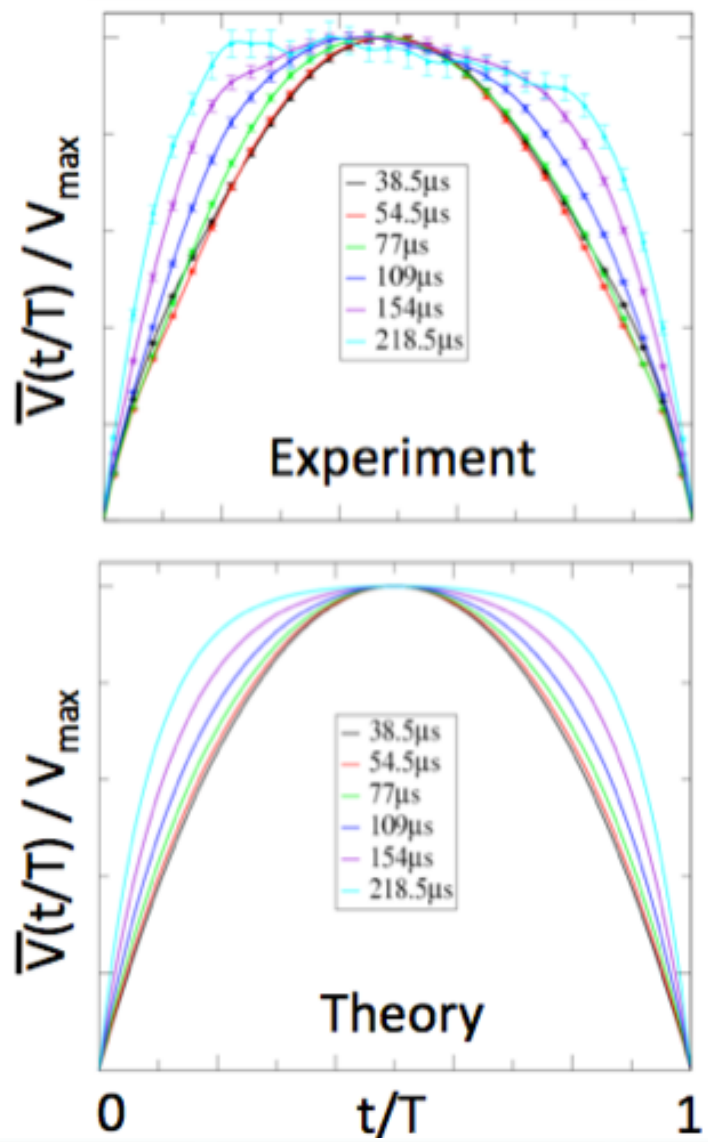
Cornell University

From power laws to shapes: Identifying non-equilibrium universality classes

Stefanos Papanikolaou



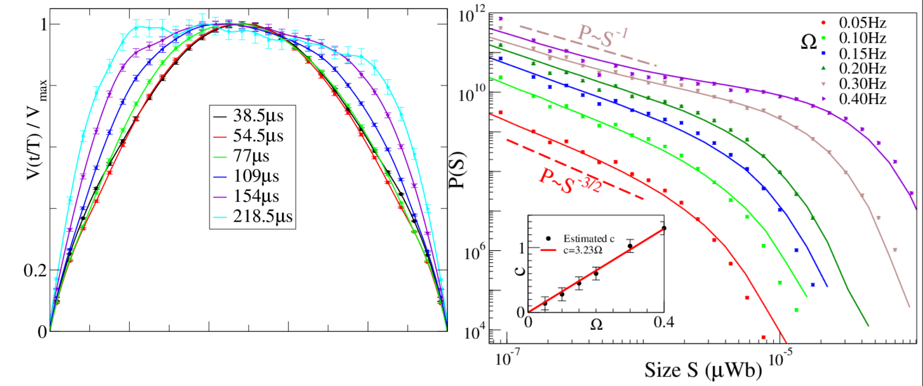




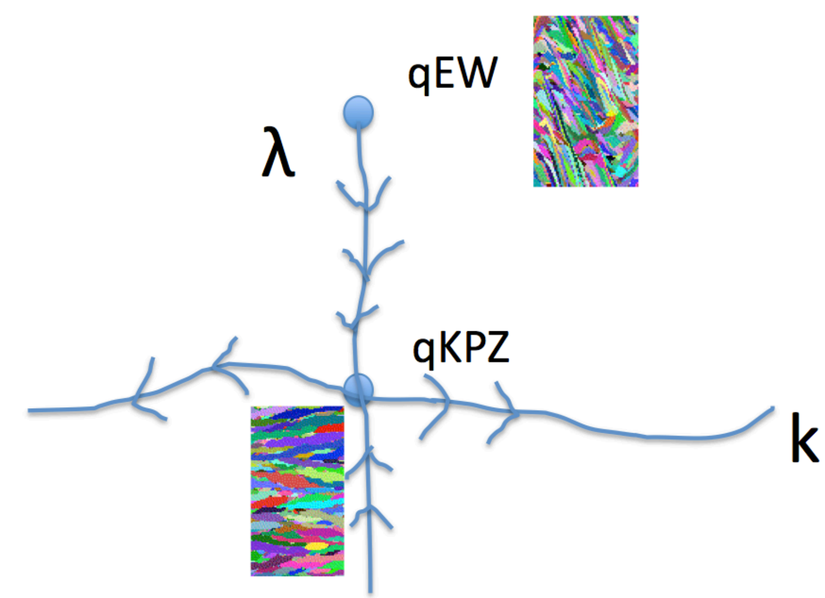
Collaborators: F. Bohn, Y-J. Chen, G. Durin, J. P. Sethna, S. Zapperi

Outline

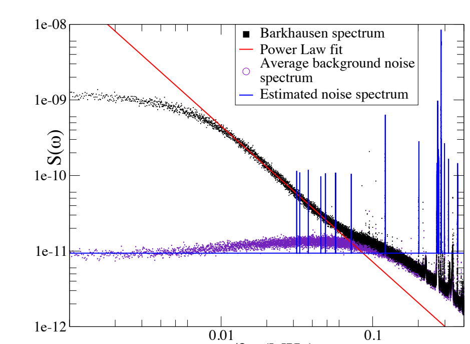
- Universality in theory and experiments and identifying a universality beyond ν, β, σ, κτλ(etc)

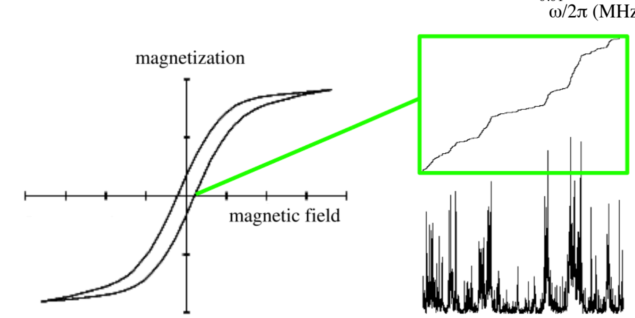


- Delving into theory: directed percolation depinning, windowing and multiple scaling variables



- Merging theory with experiment: Barkhausen crackling noise in the mean-field universality class





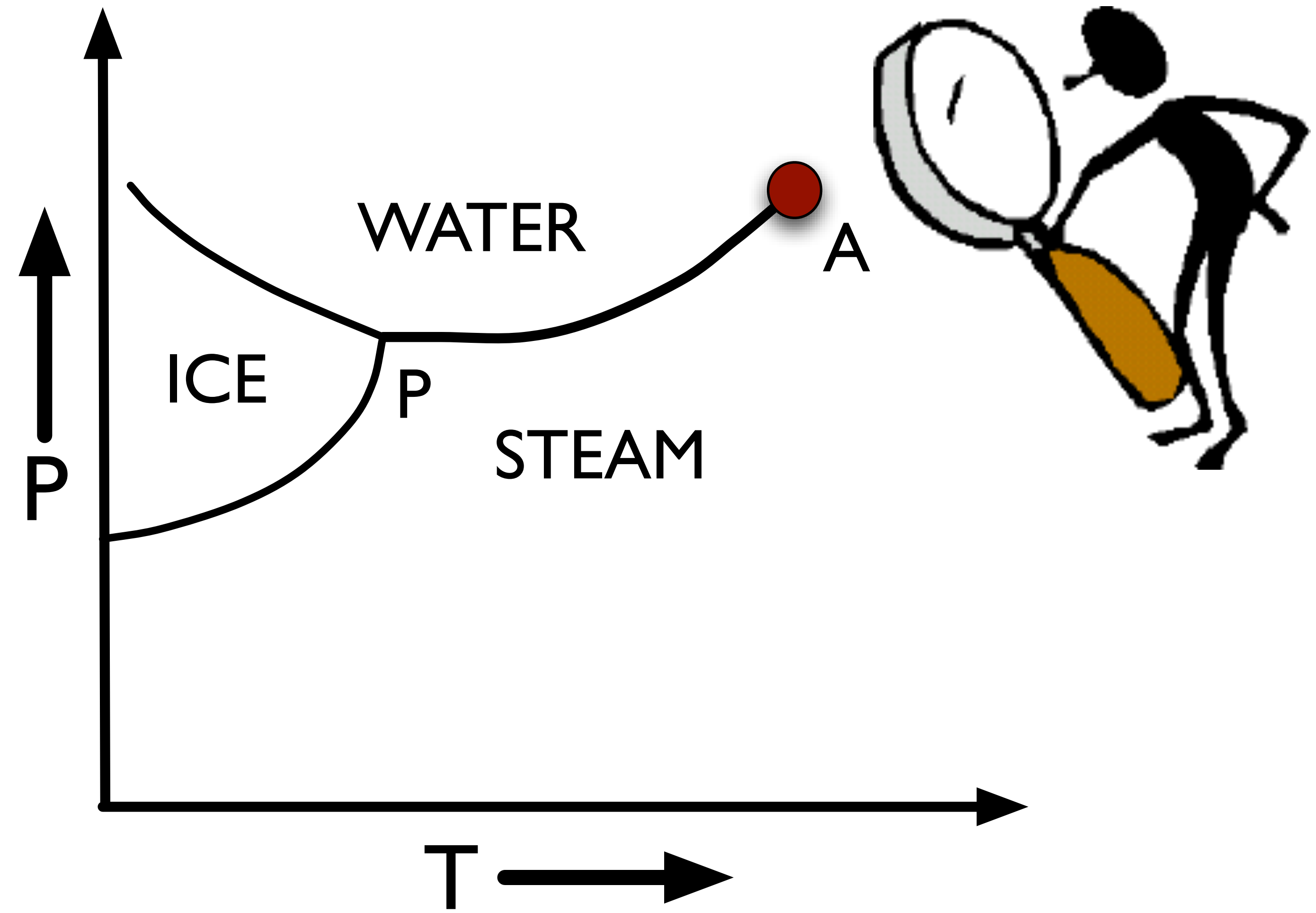
Universality and Scaling: What is it?

- Scaling exists when observables of systems with many degrees of freedom show power law (scale-free) behavior:

$$C(r,t,h) = \langle S(0)S(r)\rangle_{t,h} = r^{-\eta}\mathcal{C}(r/\xi, \xi t^{\nu}, \xi h^{\frac{\nu}{\beta\delta}})$$

- Why so important?
  a) Classifiable “universal” behavior, according to symmetries,
  b) Low energy behavior of nearby phases “controlled” by it...

Universality and Scaling: All over us or maybe not?

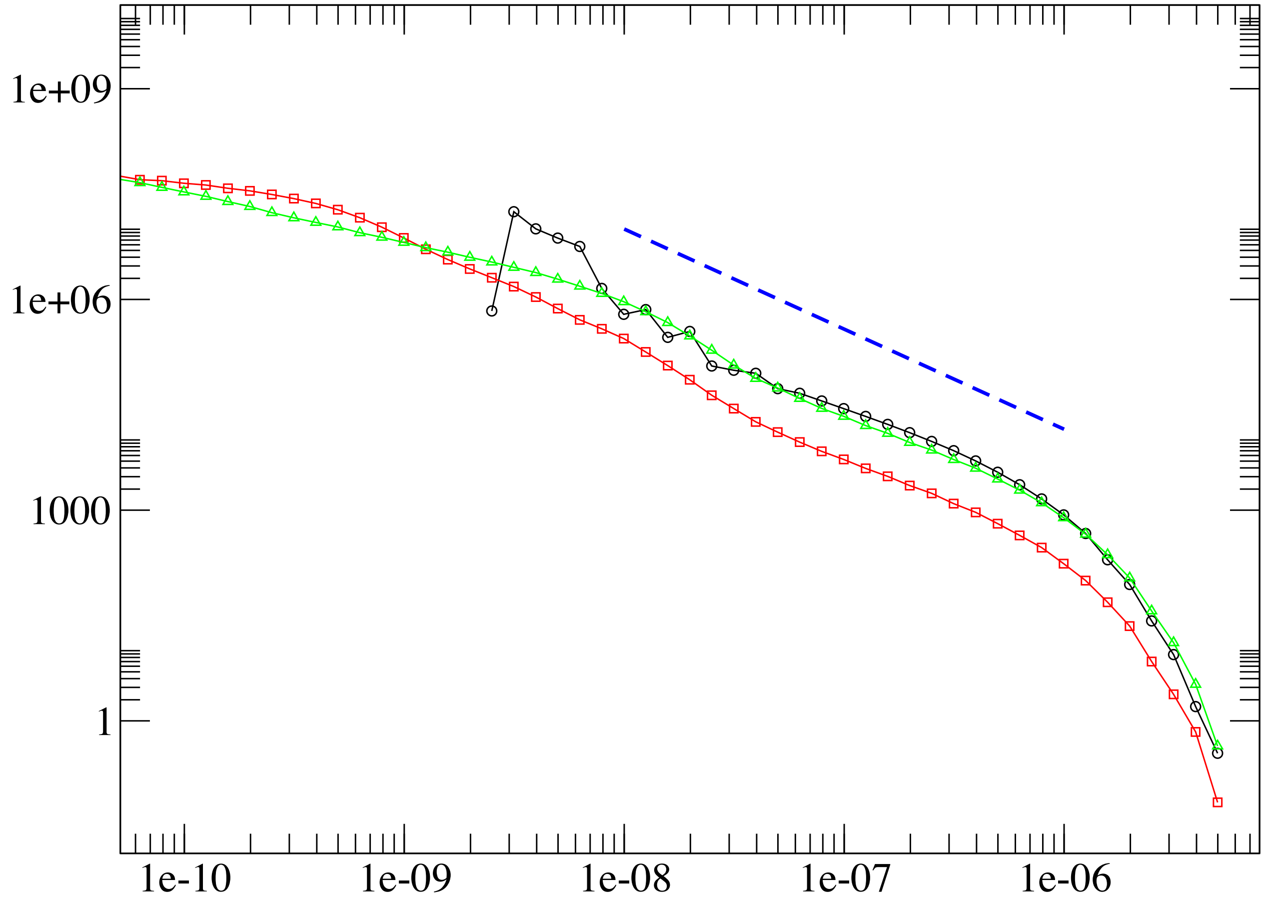


- Single points in parameter space separating different phases of matter

- Even though hard to find, they control their surrounding phase diagram: Understanding a single point is equivalent to looking everywhere...

Scaling in Experiments: Science vs. Art ?

- Looking at slopes and guessing:



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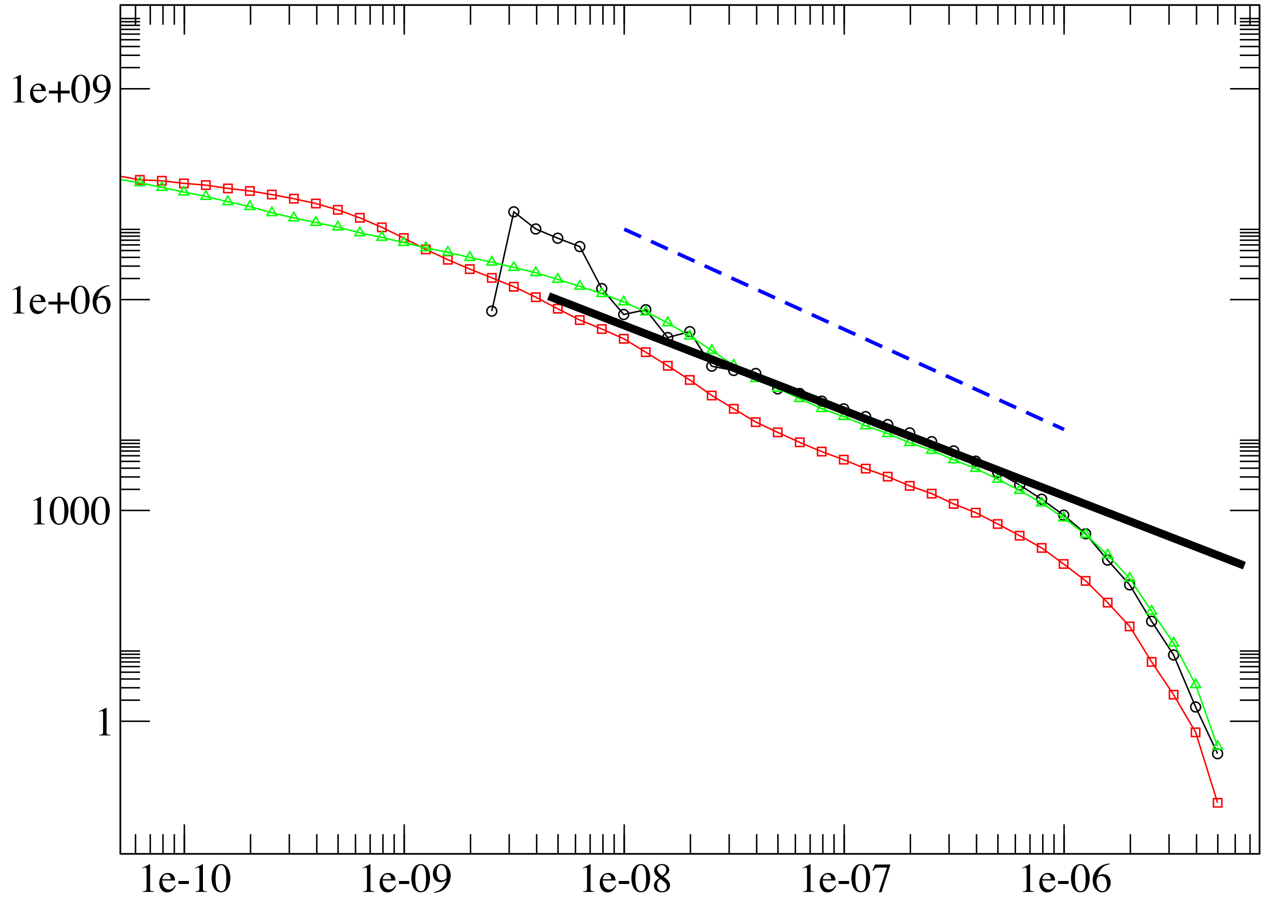

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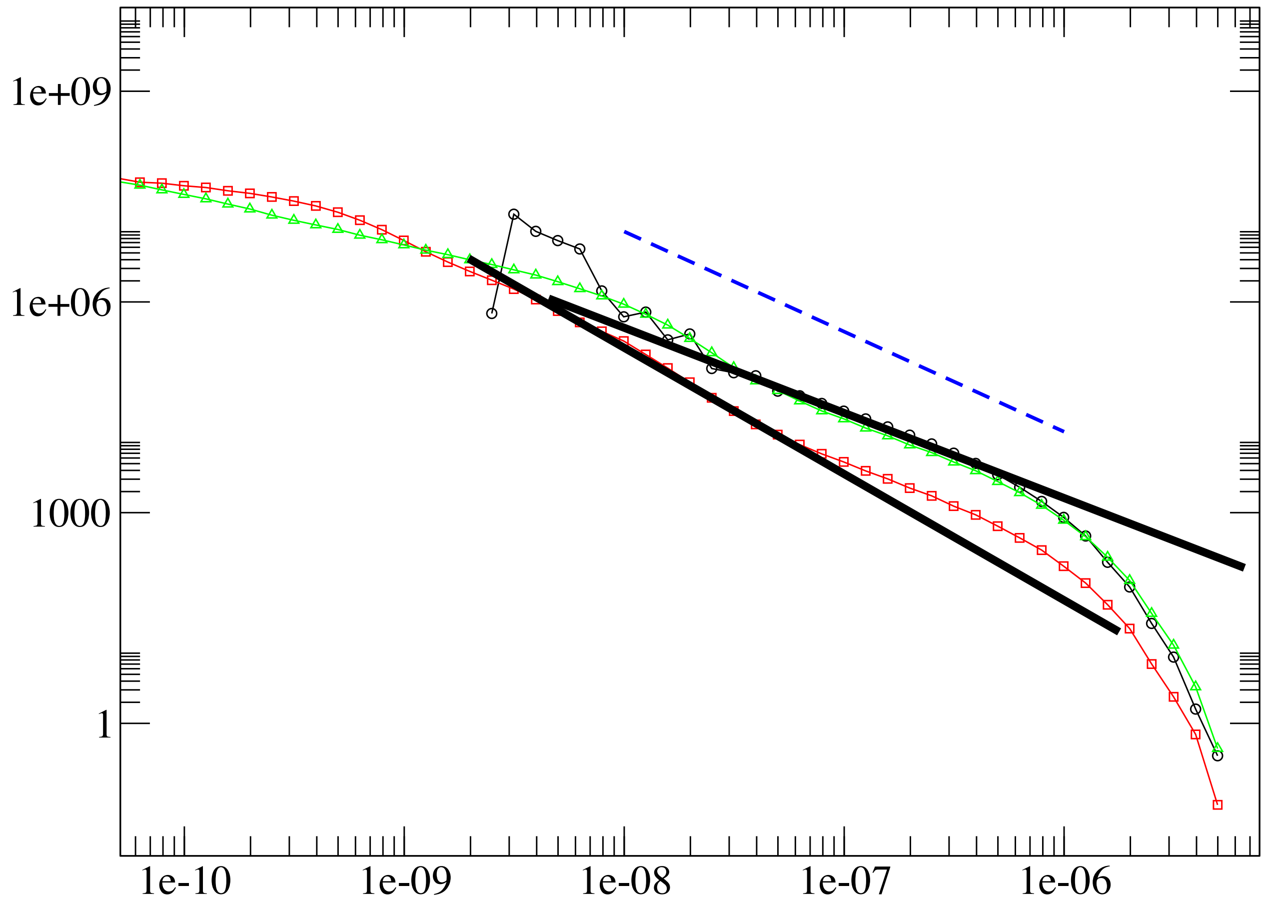

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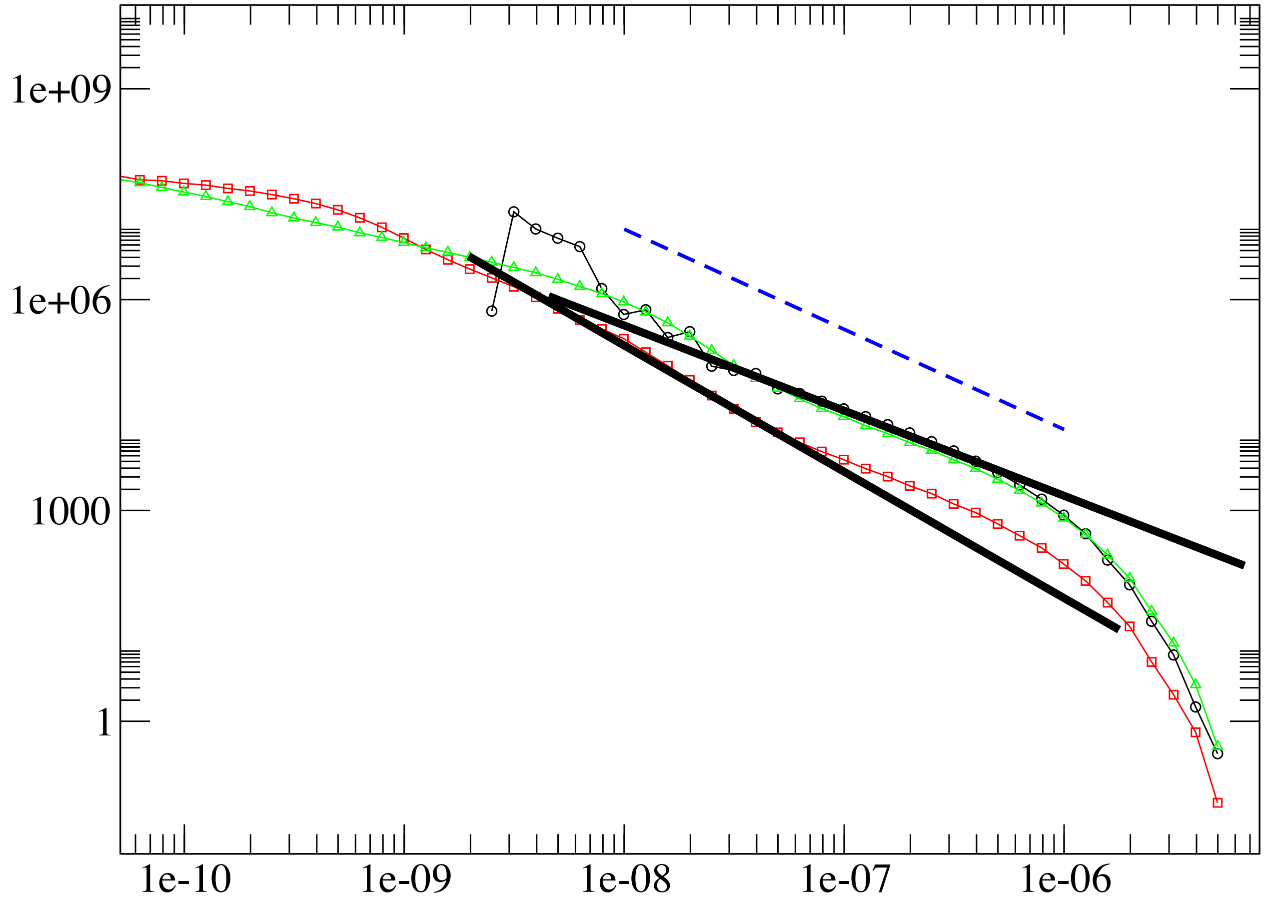
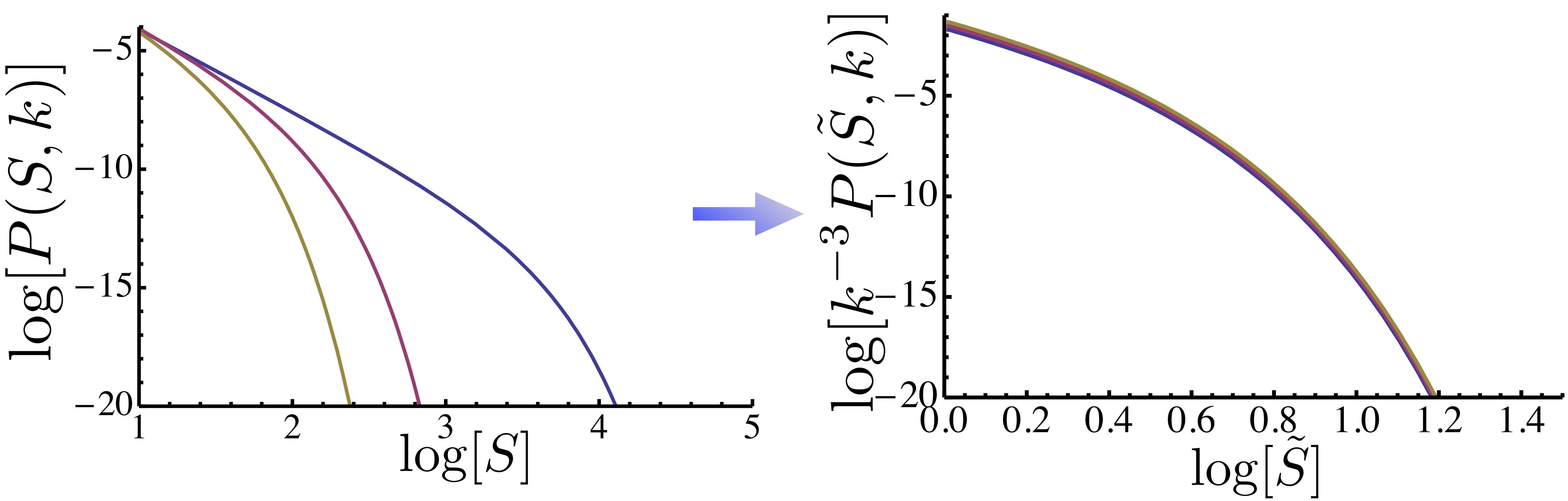


- Exponents with 20% error are not well defined...

Scaling in Experiments: Science vs. Art ?

- Performing scaling collapses and figuring out a part of the whole story...

$$P(S,k) = \frac{1}{\mathcal{N}} S^{-3/2} e^{-k^2 S} = k^3 \tilde{S}^{-3/2} \mathcal{P}(\tilde{S} \equiv S k^2)$$



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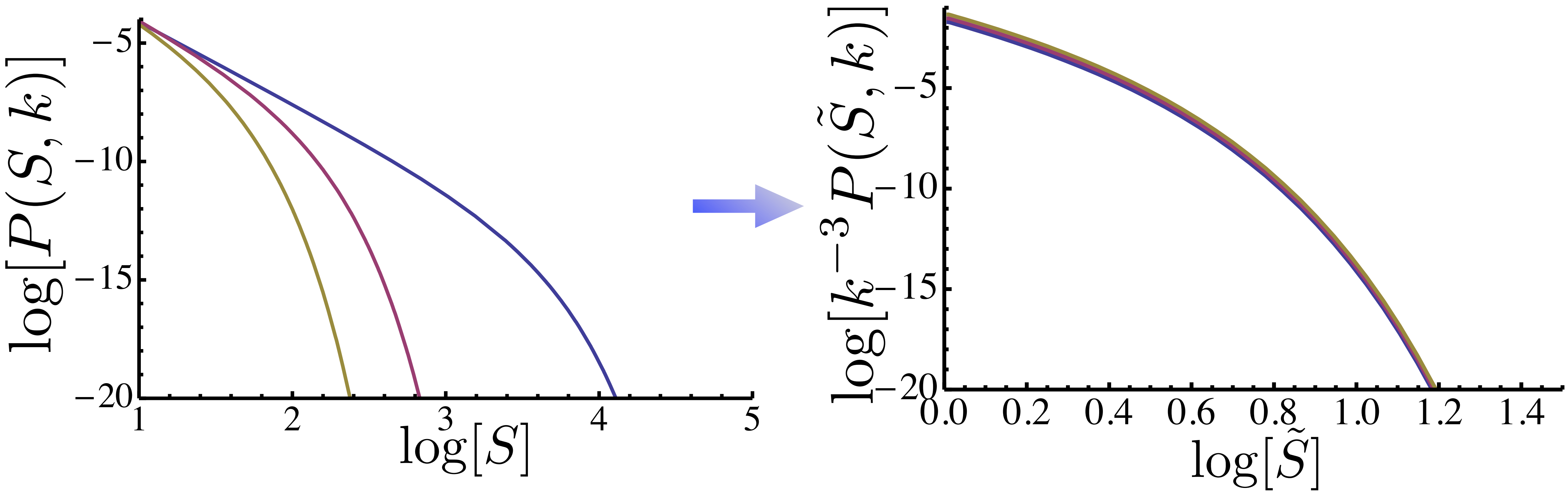
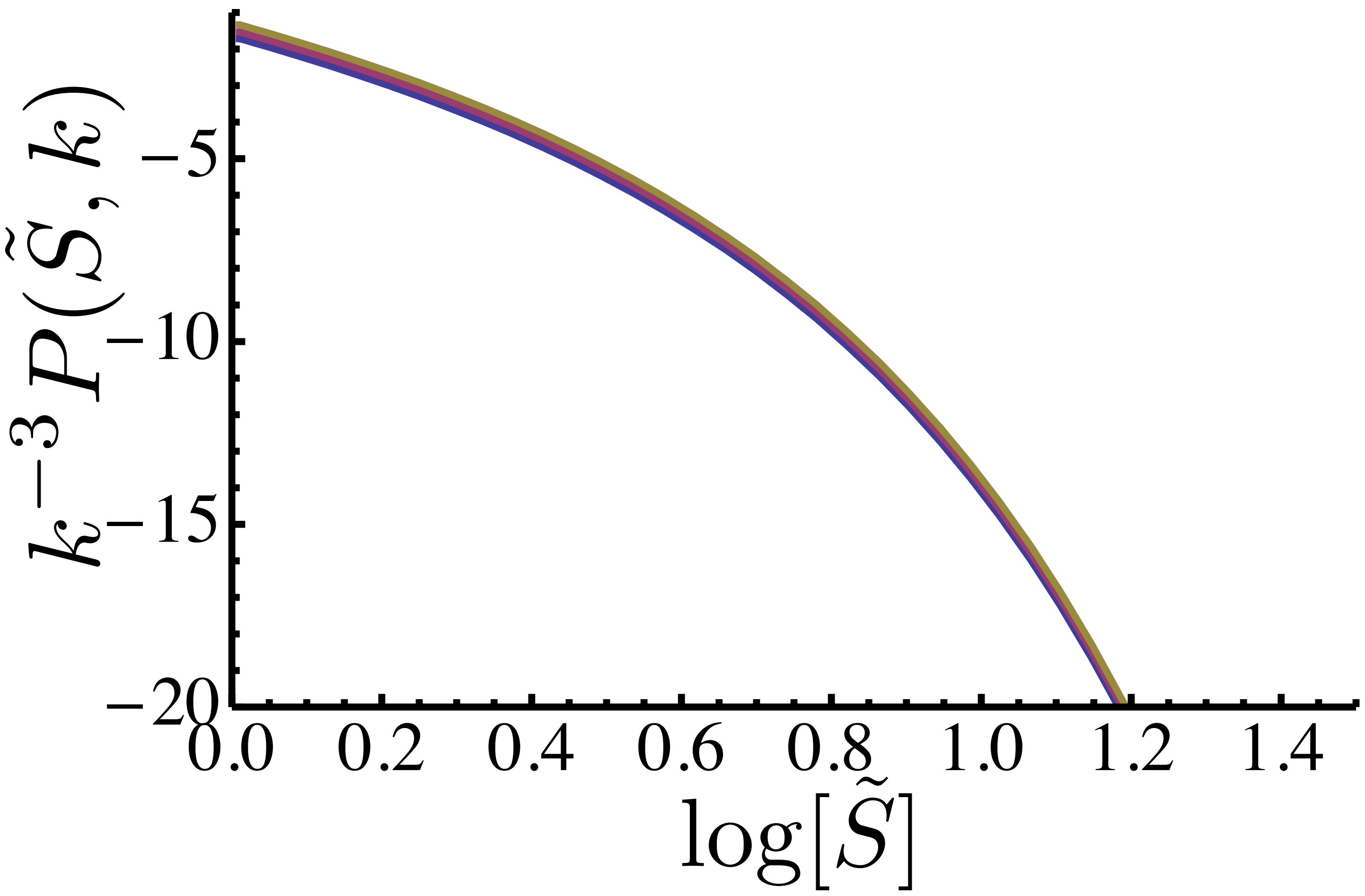


- A first access to shapes...

Shapes better than Slopes: When and Why?

- Differences are small but there are many points of comparison, in contrast to 2 exponent values,

- Strong check of a universality class...



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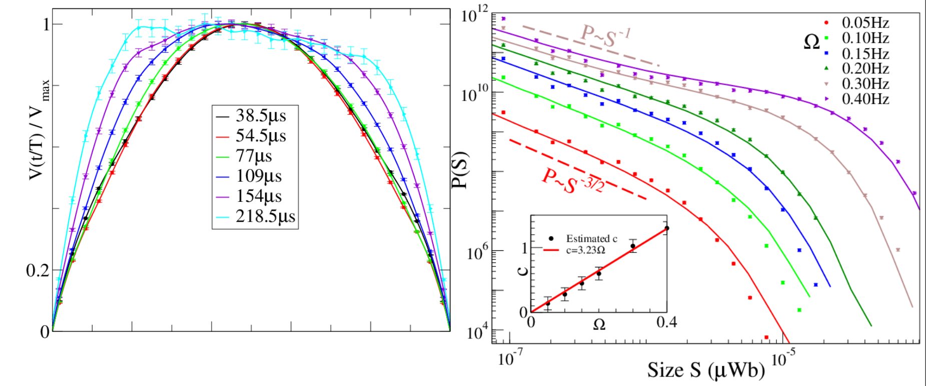

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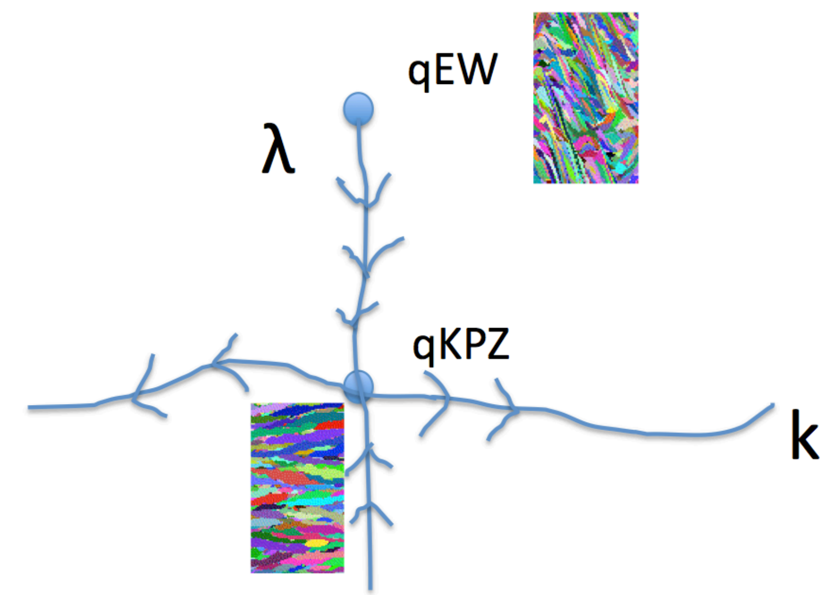
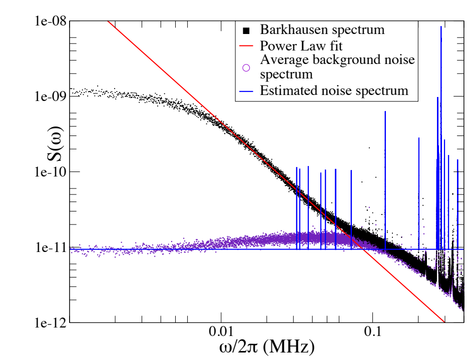
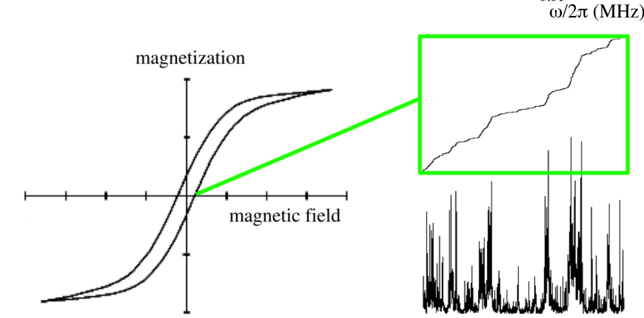
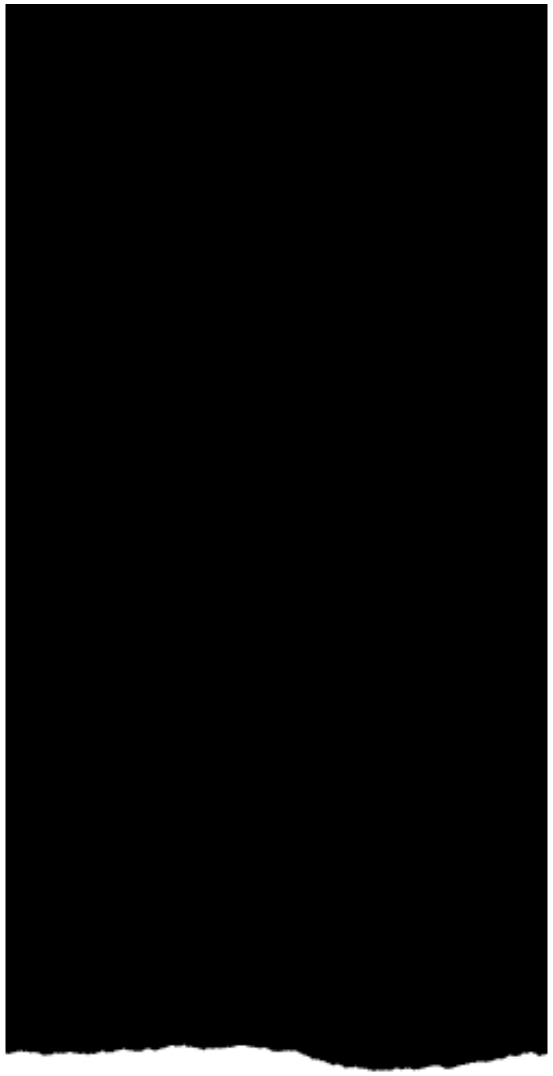


- Delving into an experiment: Barkhausen crackling noise, a problem that gave us insights and tools





Quenched Kardar-Parisi-Zhang Equation



- General interface modeling:

$$\frac{\partial h(x,t)}{\partial t} = F - k\langle h\rangle + \gamma\nabla^2 h + \lambda(\nabla h)^2 + \eta(x,h)$$

relevant parameter (e.g. gravity): $k\langle h\rangle$

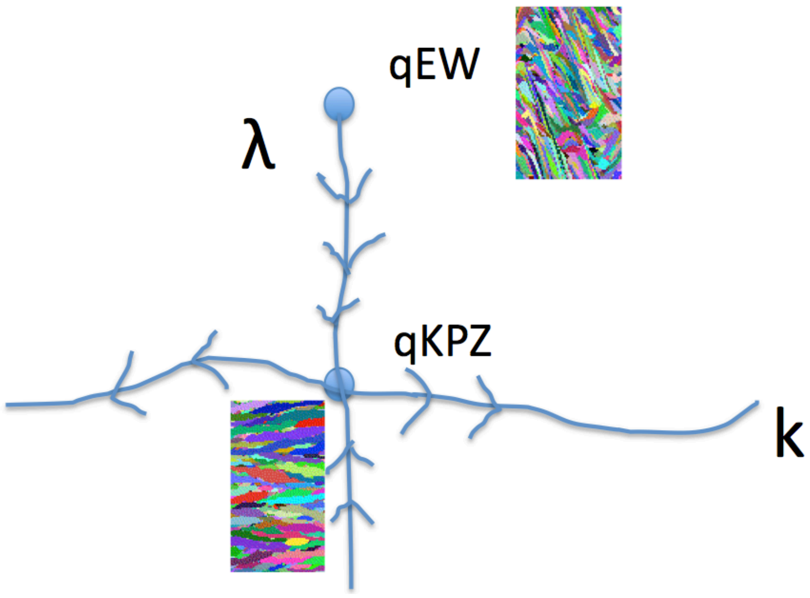
quenched noise: $\eta(x,h)$

KPZ terms: $\gamma\nabla^2 h + \lambda(\nabla h)^2$

external force: F

- With k > 0, the interface is always stationary; avalanches as F increases

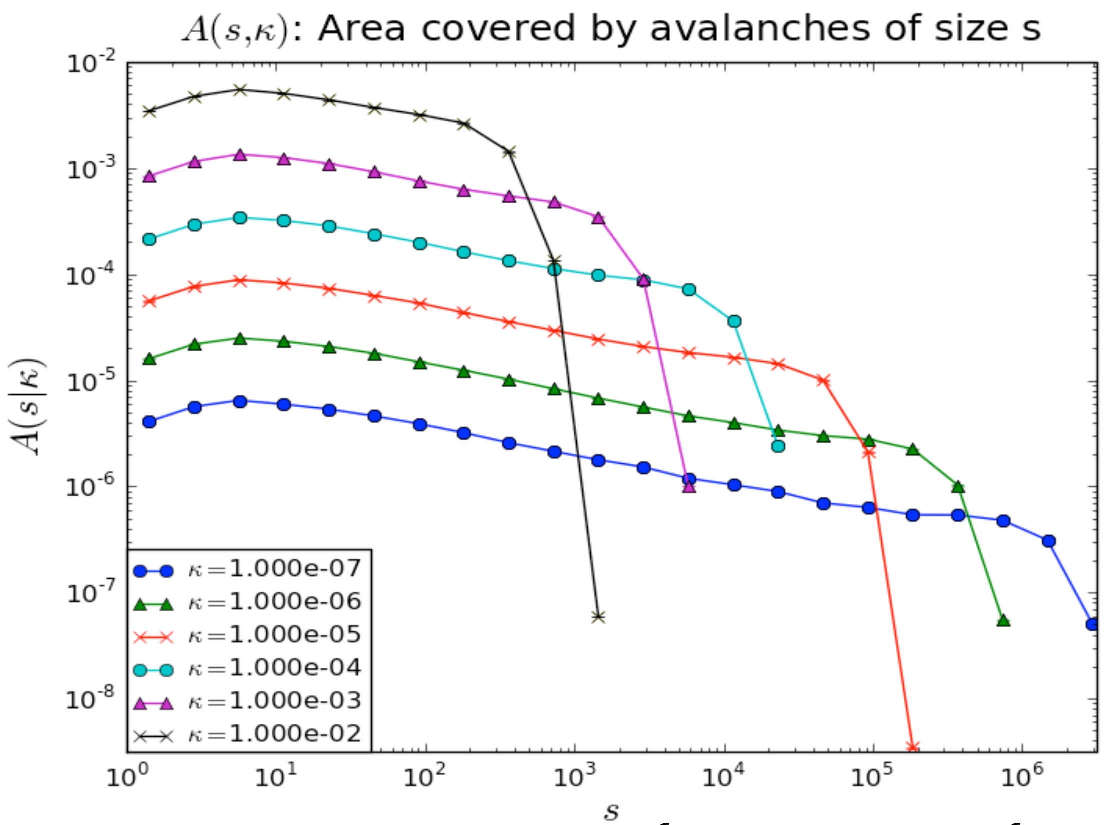
- As λ -> 0, crossover to the Eduards - Wilkinson model

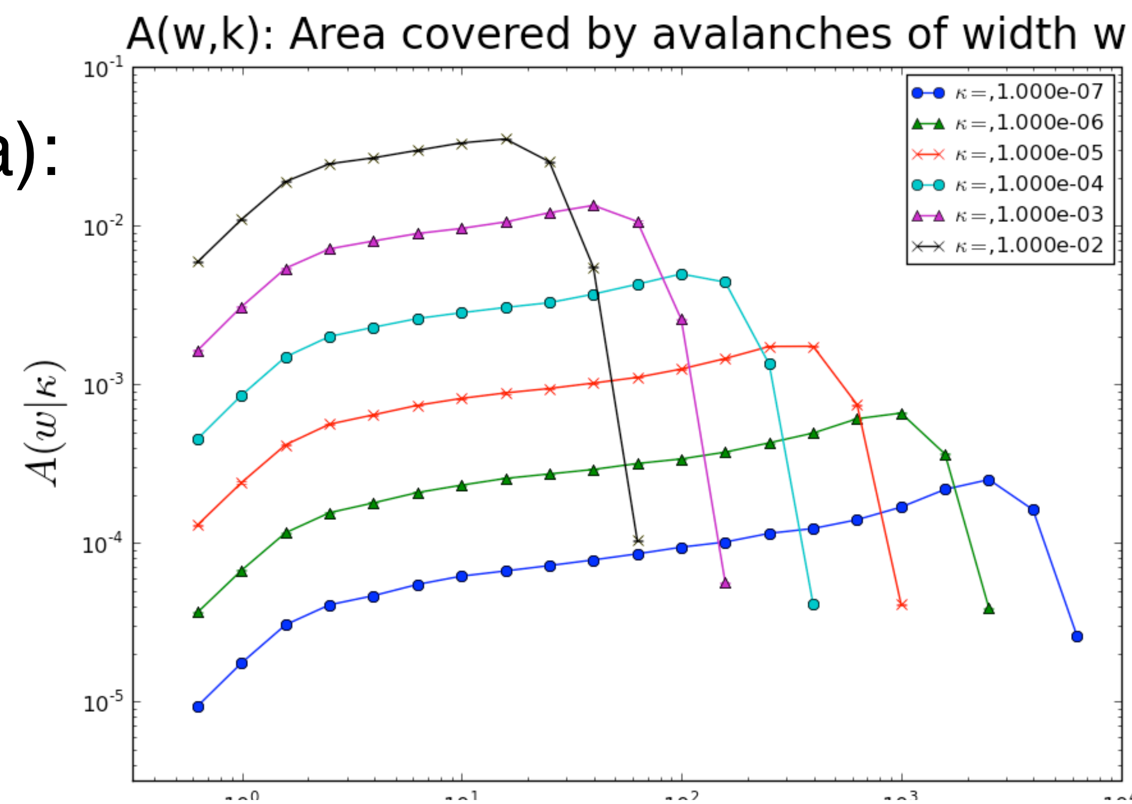


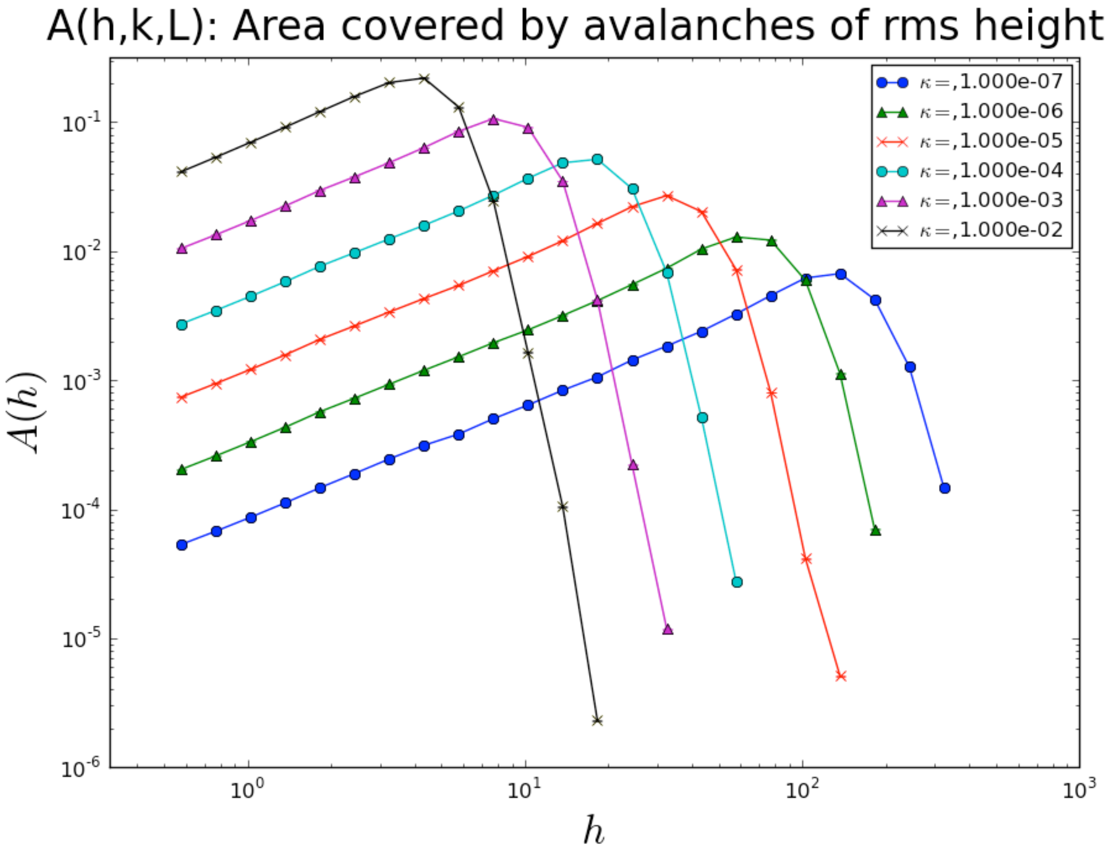
Avalanche spatial structure

- Multivariate distributions (s,k,h,w)
- automatic fits, free software (still beta):

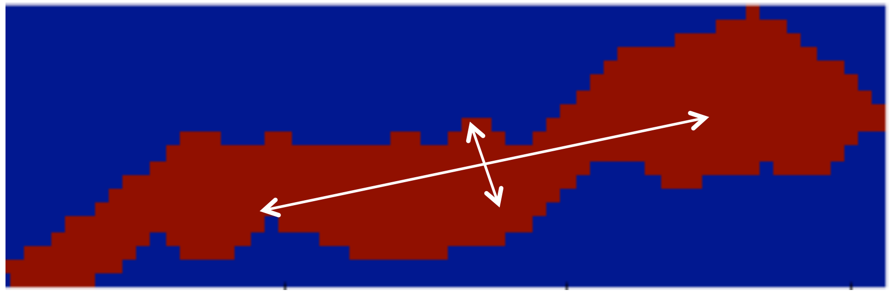
google SloppyScaling



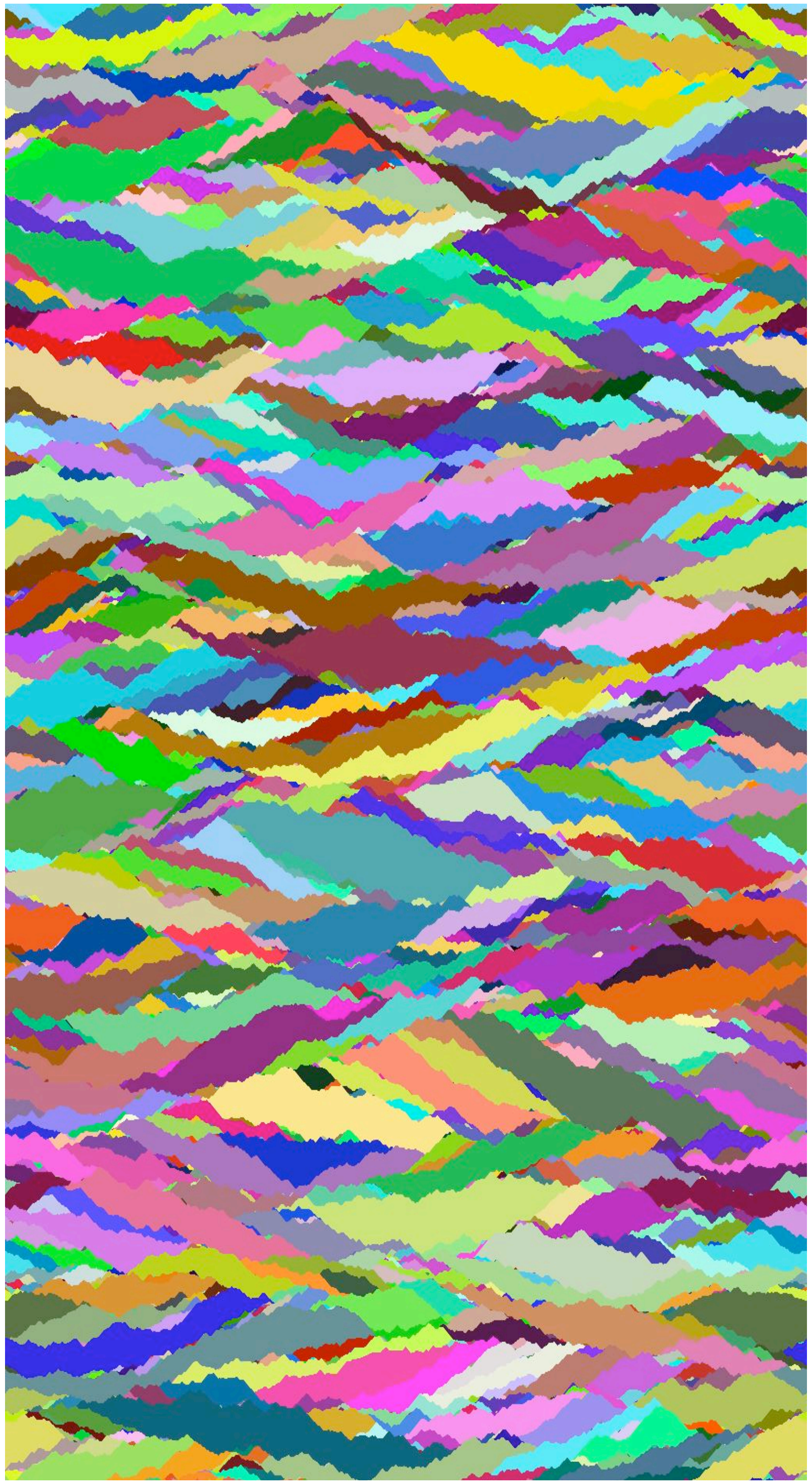


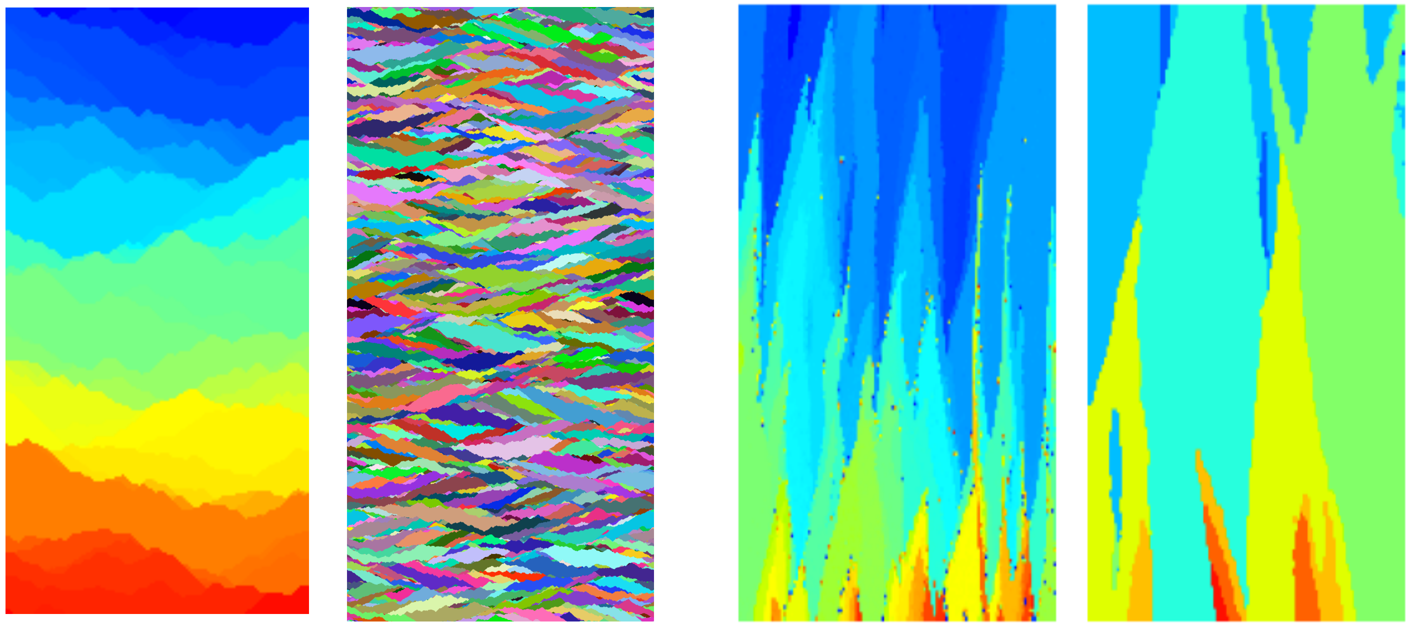


Self-affine: $w \sim h^{\zeta} \quad s \sim wh^{\zeta} \sim w^{1+\zeta}$

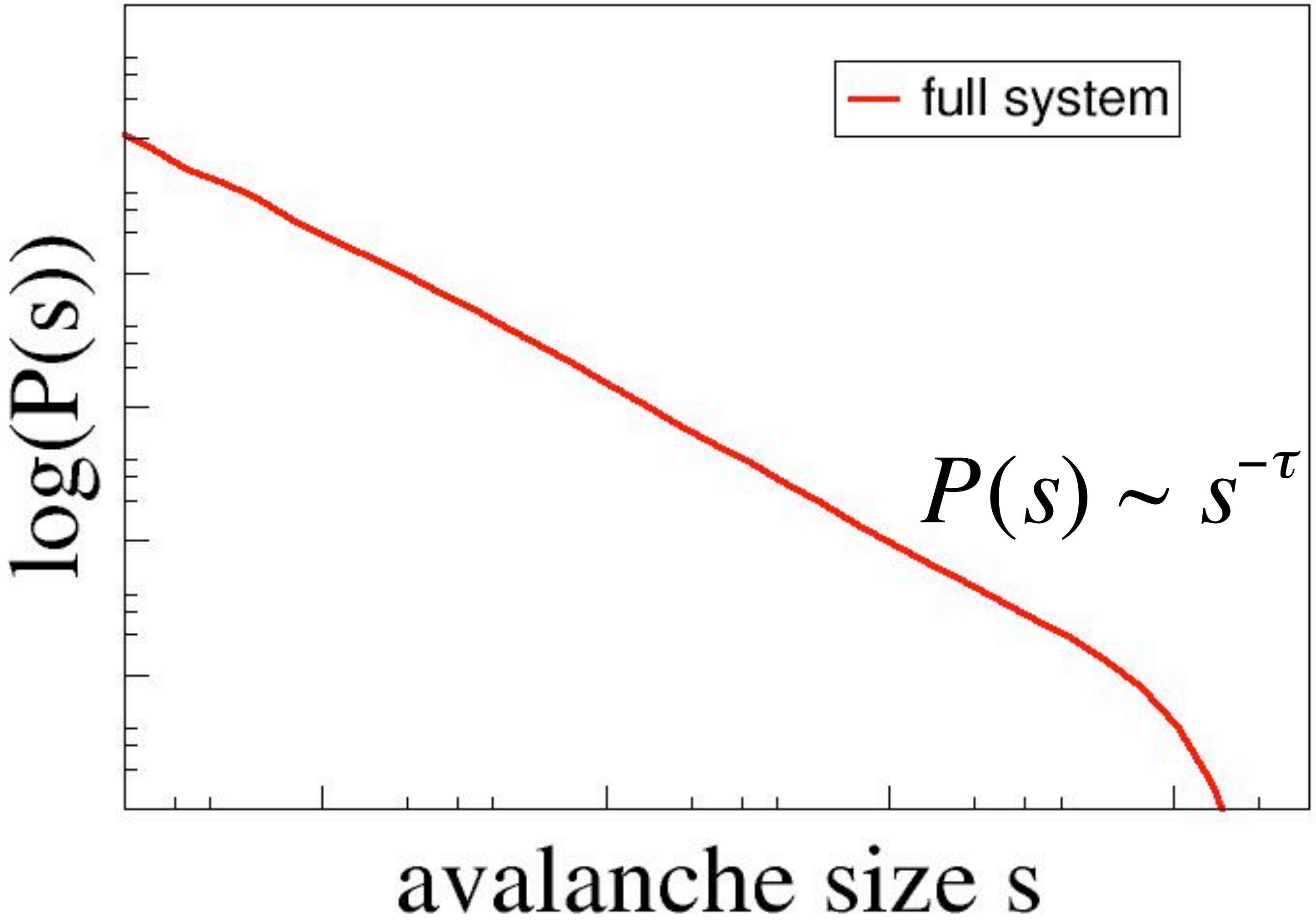


Emulating experimental distortions: Windows & Inference of Scaling Behavior





[Magni et al.,*J. Stat. Phys* (2009)]



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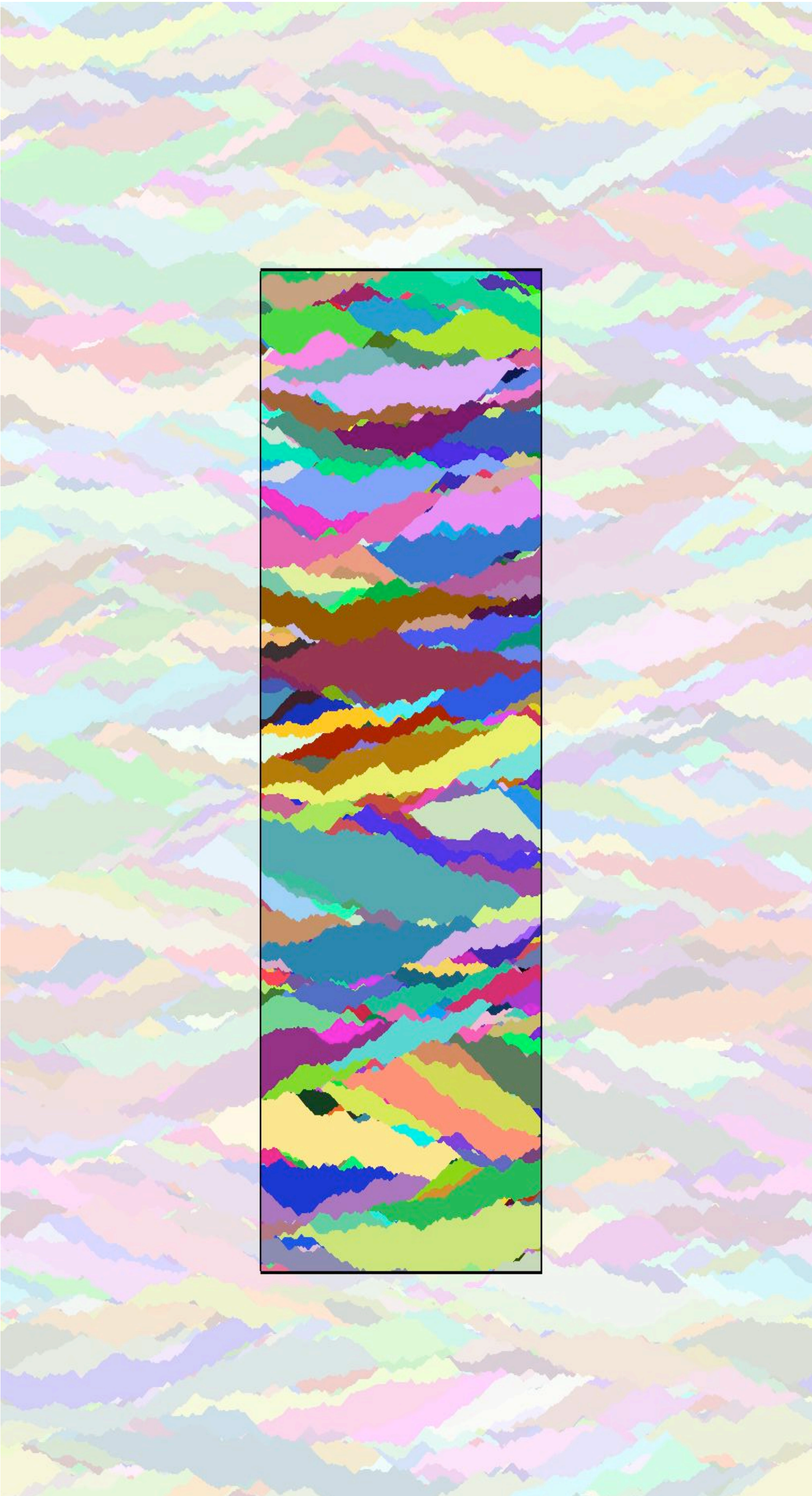

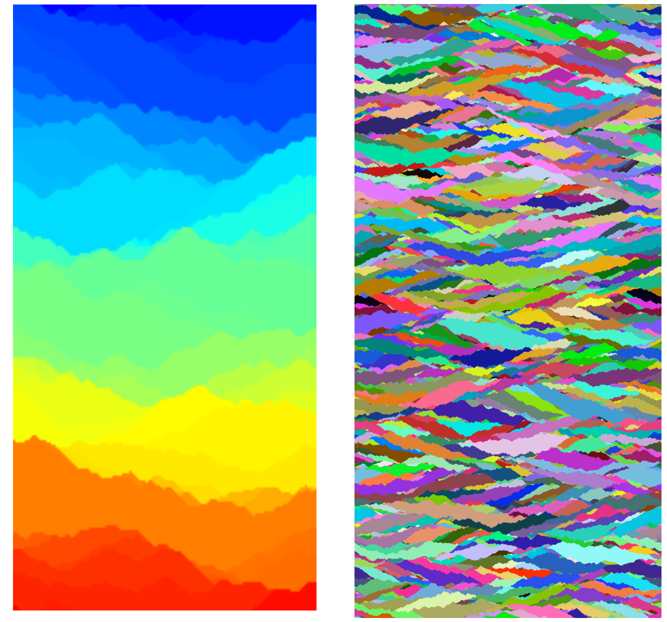

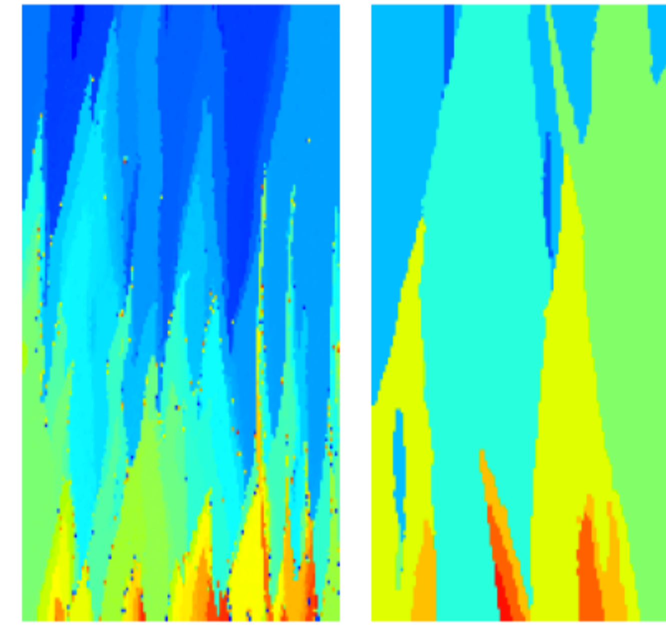

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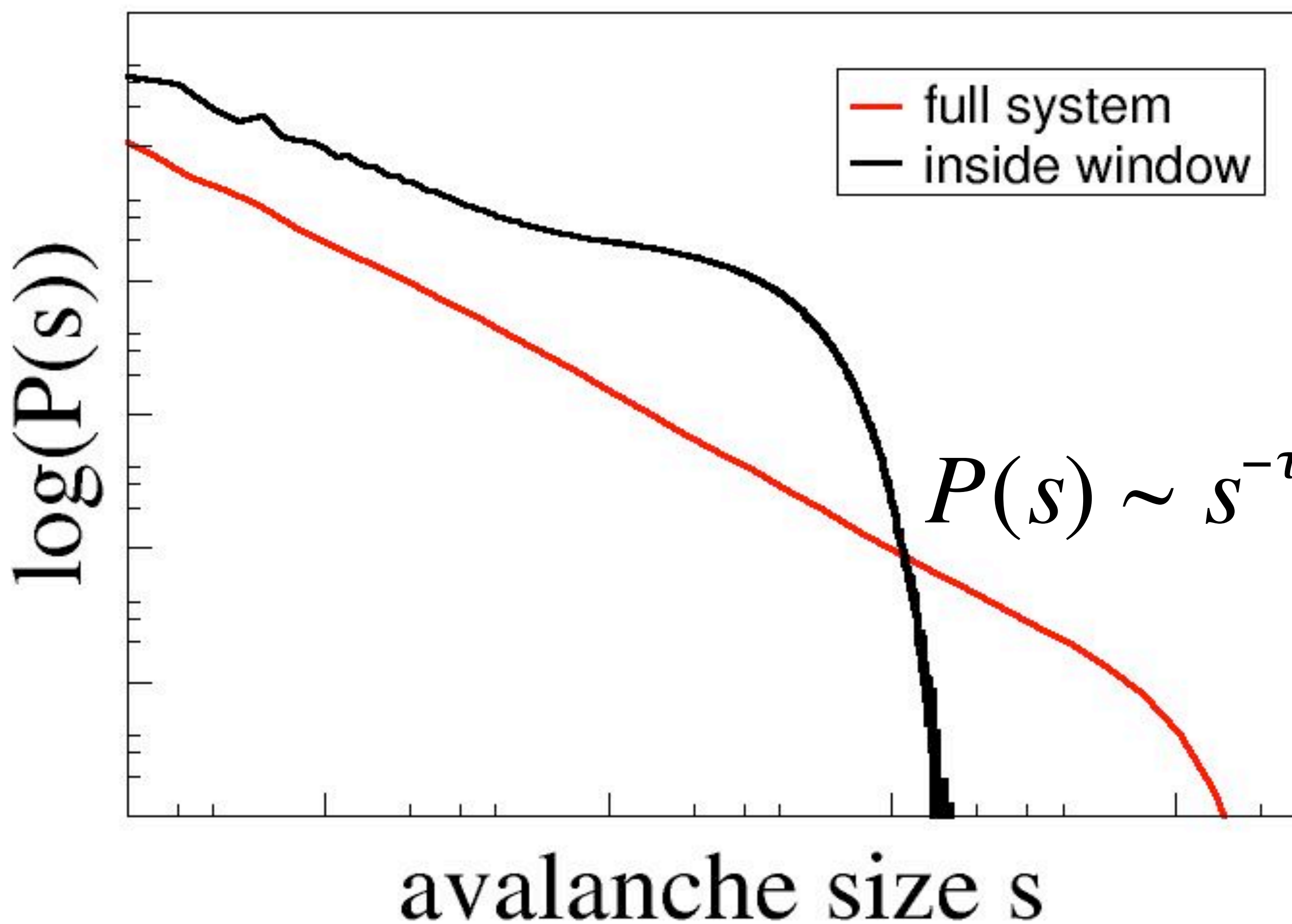

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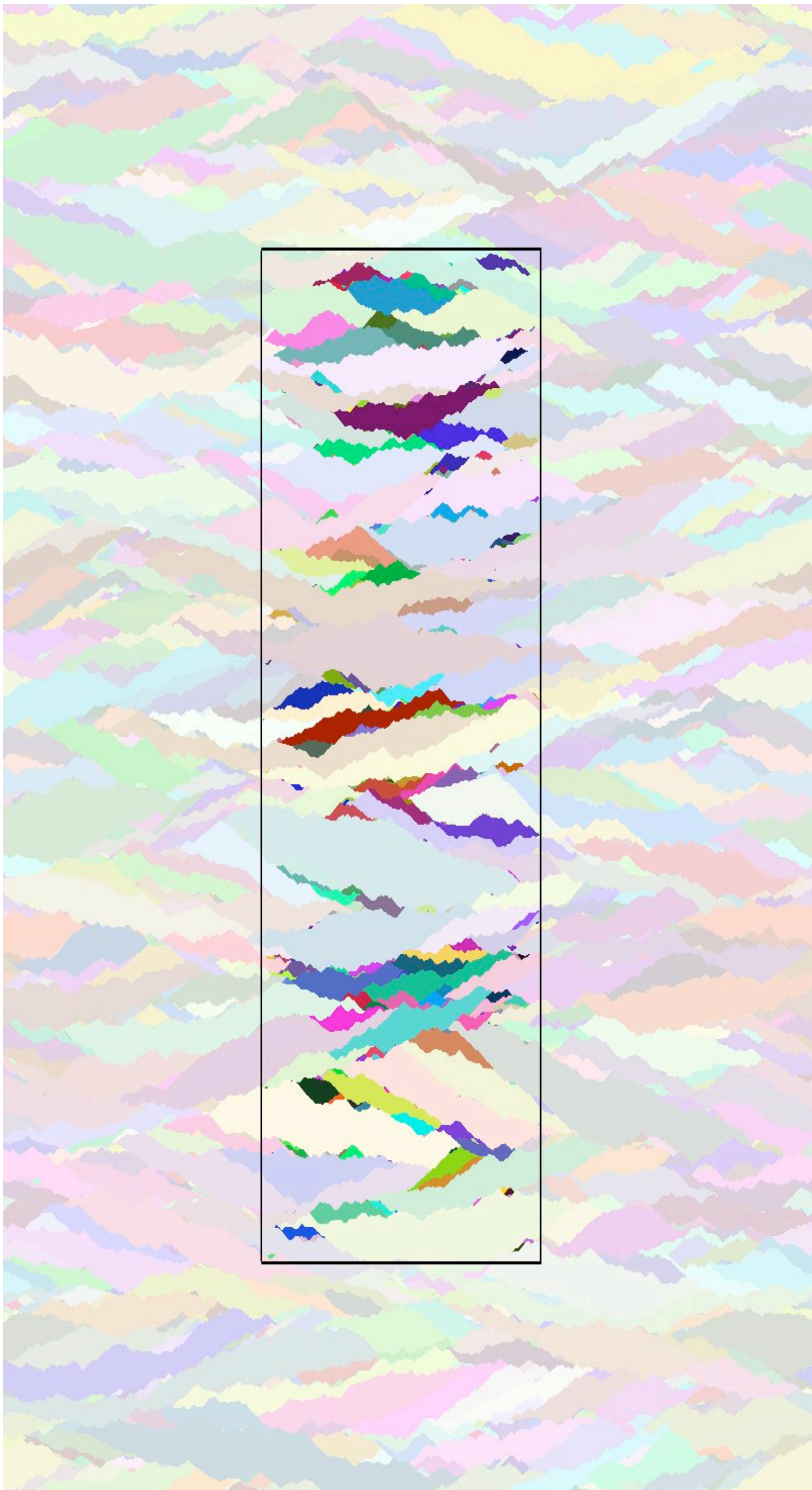

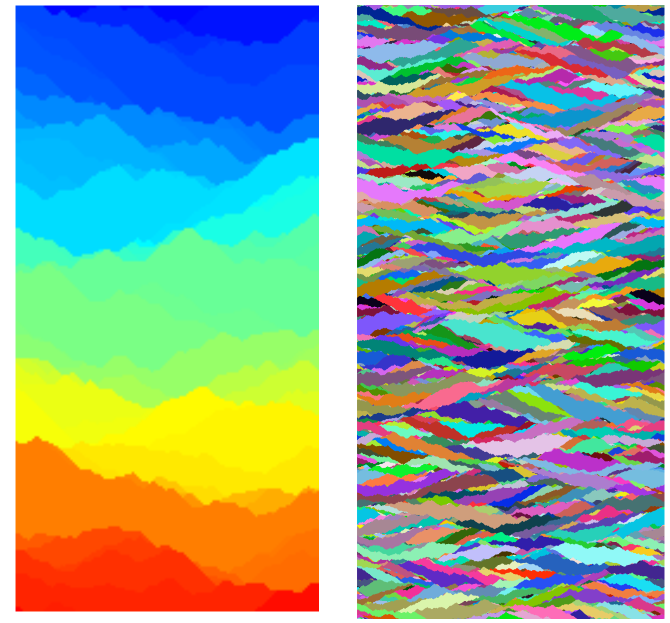

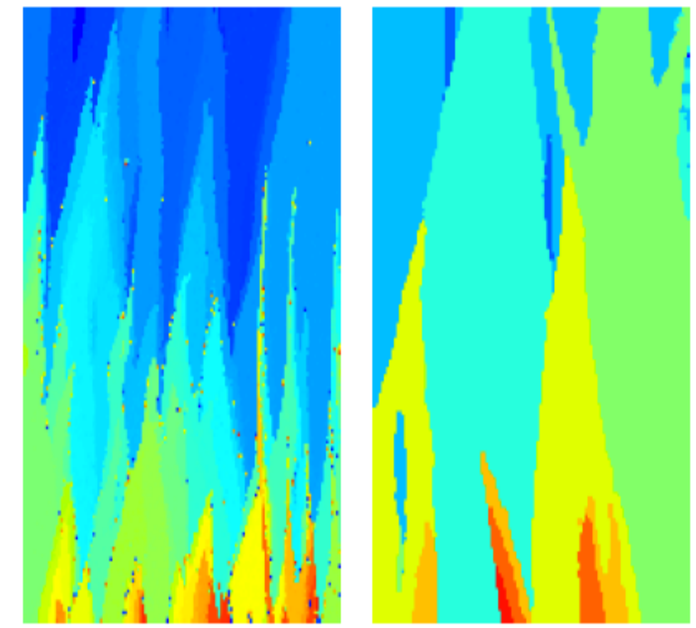

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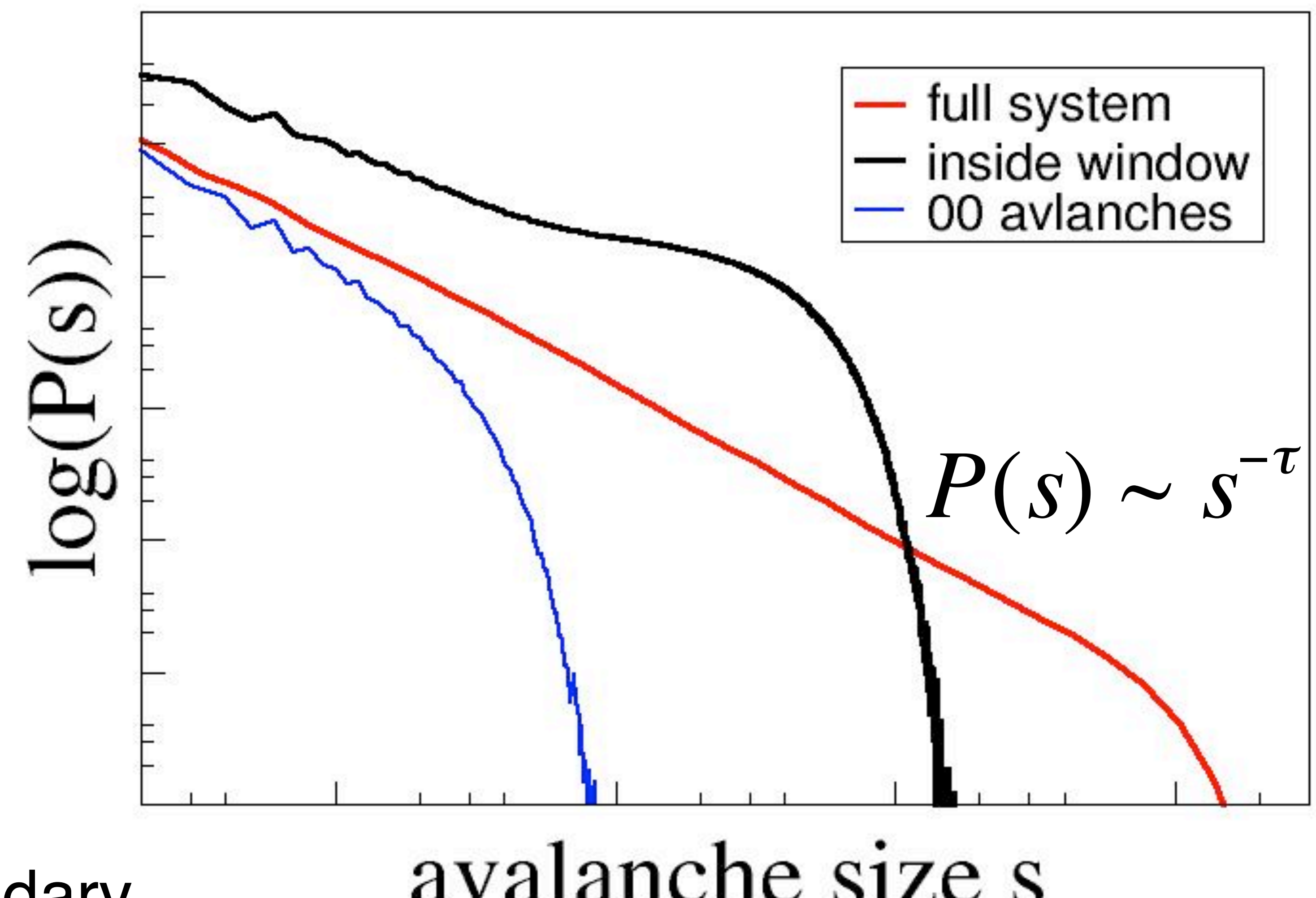
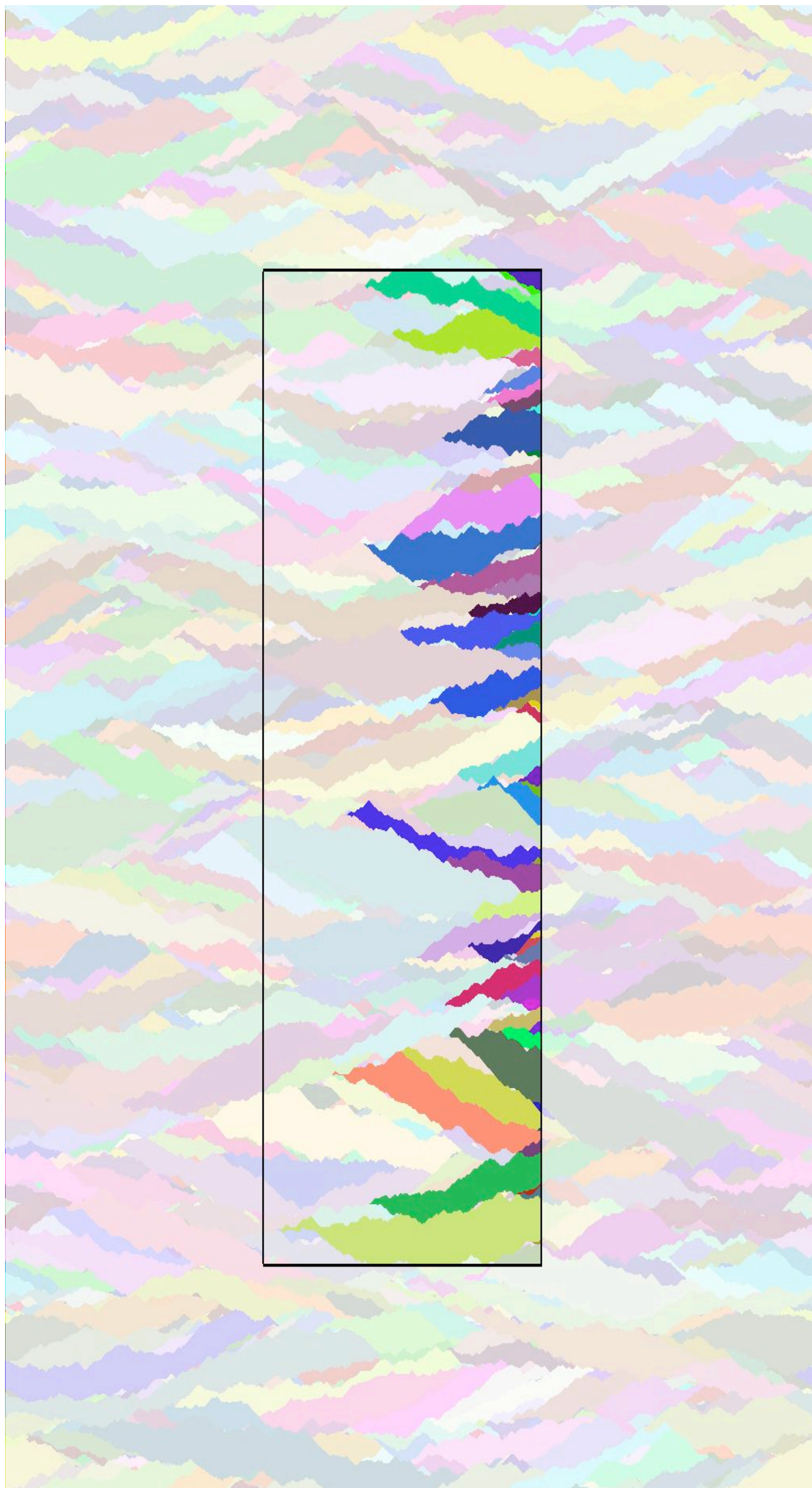
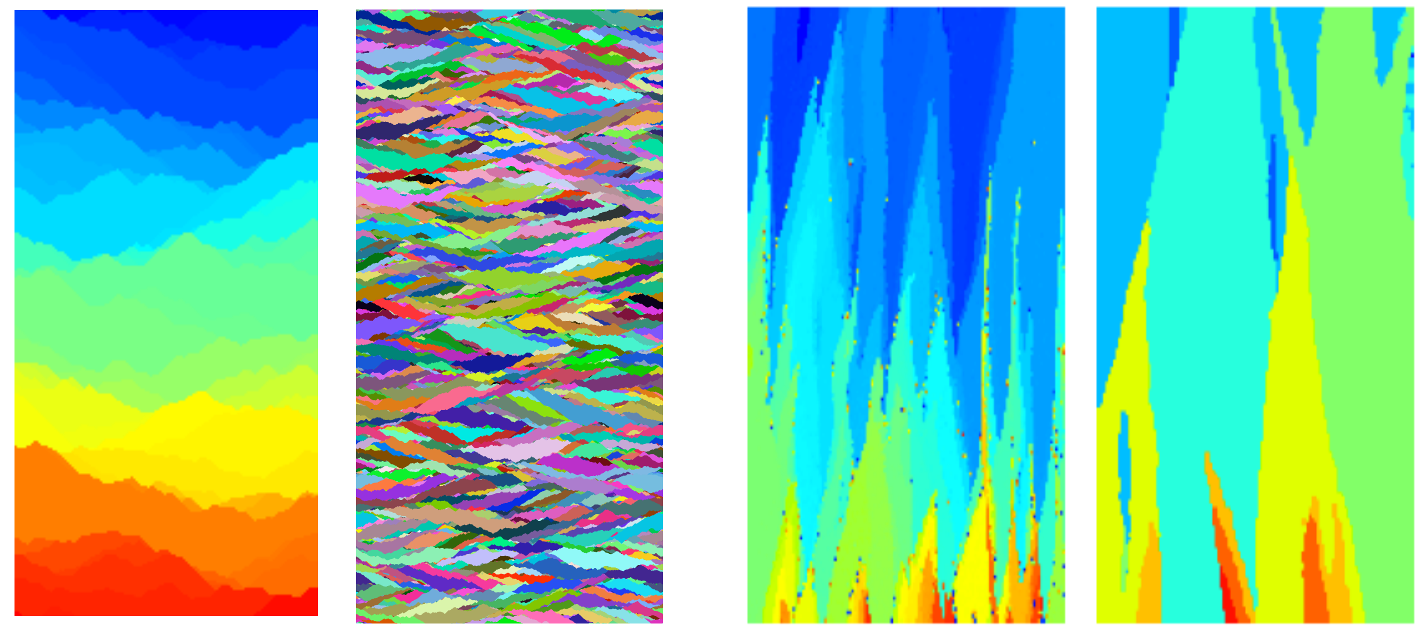


00 avalanches touch neither boundary

Emulating experimental distortions: Windows & Inference of Scaling Behavior



01 avalanches touch right boundary



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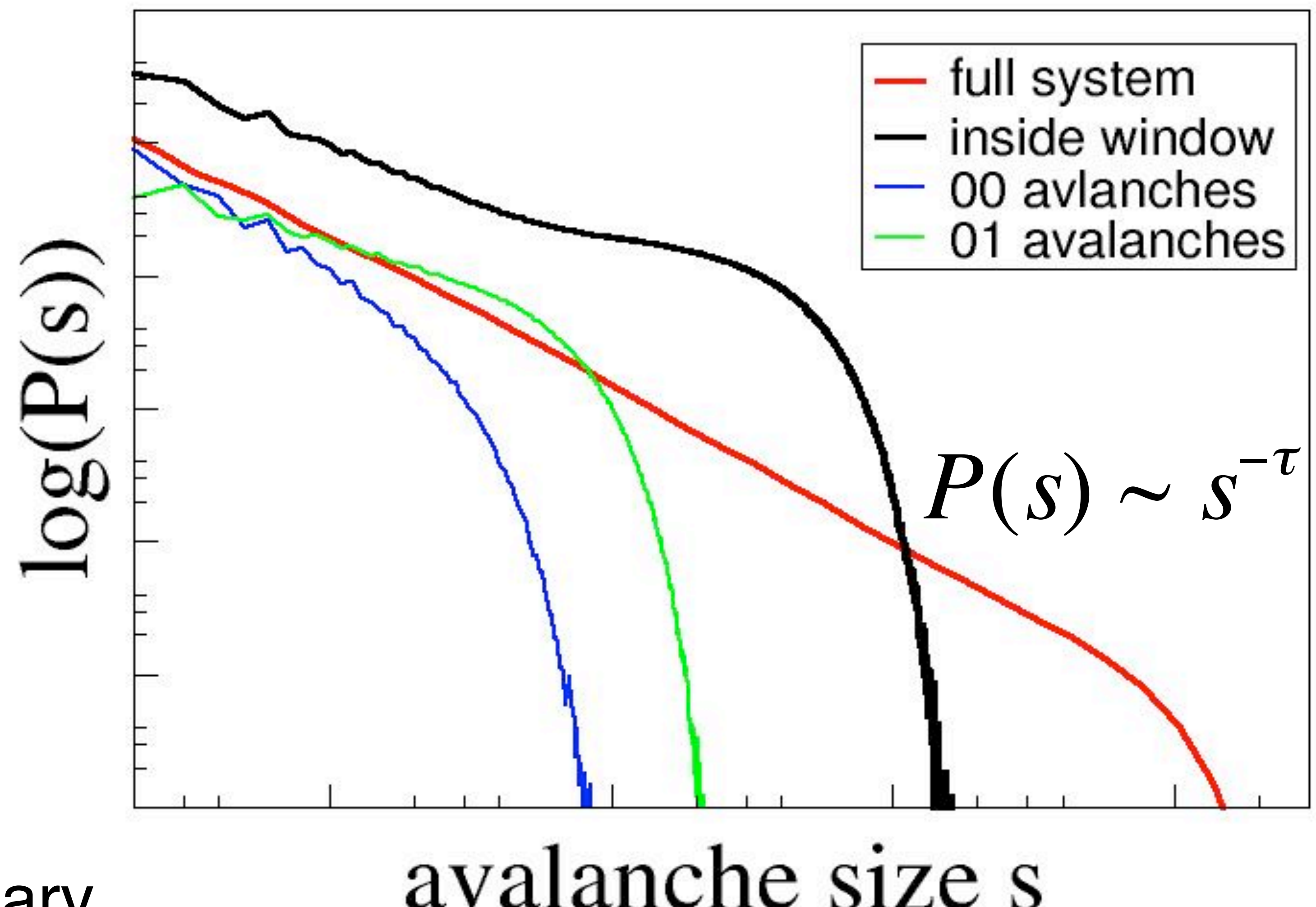

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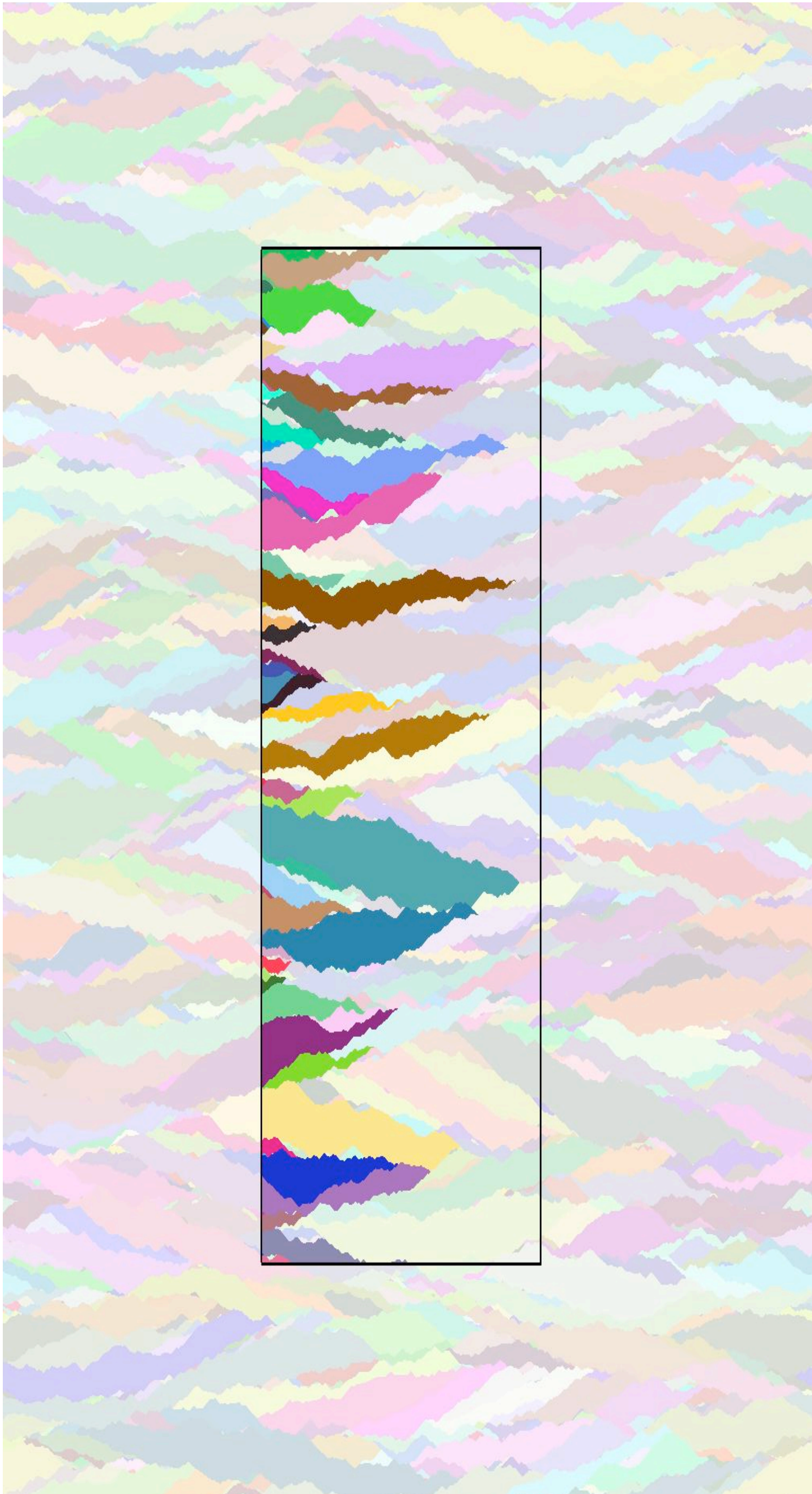

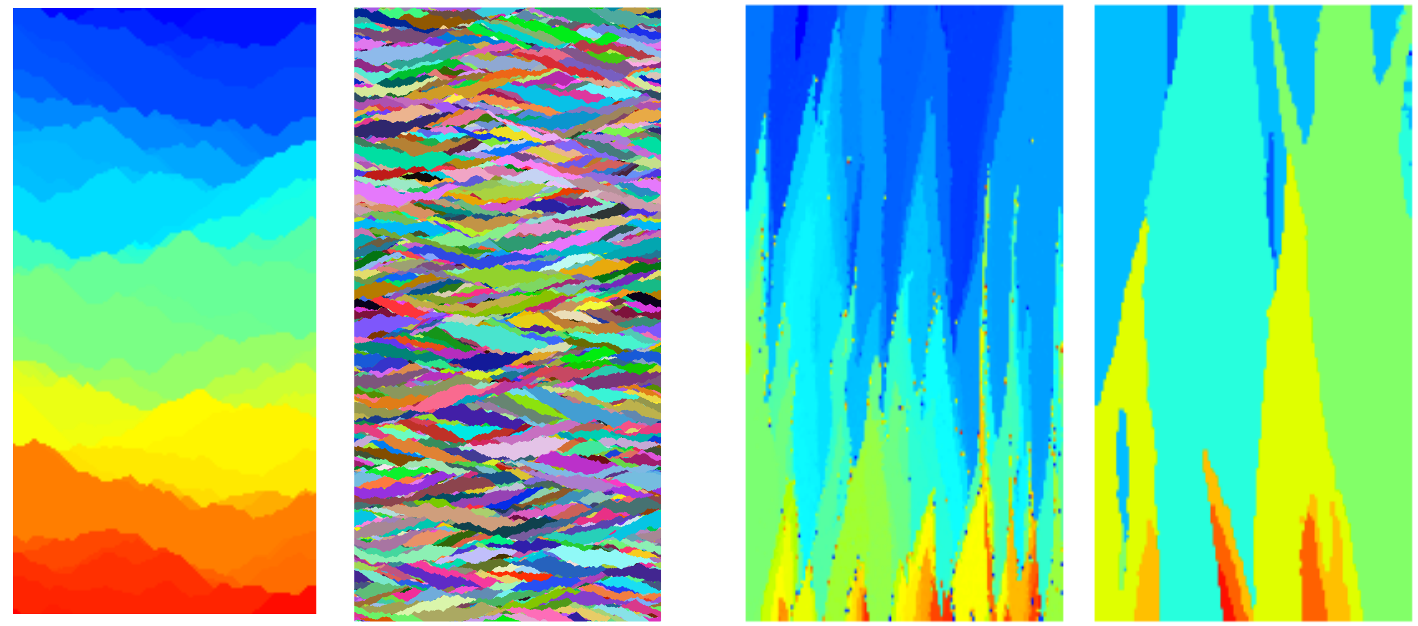

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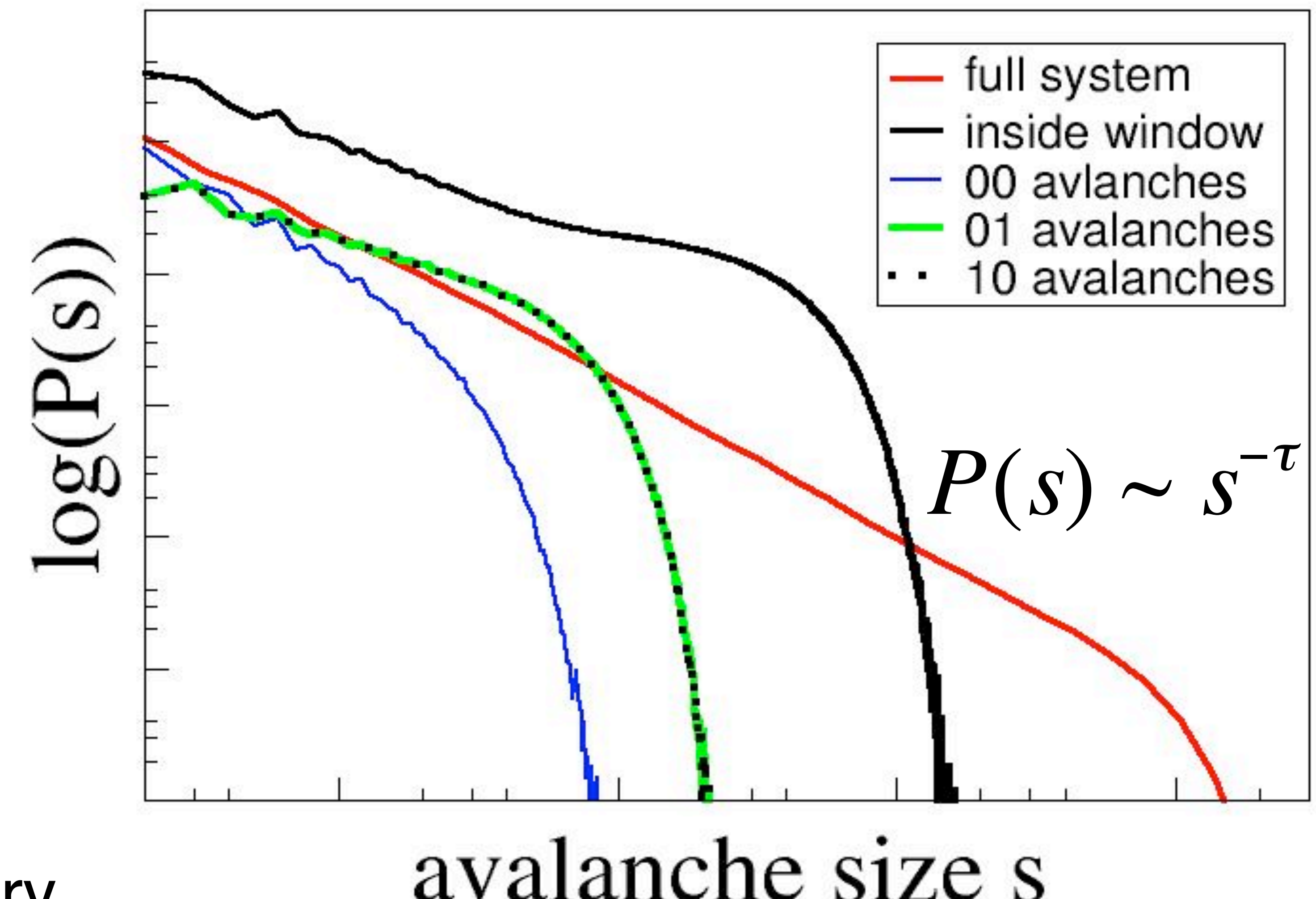

10 avalanches touch left boundary

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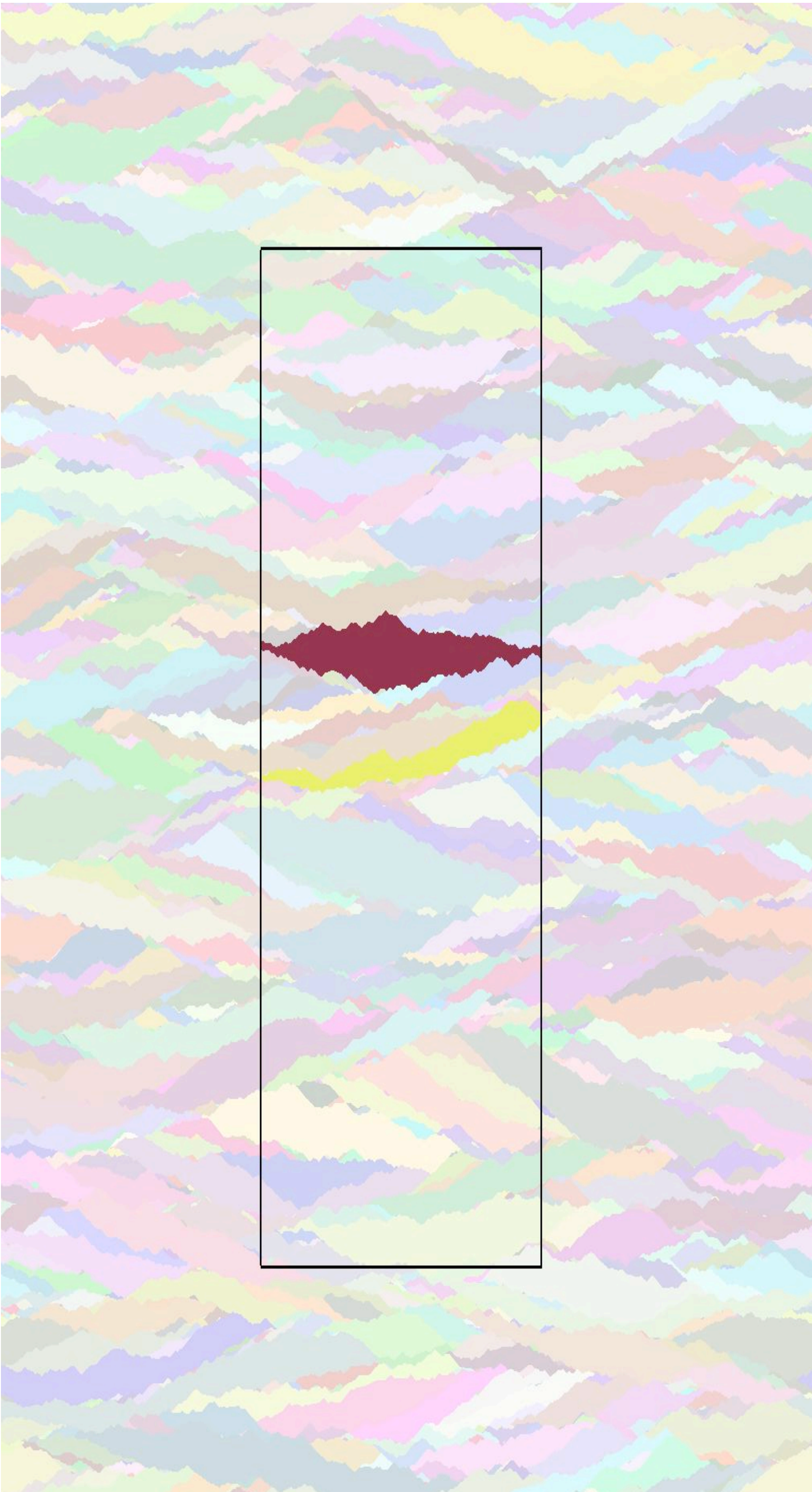

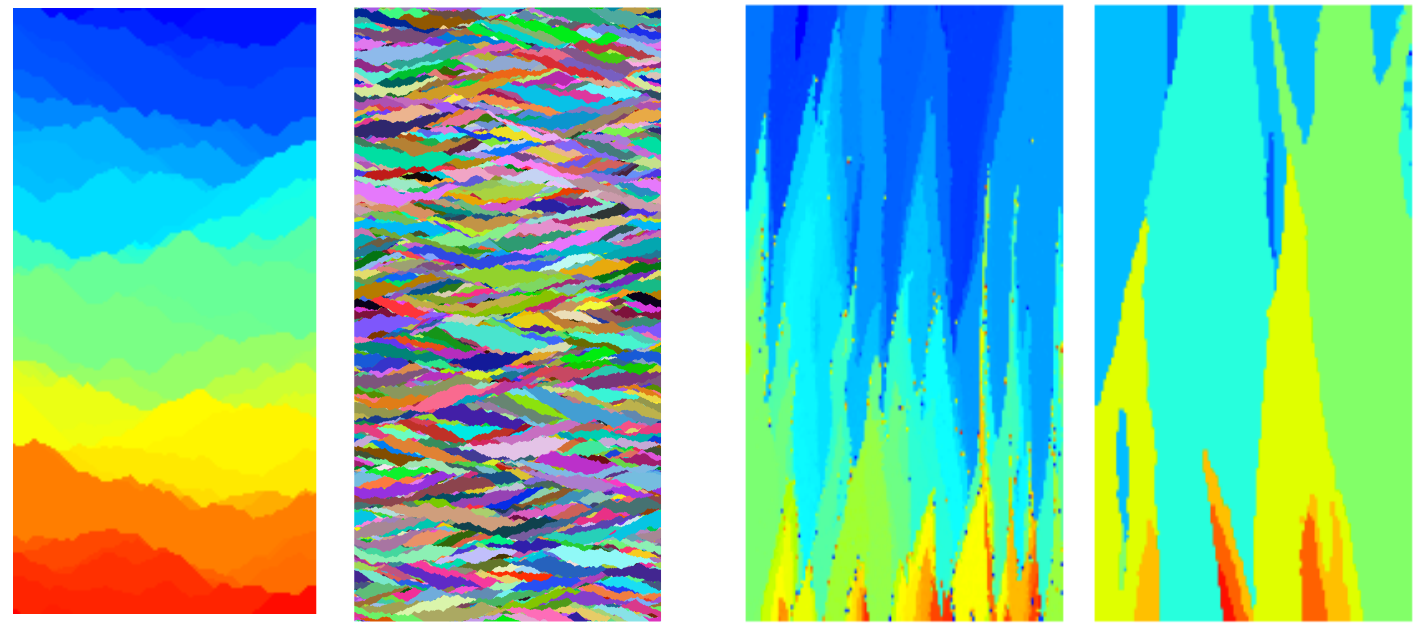

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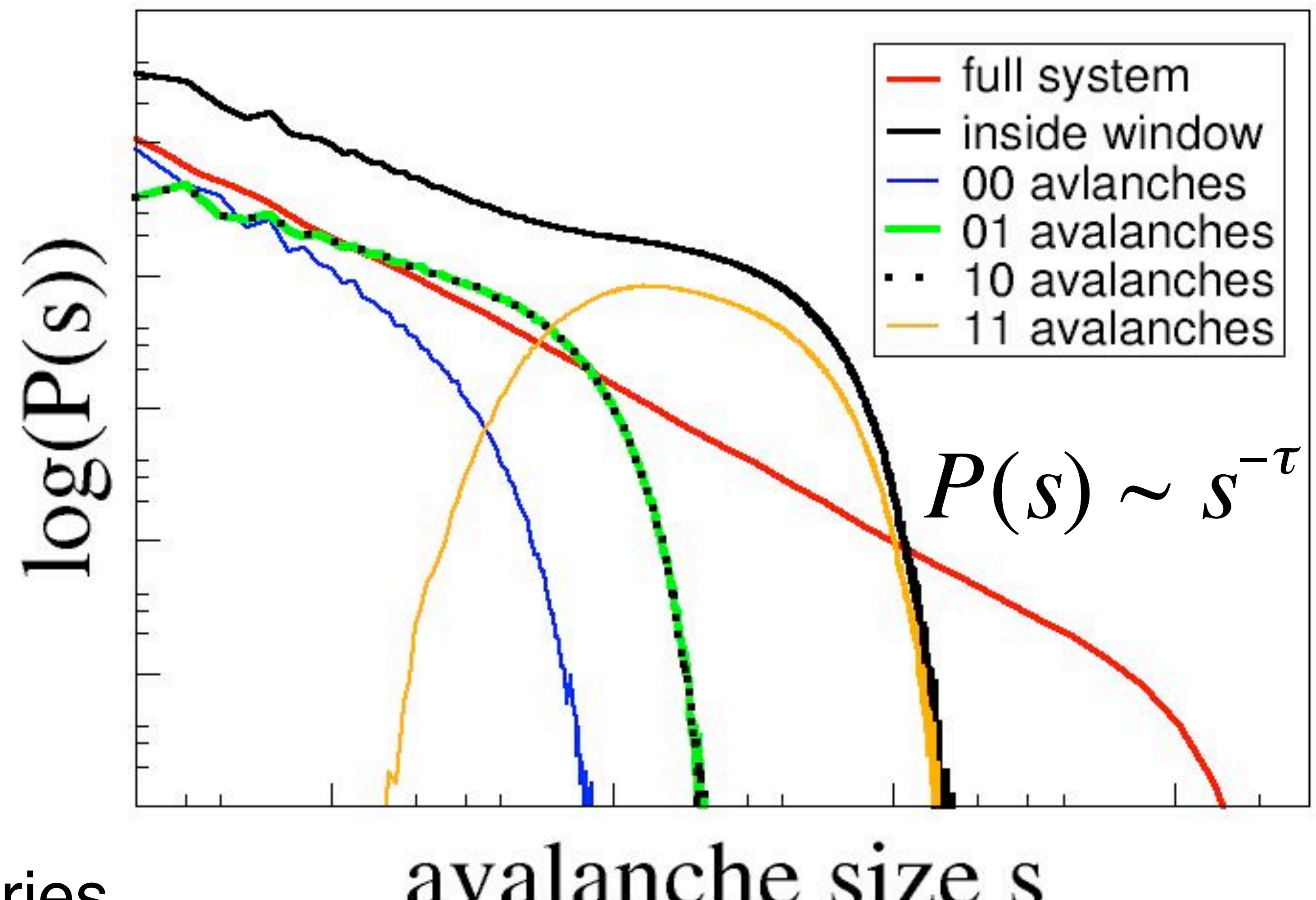

11 avalanches touch both boundaries

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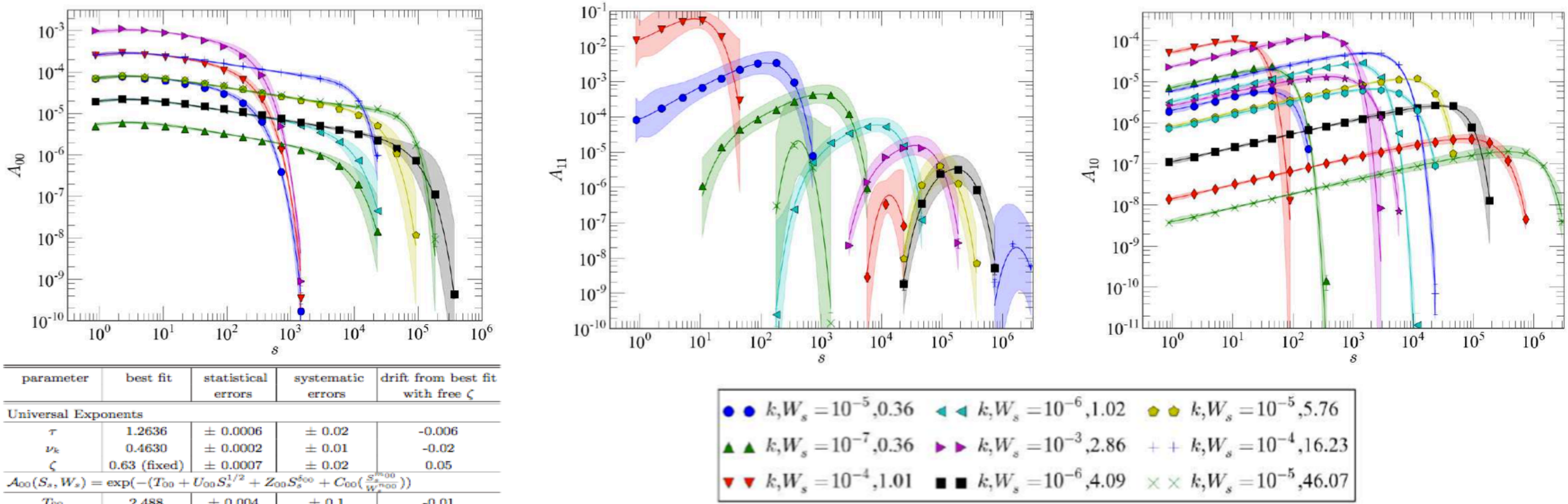

parameter	best fit	statistical errors	systematic errors	drift from best fit with free ζ
Universal Exponents				
τ	1.2636	± 0.0006	± 0.02	-0.006
ν_k	0.4630	± 0.0002	± 0.01	-0.02
ζ	0.63 (fixed)	± 0.0007	± 0.02	0.05
$\mathcal{A}_{00}(S_s, W_s) = \exp(-(T_{00} + U_{00} S_s^{1/2} + Z_{00} S_s^{\delta_{00}} + C_{00}(\frac{S_s^{m_{00}}}{W_s^{n_{00}}}))$				
T_{00}	2.488	± 0.004	± 0.1	-0.01
U_{00}	-0.150	± 0.005	± 0.1	+ 0.04
Z_{00}	0.0040	± 0.0004	± 0.01	-0.0009
δ_{00}	2.21	± 0.03	± 0.9	+0.06
C_{00}	5.60	± 0.01	± 0.7	+1.8
m_{00}	1.371	± 0.003	± 0.1	-0.04
n_{00}	1.621	± 0.004	± 0.7	0.04
$\mathcal{A}_{10}(S_s, W_s) = \exp(-(T_{10} + U_{10} S_s^{1/2} + Z_{10} S_s^{\delta_{10}} + C_{10}(\frac{S_s^{m_{10}}}{W_s^{n_{10}}}))$				
T_{10}	1.437	± 0.004	± 0.1	-0.1
U_{10}	0.244	± 0.244	± 0.1	-0.03
Z_{10}	0.027	± 0.001	± 0.03	+0.005
δ_{10}	1.64	± 0.01	± 0.4	-0.06
C_{10}	1.153	± 0.004	± 0.2	+0.7
m_{10}	1.962	± 0.005	± 0.2	+0.04
n_{10}	1.624	± 0.004	± 0.1	+0.06
$\mathcal{A}_{11}(S_s, W_s) =$ $\exp(-(T_{11} + U_{11} S_s^{1/2} + Z_{11} S_s^{\delta_{11}} + D_{11}(\frac{S_s}{W_s})^{m1} + C_{11}(\frac{S_s}{W_s^{p1}})^{-m2})$				
T_{11}	0.42	± 0.04	± 1.2	-0.3
U_{11}	-0.5	± 0.1	± 3.6	-0.5
Z_{11}	0.21	± 0.06	± 1.7	0.4
δ_{11}	1.102	± 0.03	± 1.0	-0.12
D_{11}	0.54	± 0.03	± 1.0	+0.1
C_{11}	0.86	± 0.05	± 1.6	-0.3
m1	1.47	±0.01	± 0.4	-0.0008
m2	1.61	± 0.2	± 0.6	-0.02
p1	1.653	± 0.004	± 0.1	+0.02
Corrections to Scaling $A_1^{xx}/s + A_2^{xx}/s^2$				
A_1^{00}	-0.94	± 0.02	± 0.5	+0.06
A_2^{00}	0.27	± 0.01	± 0.4	-0.04
A_1^{10}	0.15	± 0.01	± 0.4	-0.2
A_2^{10}	-0.07	± 0.01	± 0.3	+0.1
A_1^{11}	0.2	± 0.3	± 7.9	-0.6
A_2^{11}	0.6	± 0.2	± 7.1	+0.2

- "Simple enough" scaling functions motivated by some known analytical results

  32 parameters(!), 370 data points

- Simultaneous fits of all data taken with consequent inference of full scaling behavior

- Non-trivial error bar analysis (colored regions)

Outline

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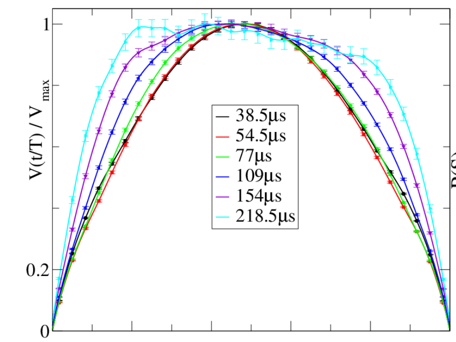

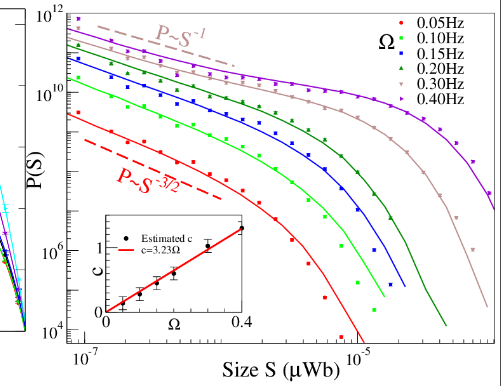

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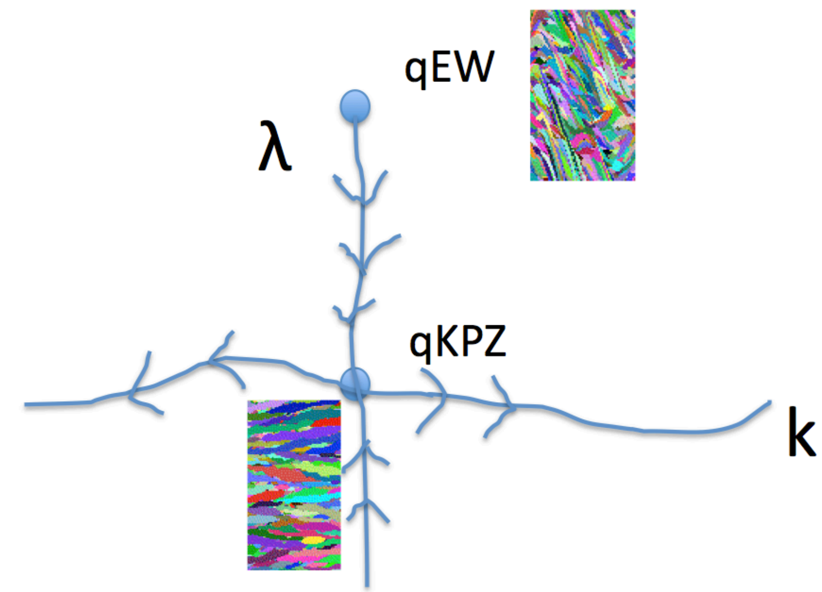
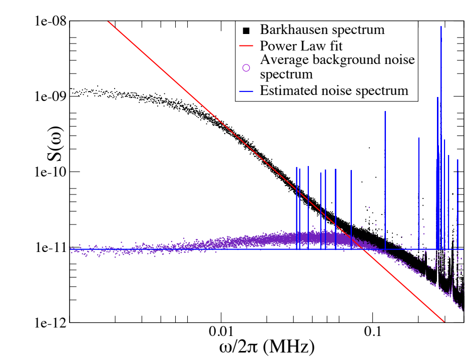
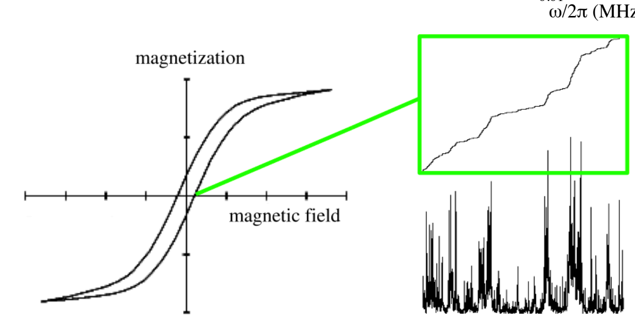


- Delving into an experiment: Barkhausen crackling noise, a problem that gave us insights and tools

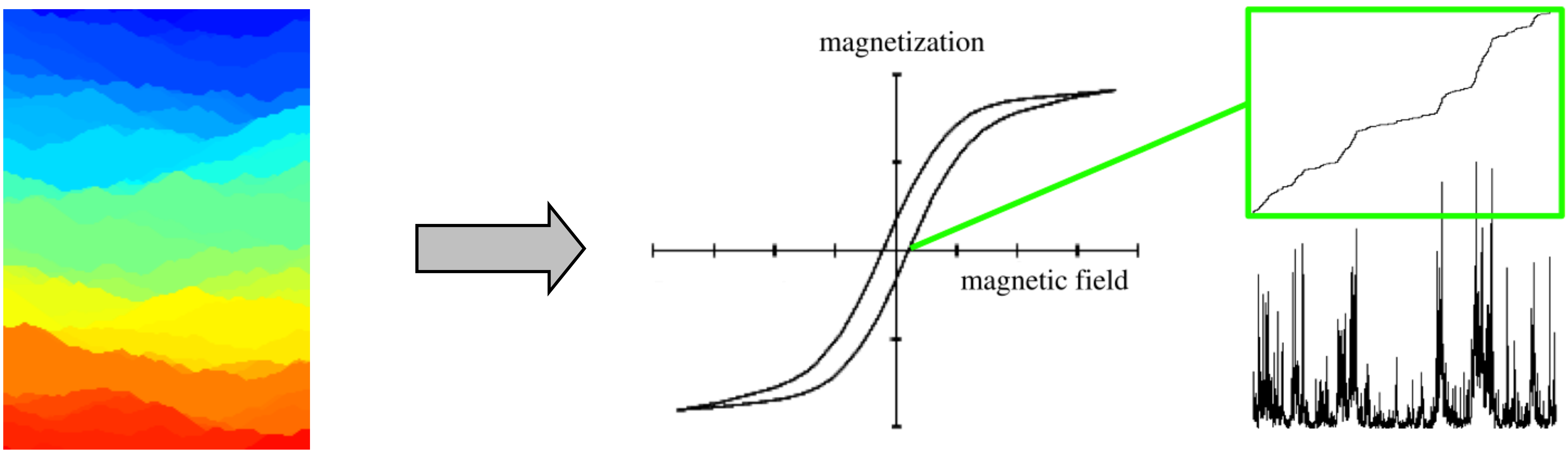




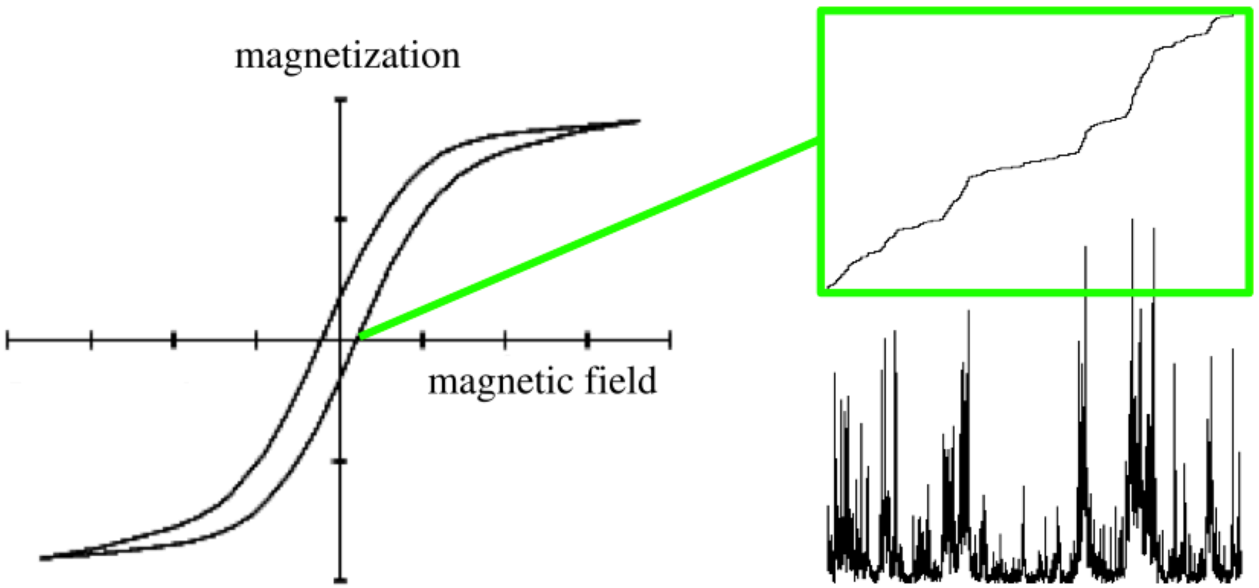
Playground for studying scaling: Barkhausen Noise

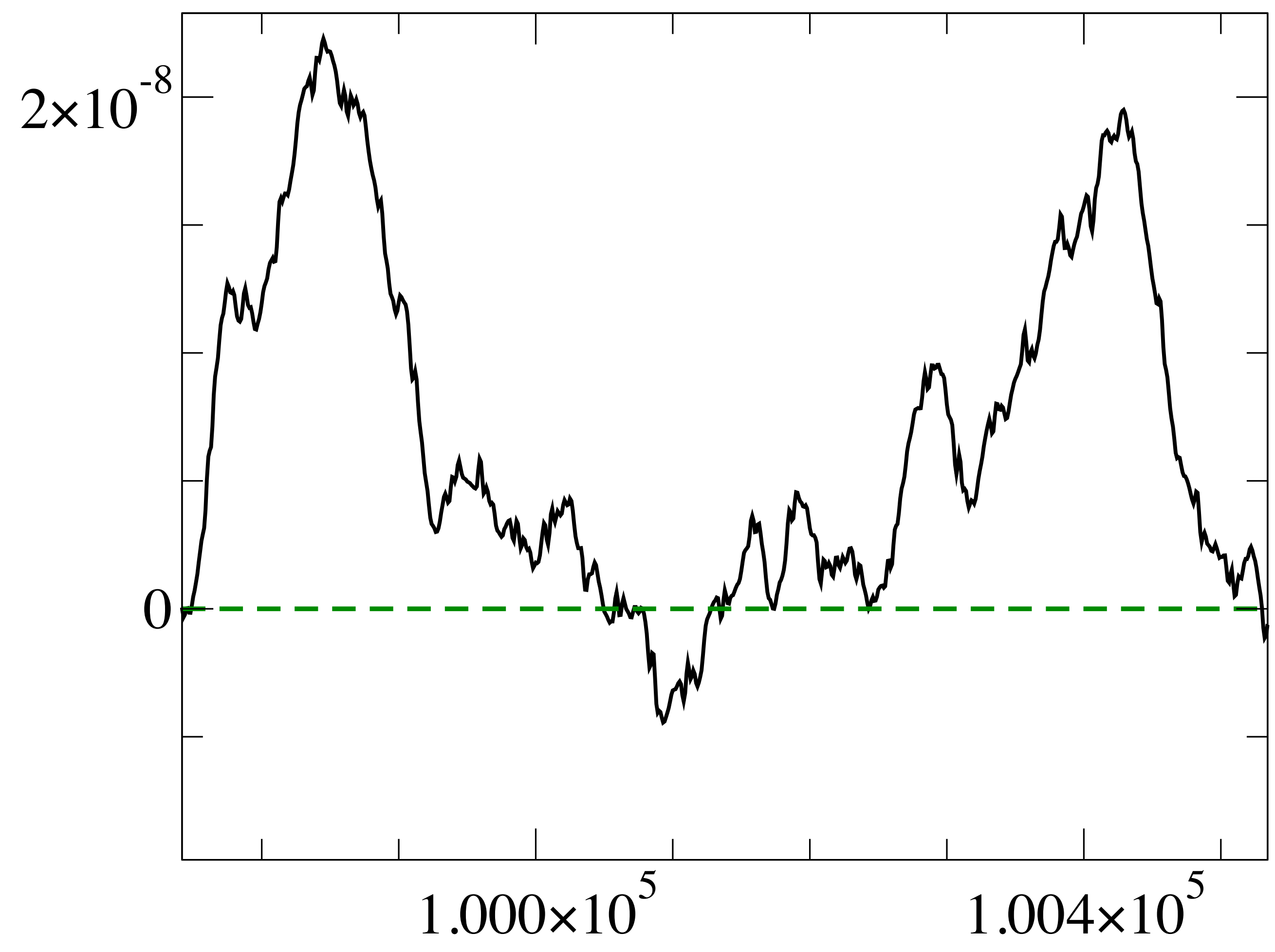
- A system where experimentalists have been seeing scaling for decades,

- From a moving interface to hysteresis and to an avalanche timeseries...

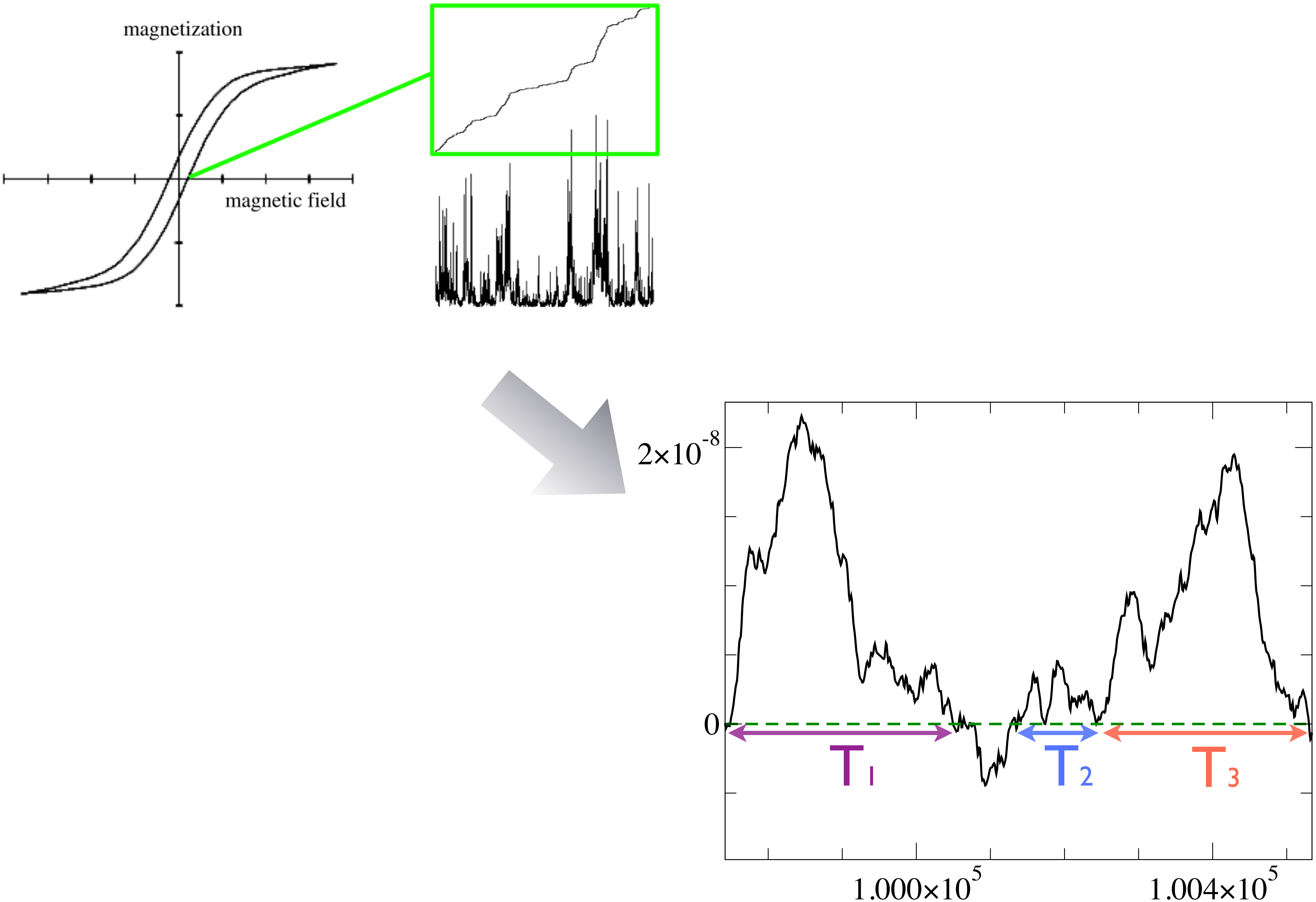


Measurements on a timeseries





Measurements on a timeseries



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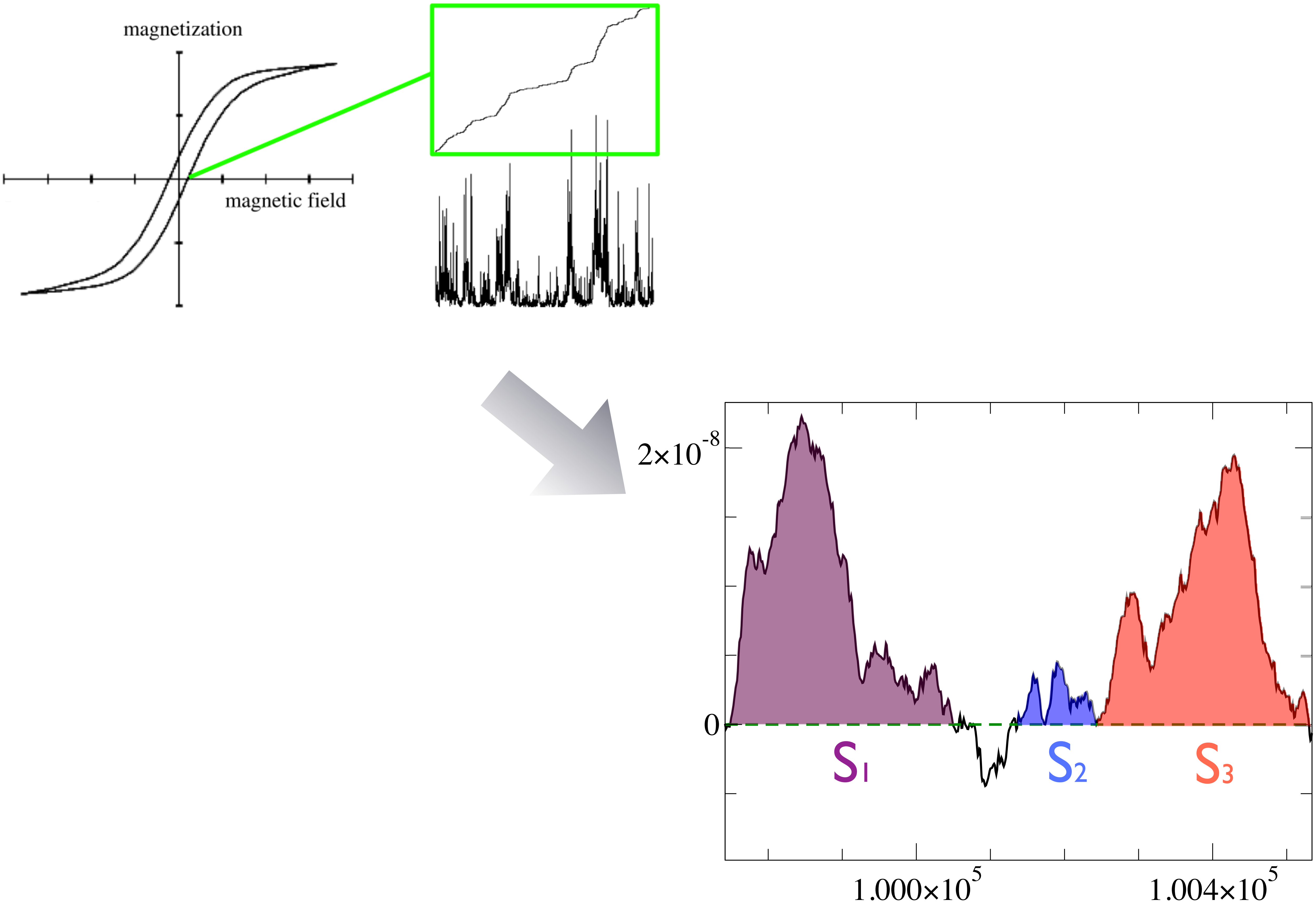

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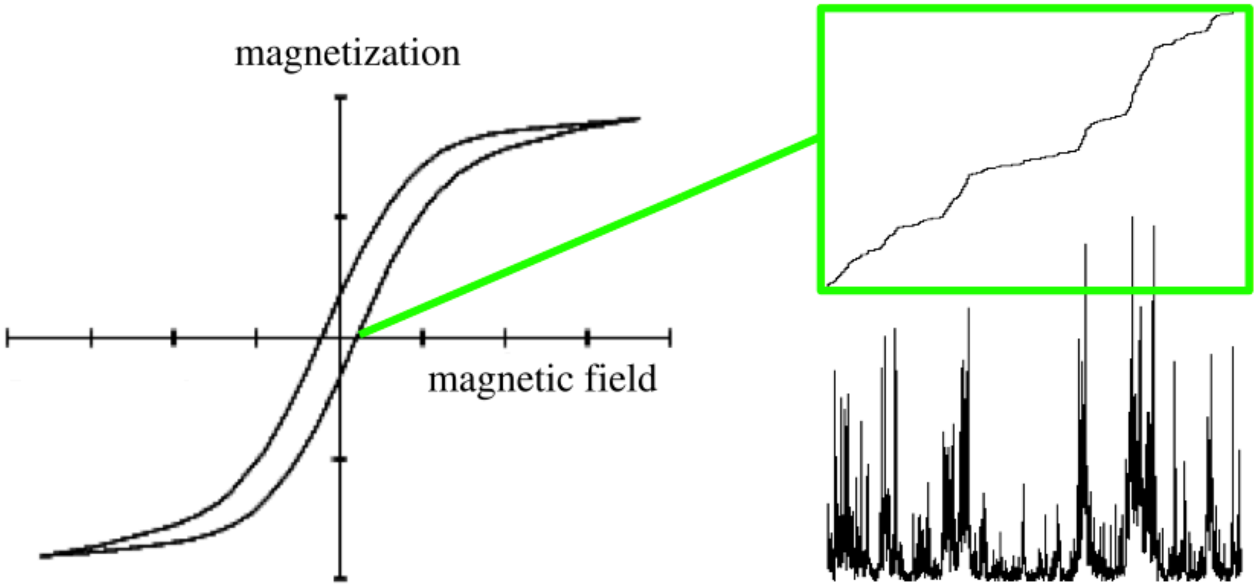

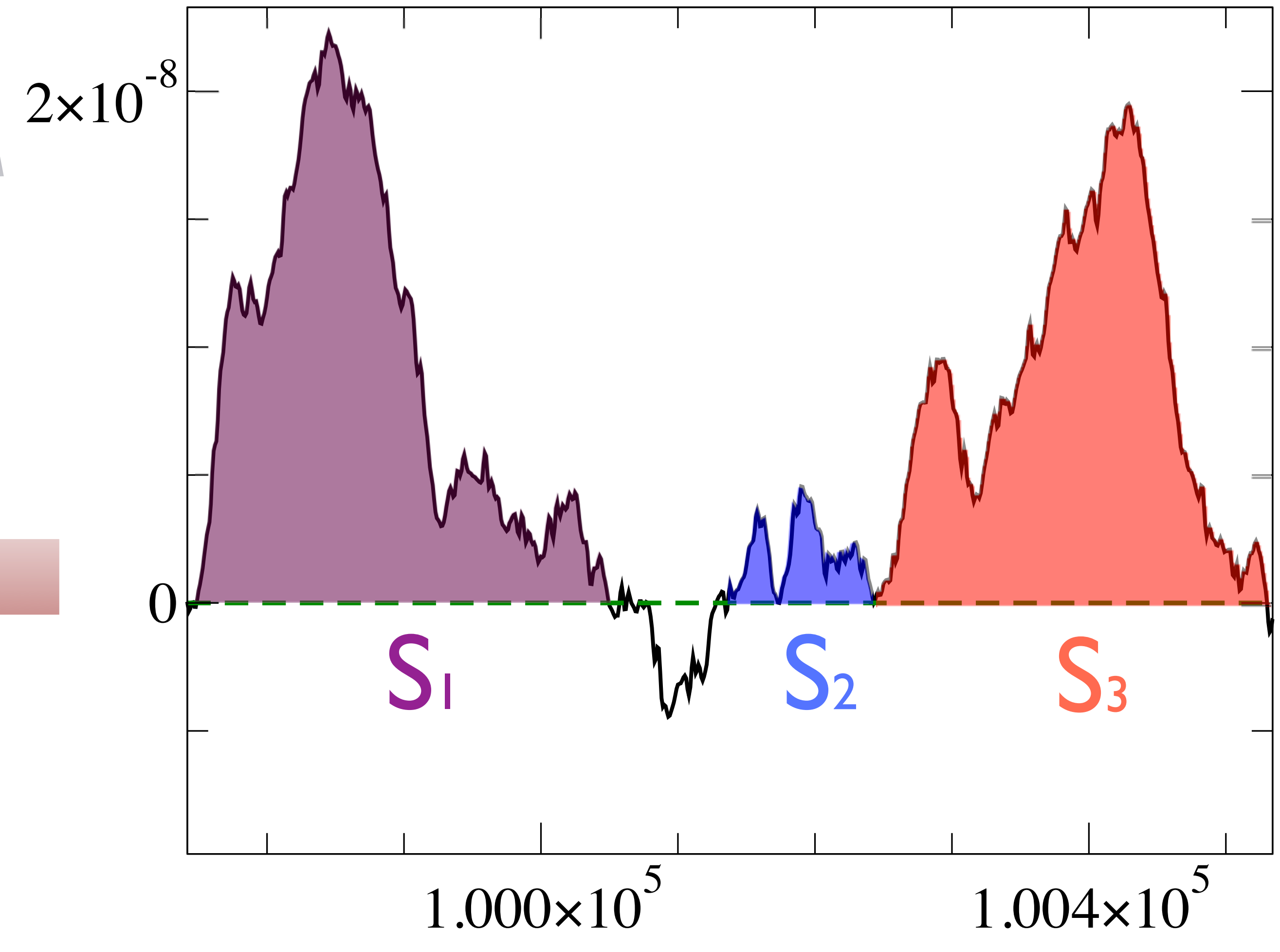

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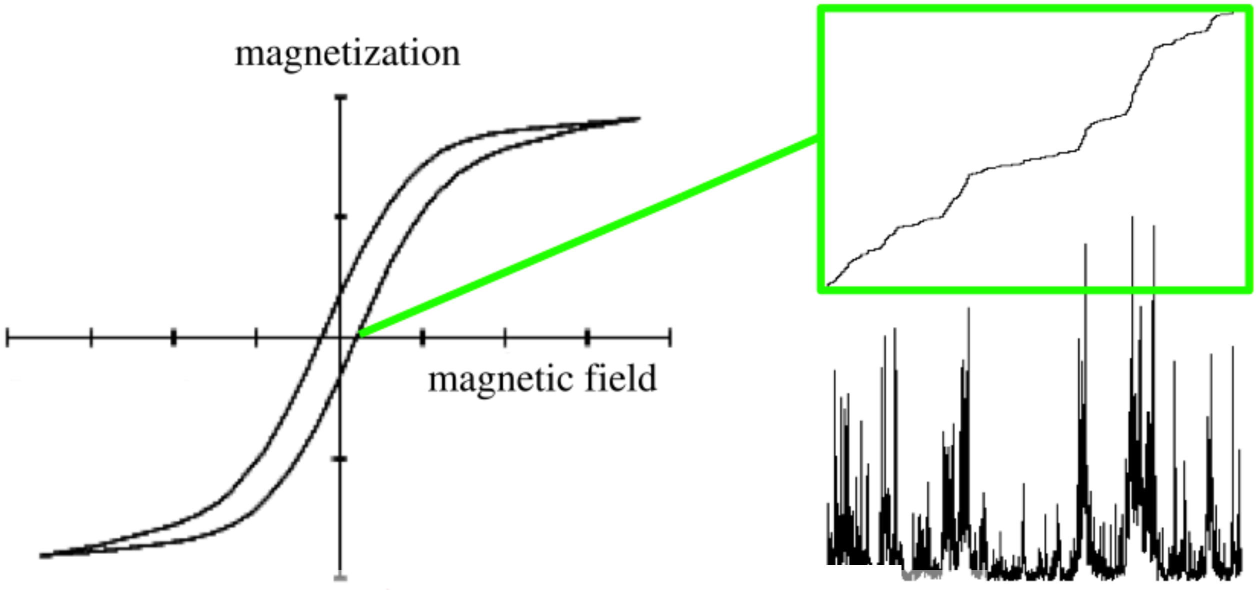

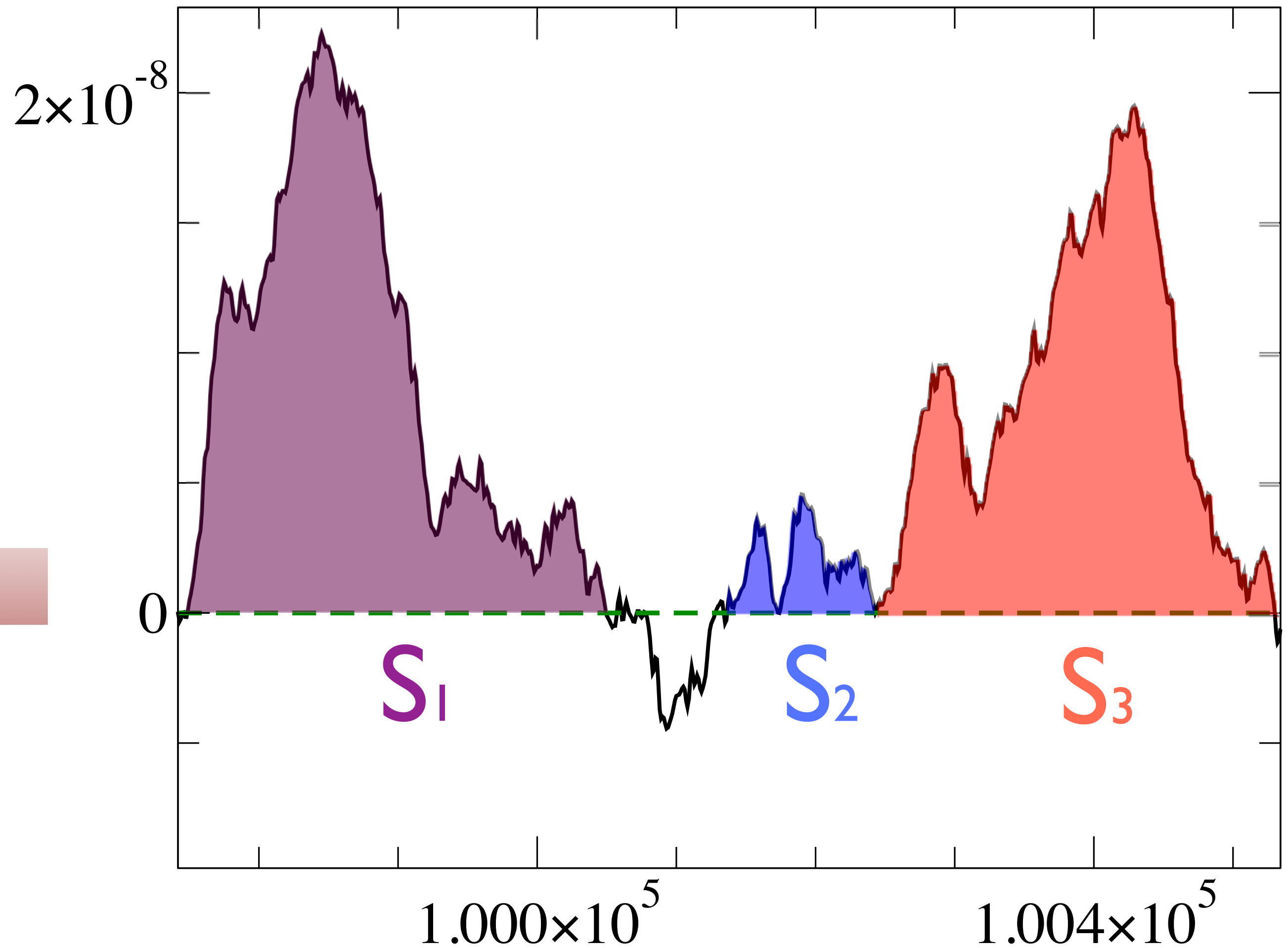

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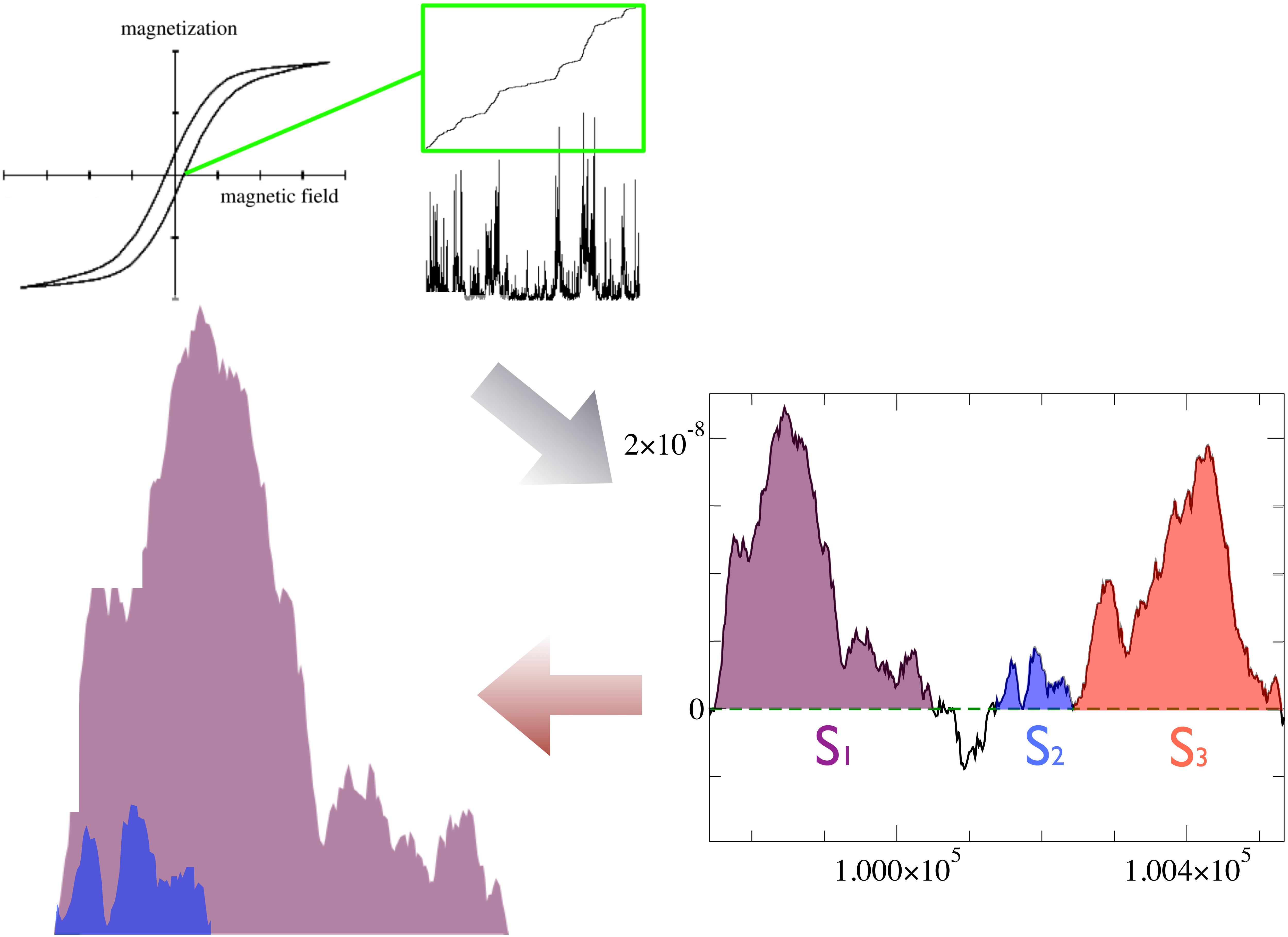

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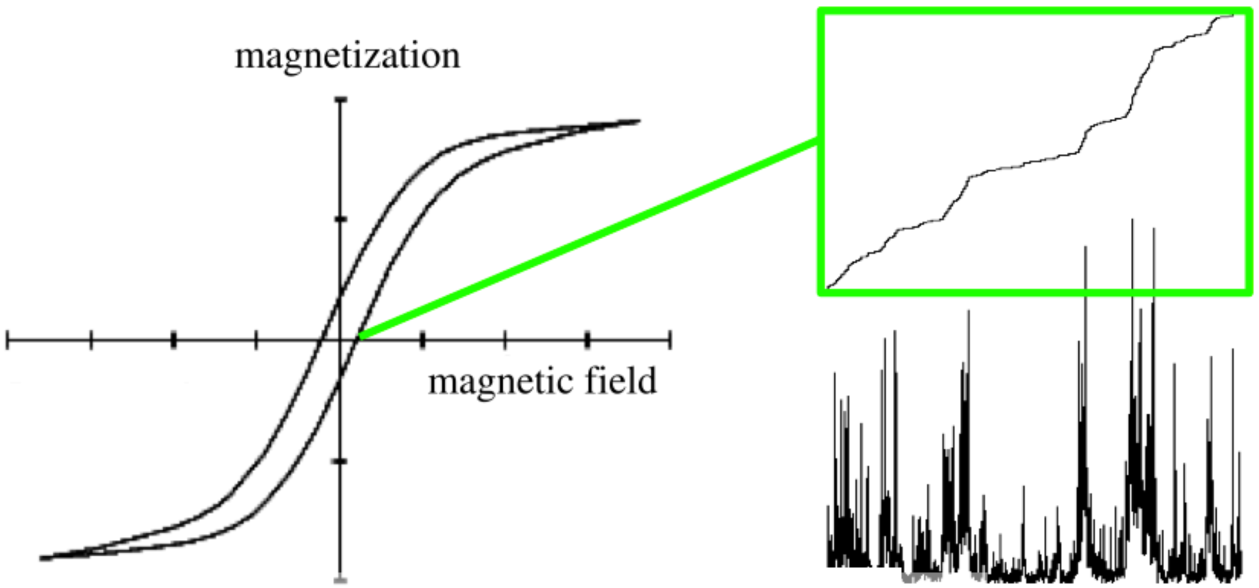

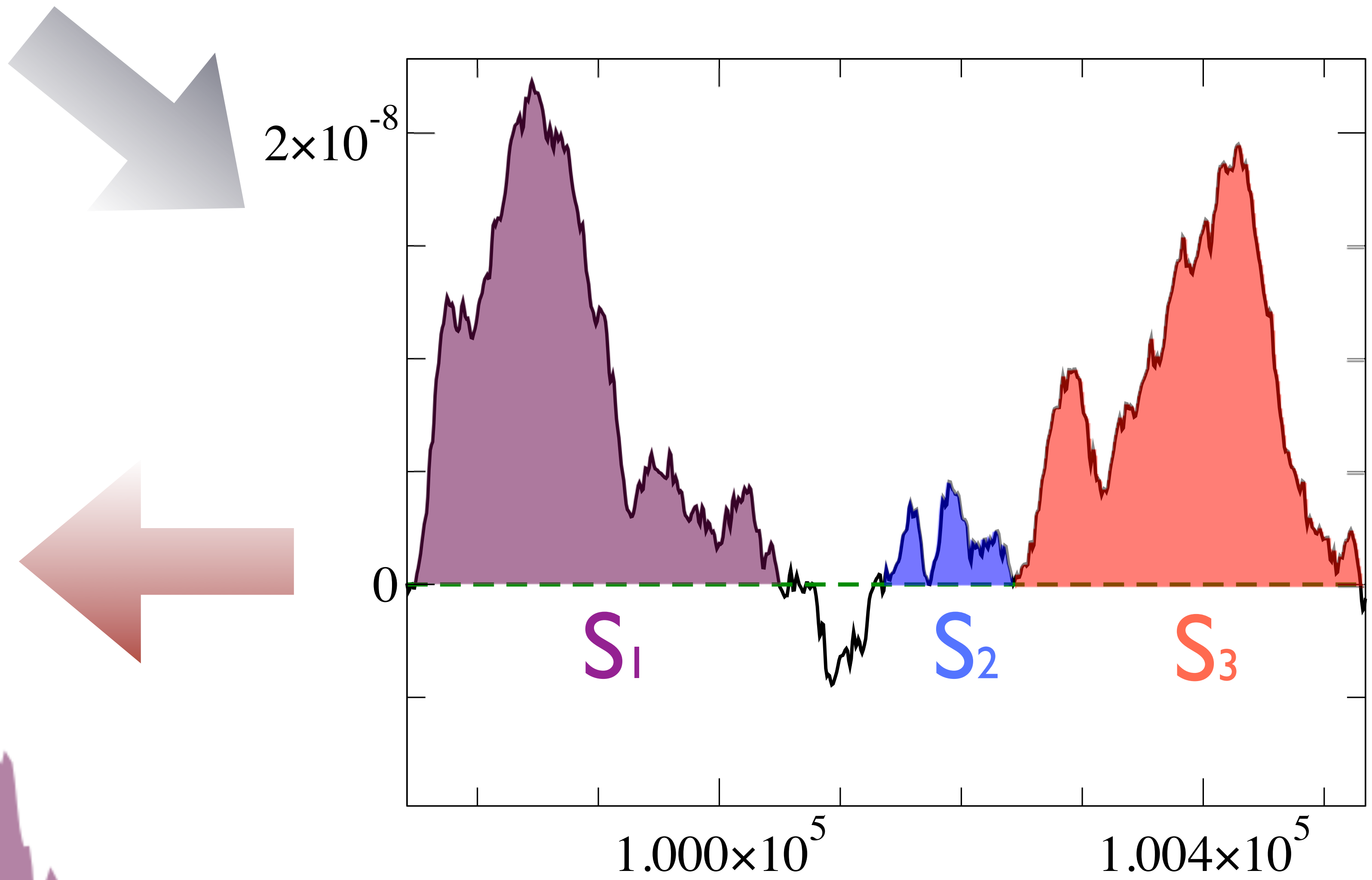

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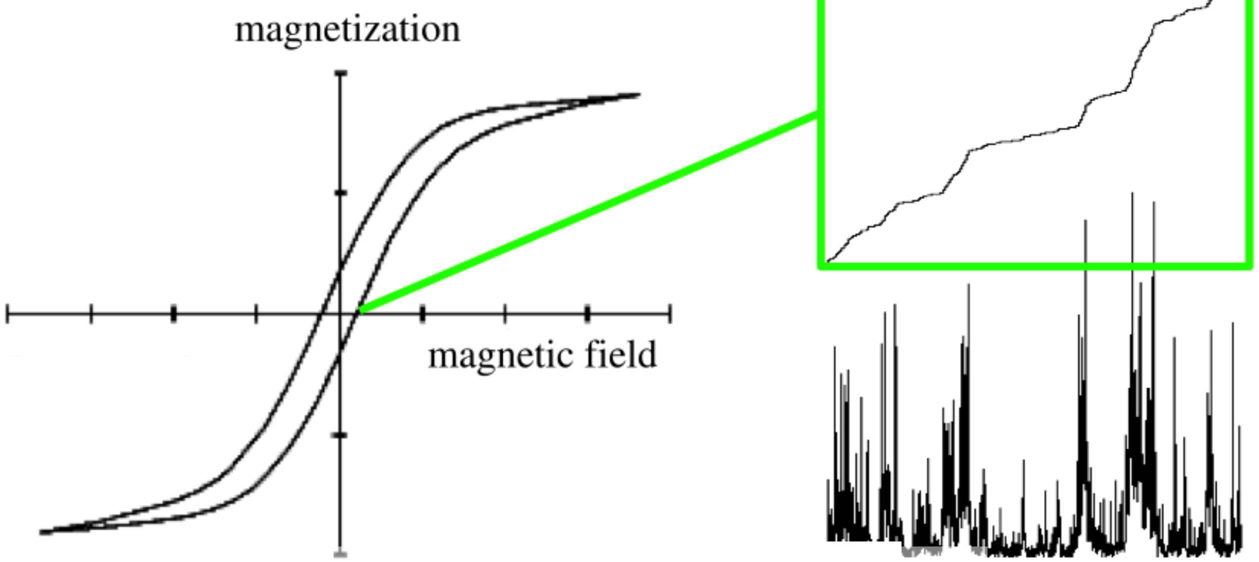

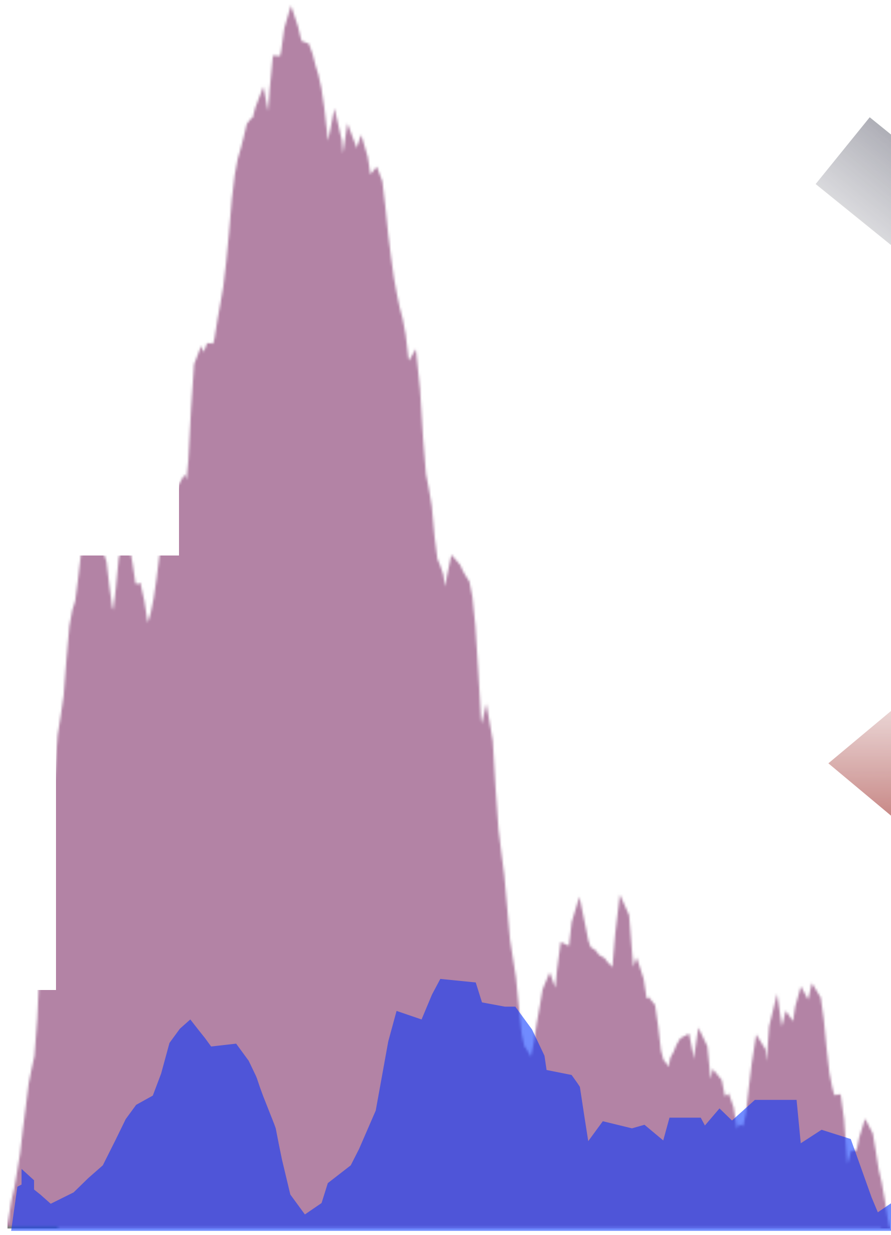

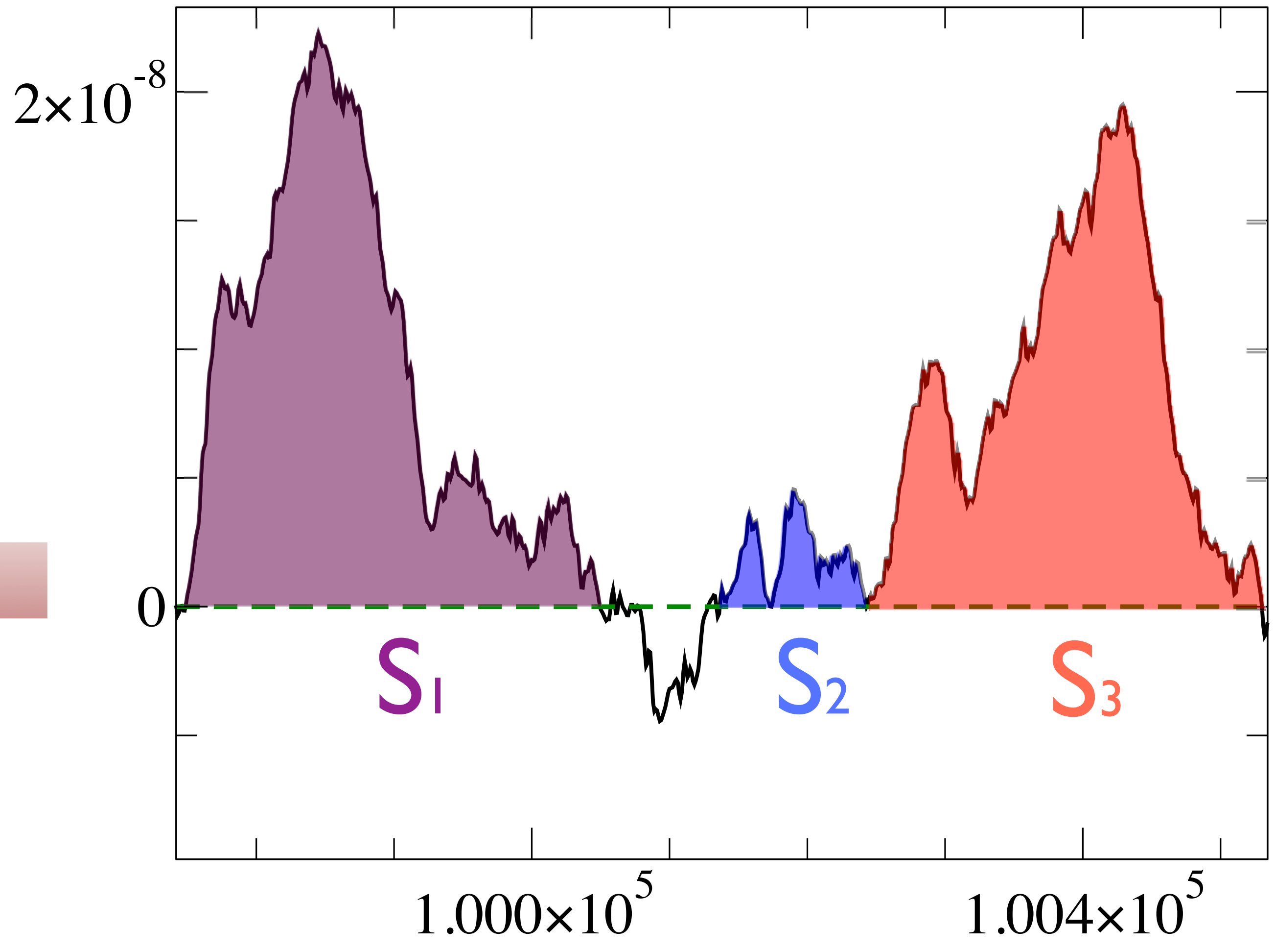

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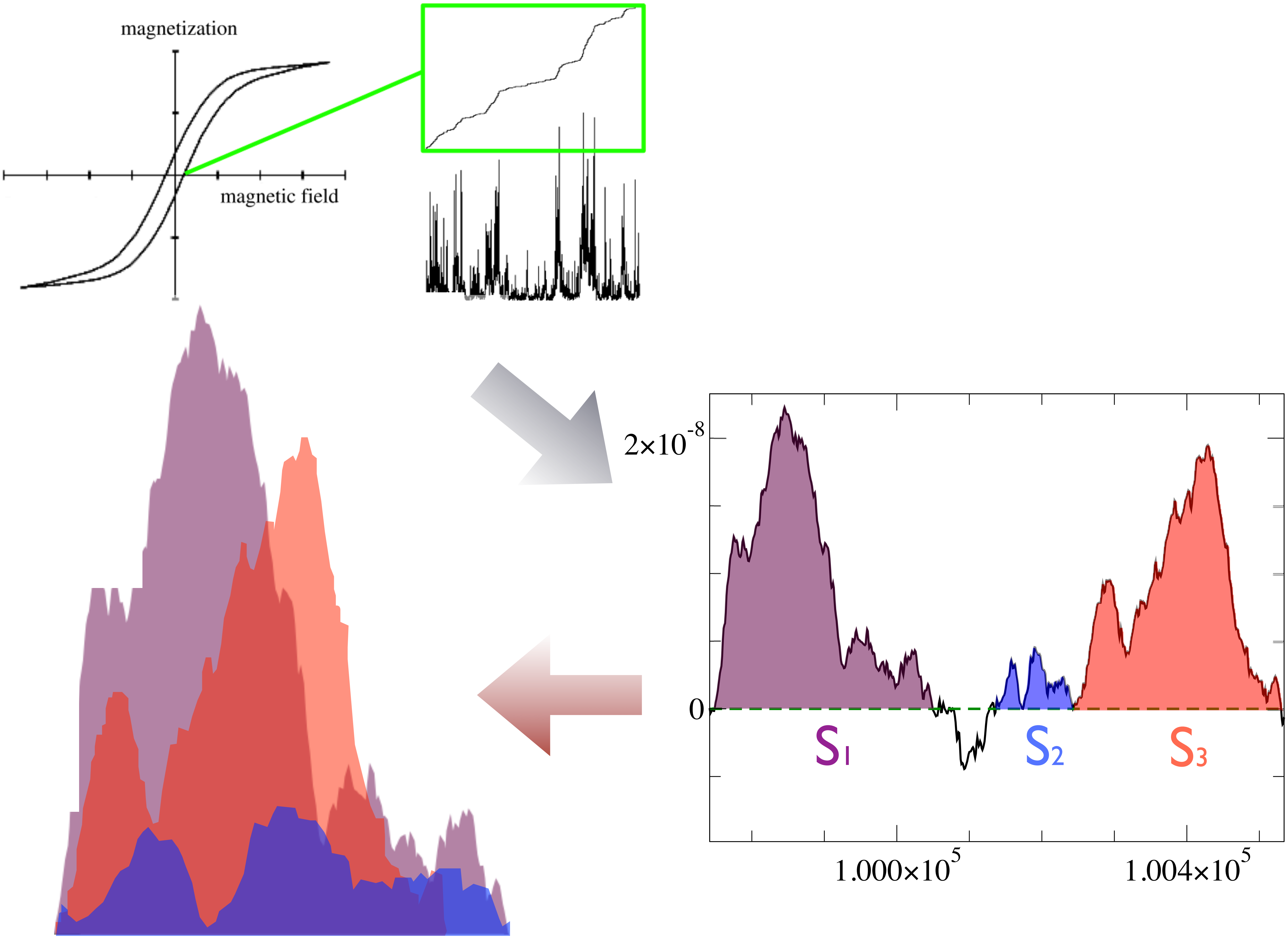

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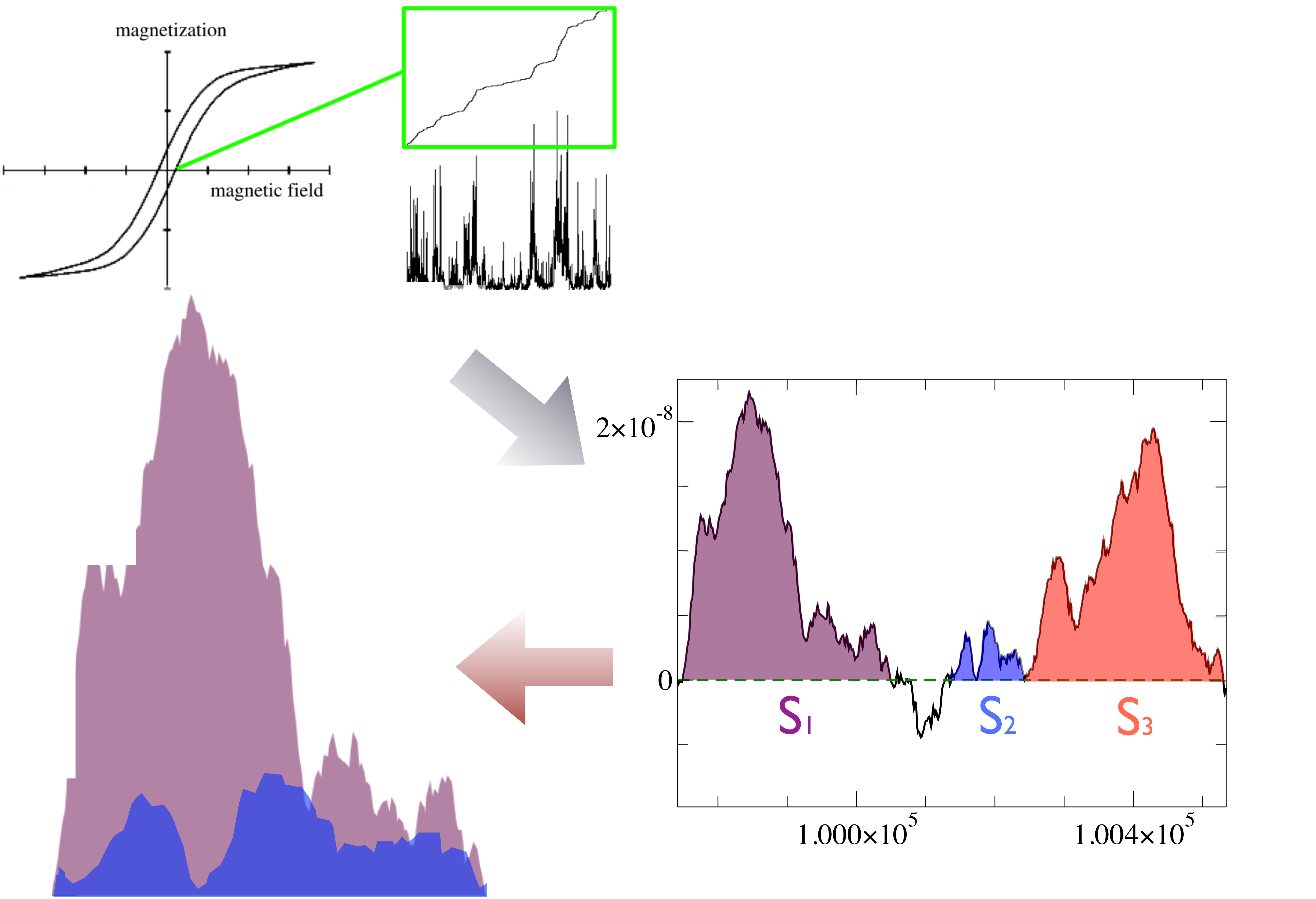

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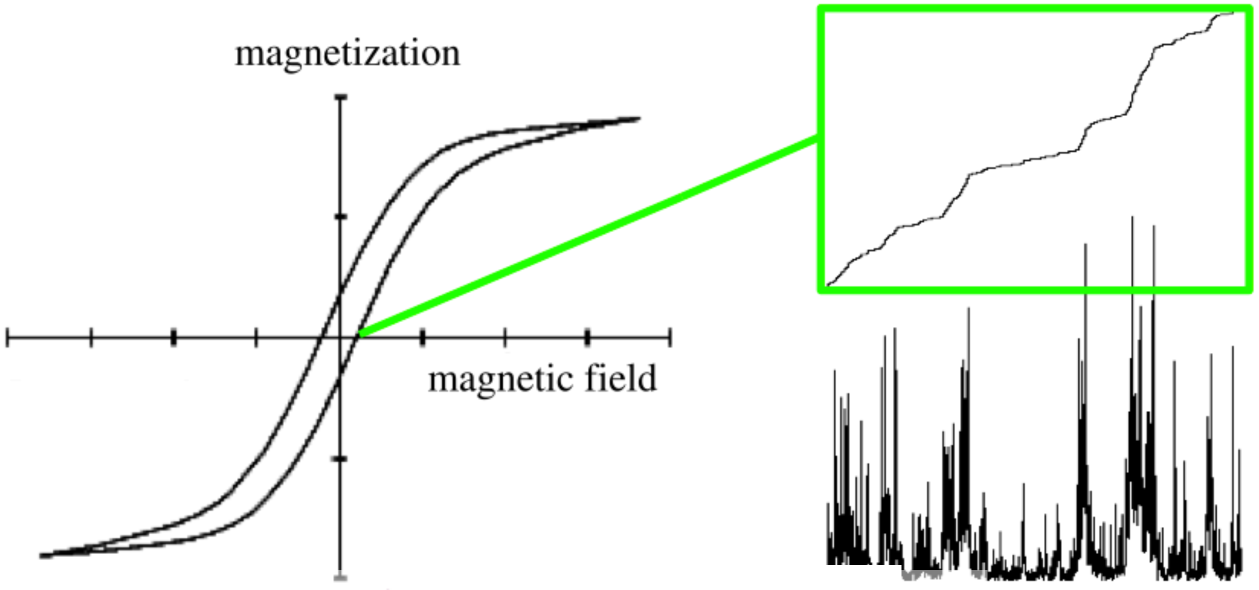

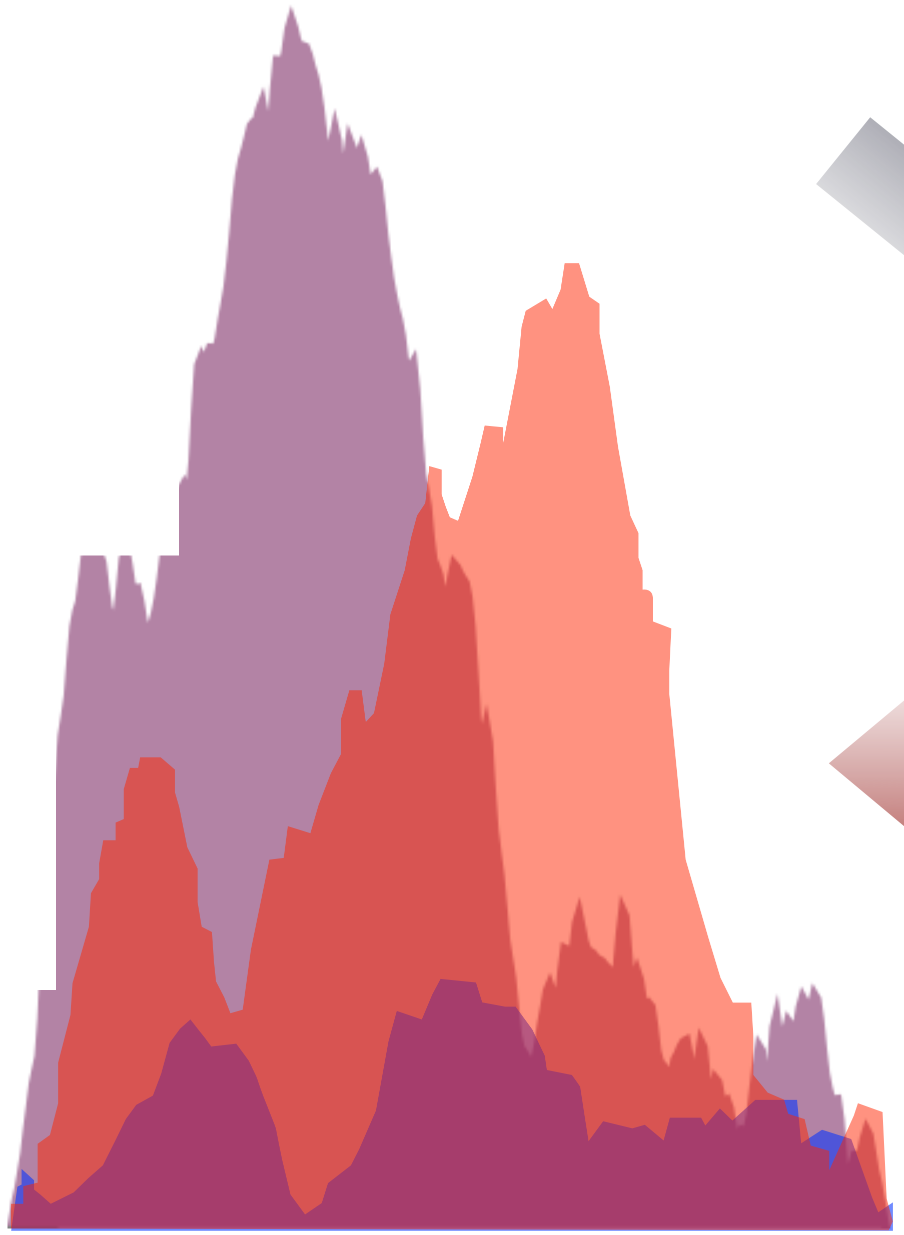

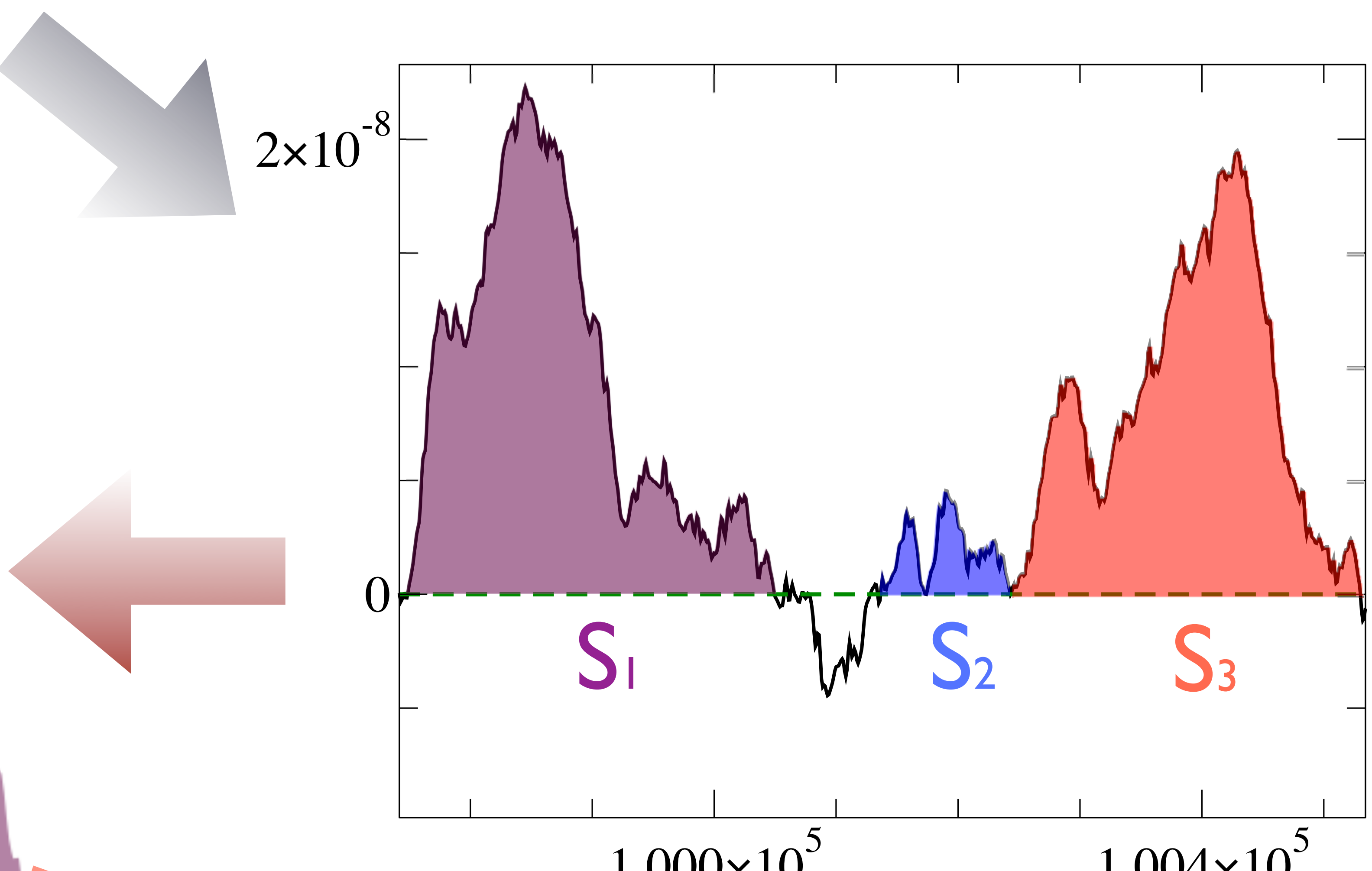

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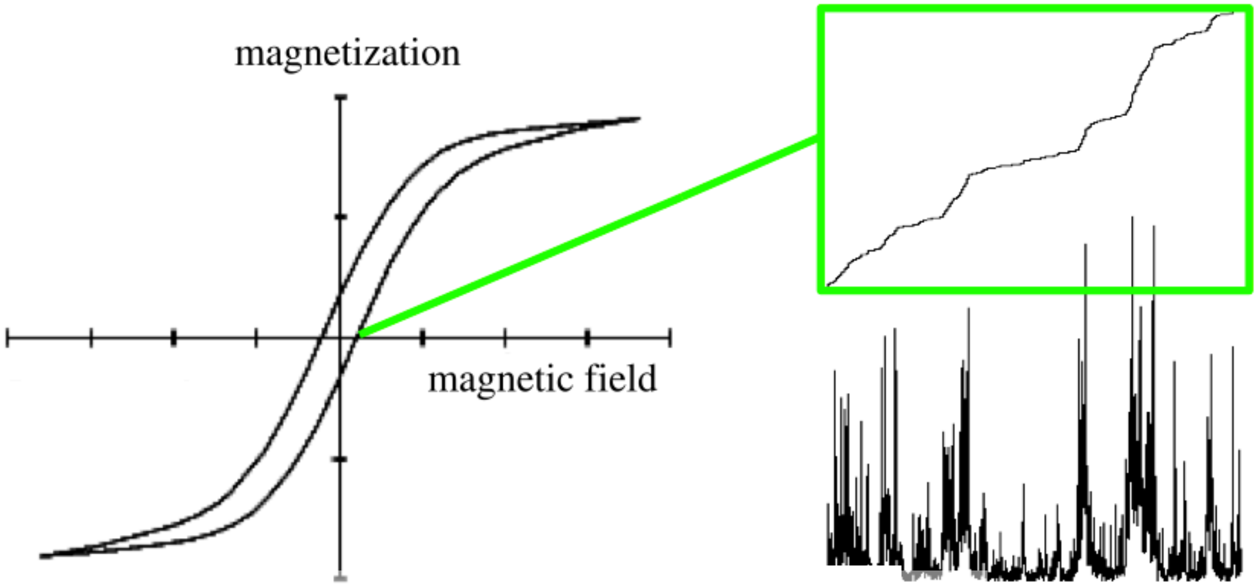
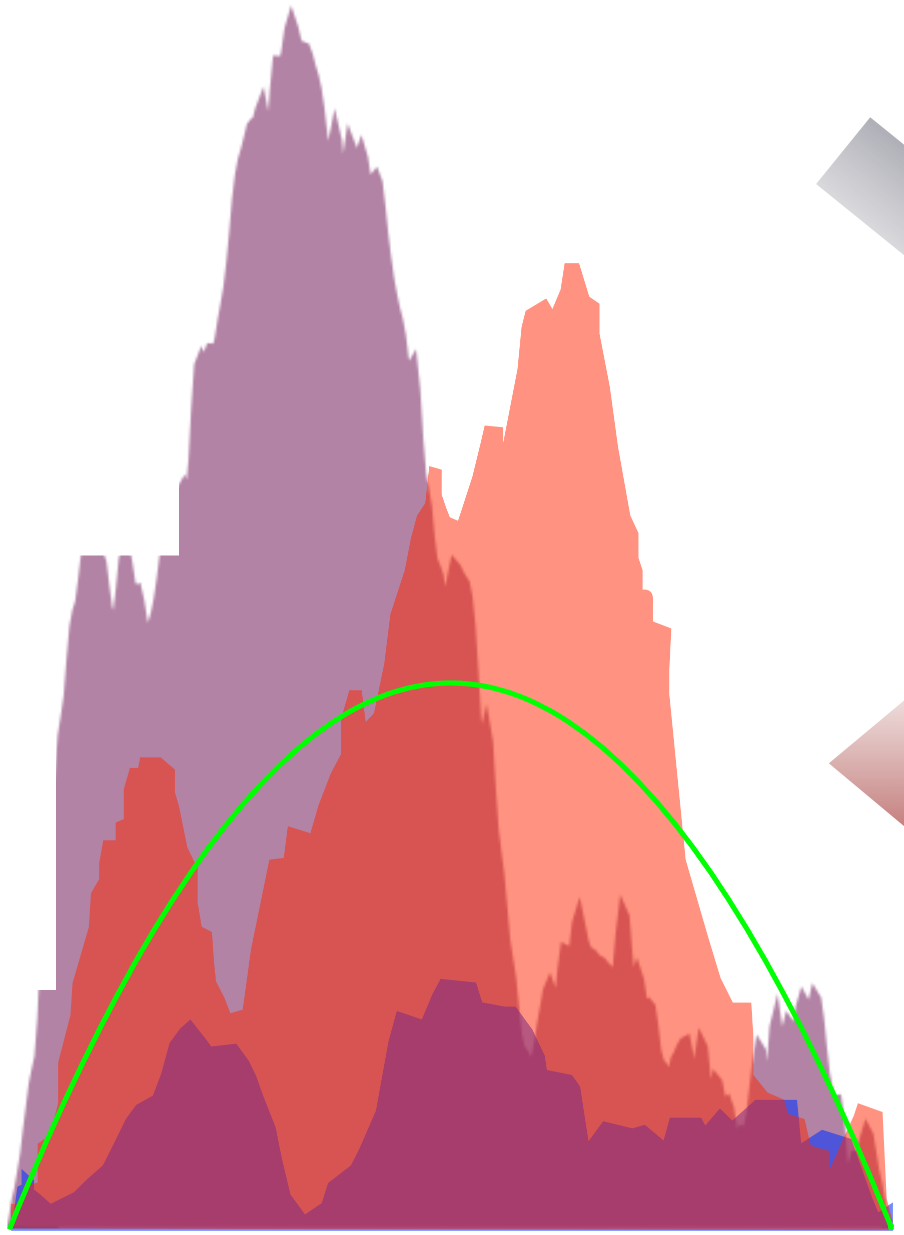
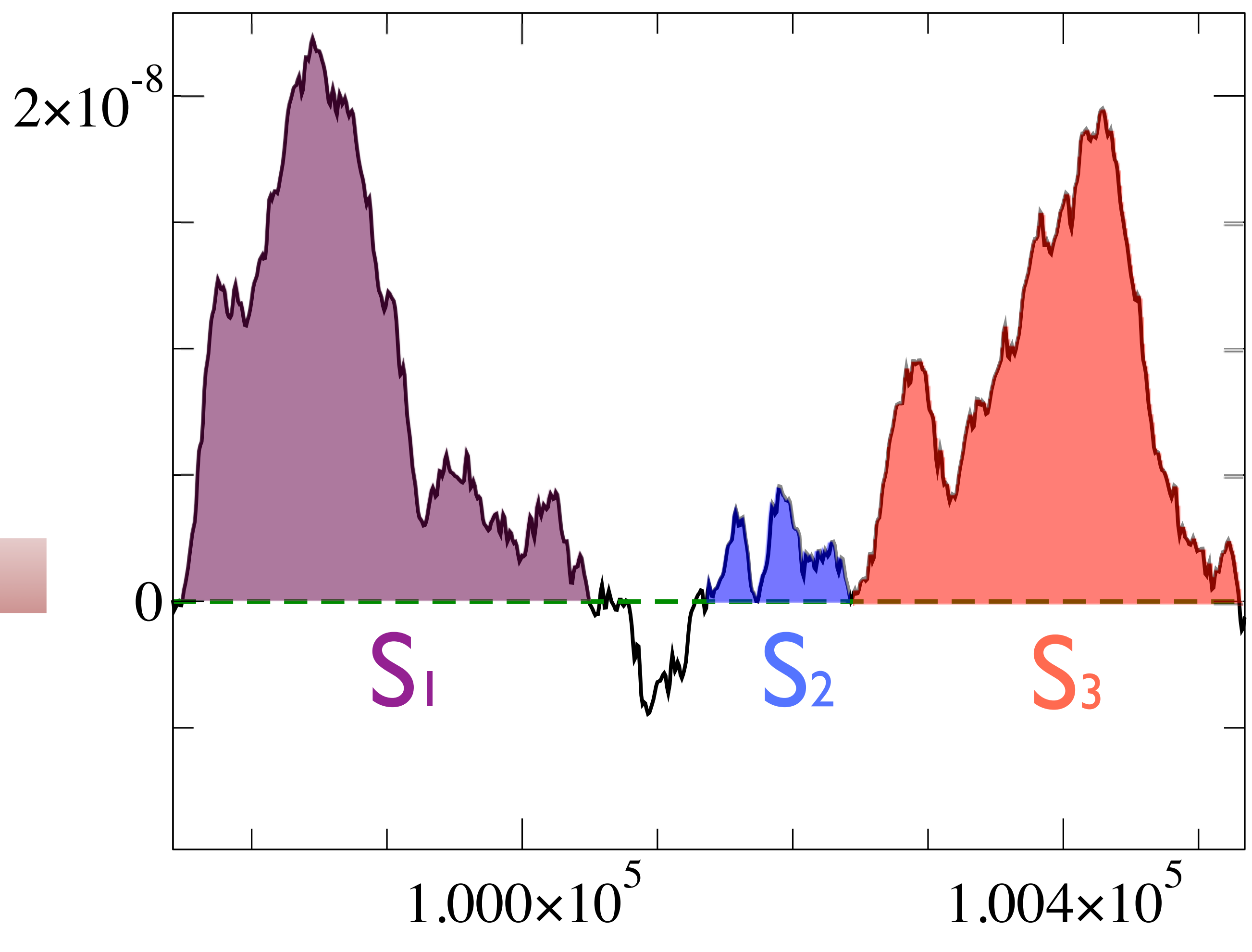
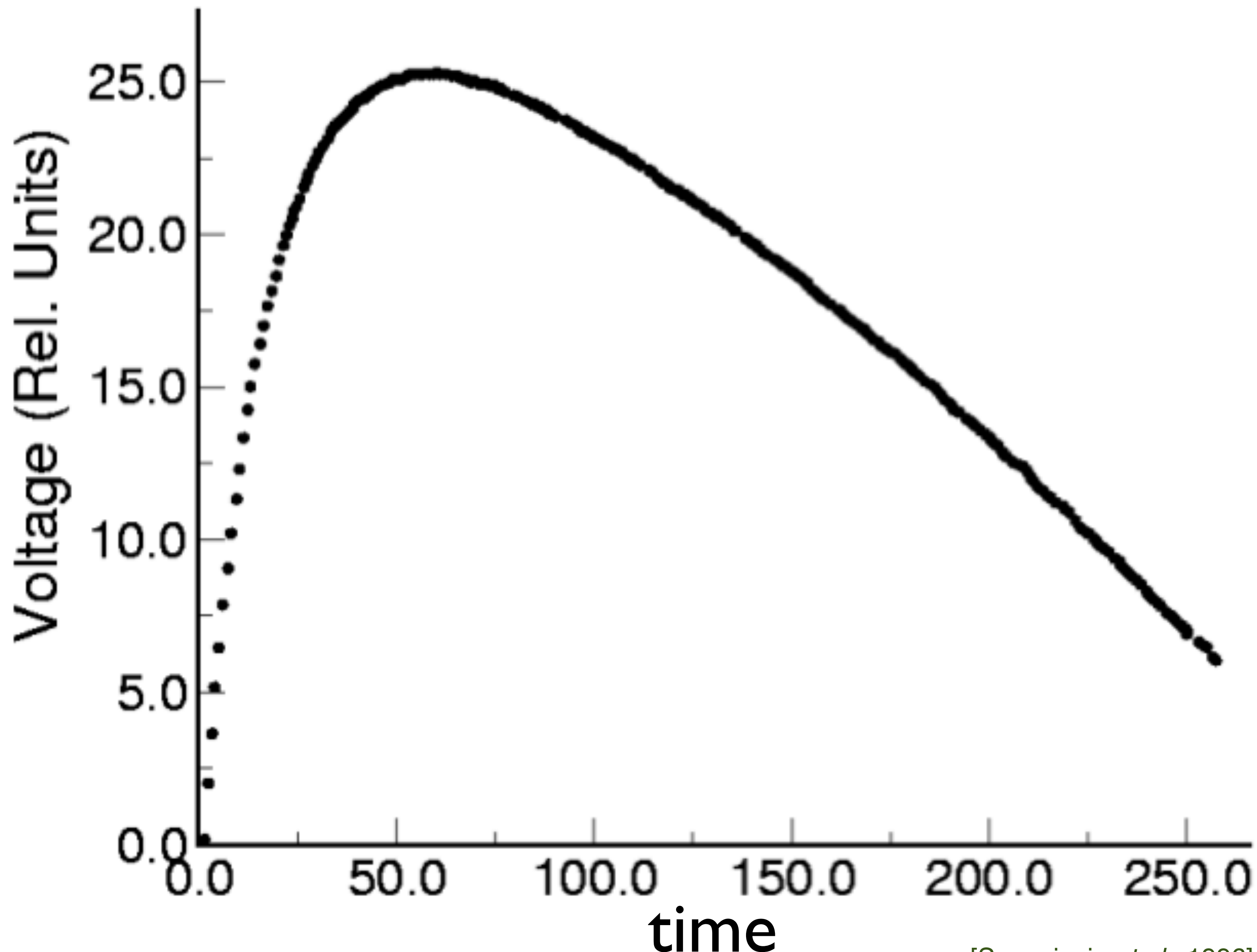






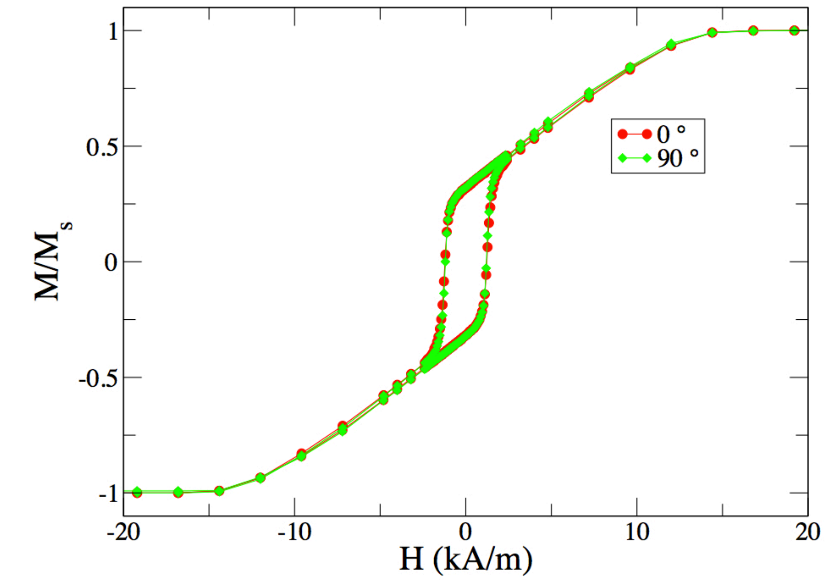
Average temporal avalanche shape & Eddy currents

- Retardation effects (eddy currents), present in thick samples, distort scaling.
- Time correlation due to the interface motion, scales as (film thickness)^2

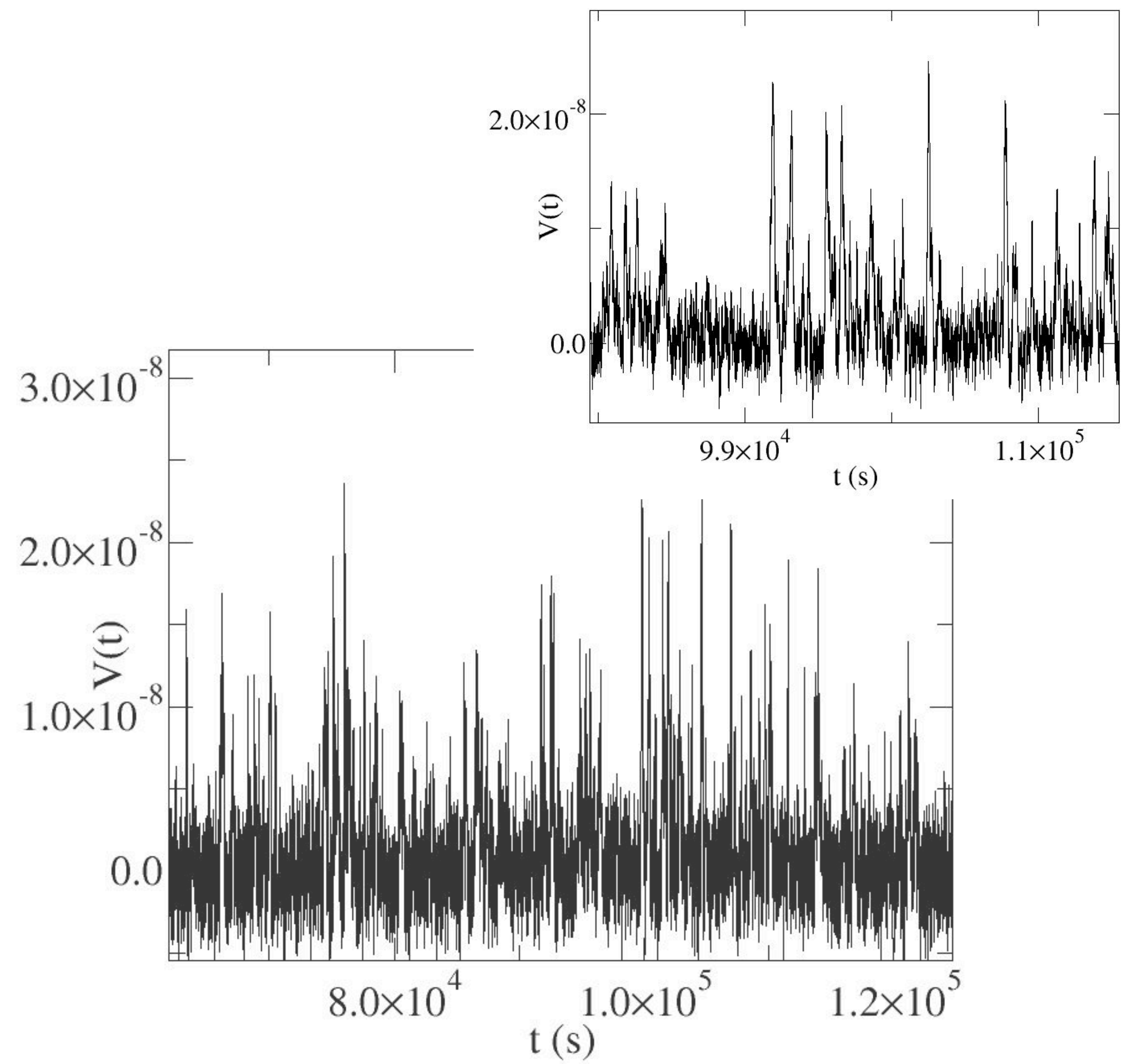


[Spasojevic *et al.*, 1996]

Experiments on thin films: “No” distortions



- Thin films have smaller signal-to-noise ratio,



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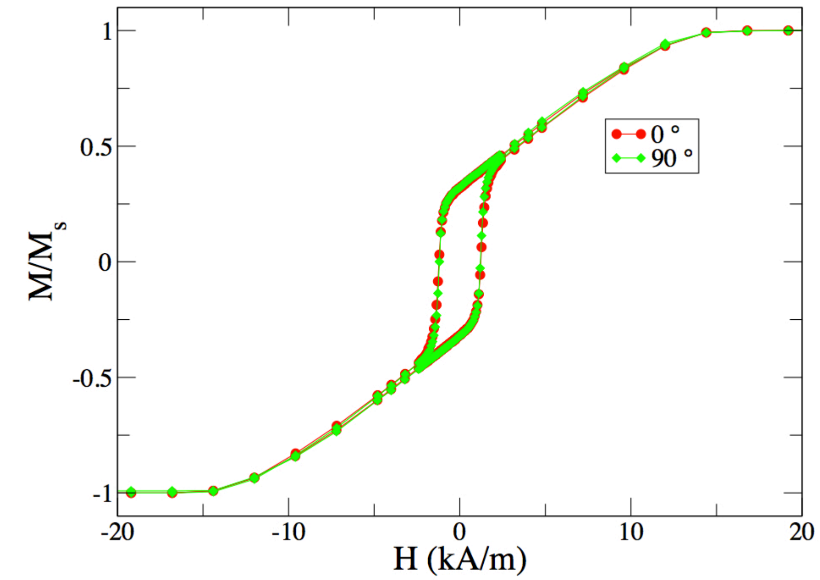

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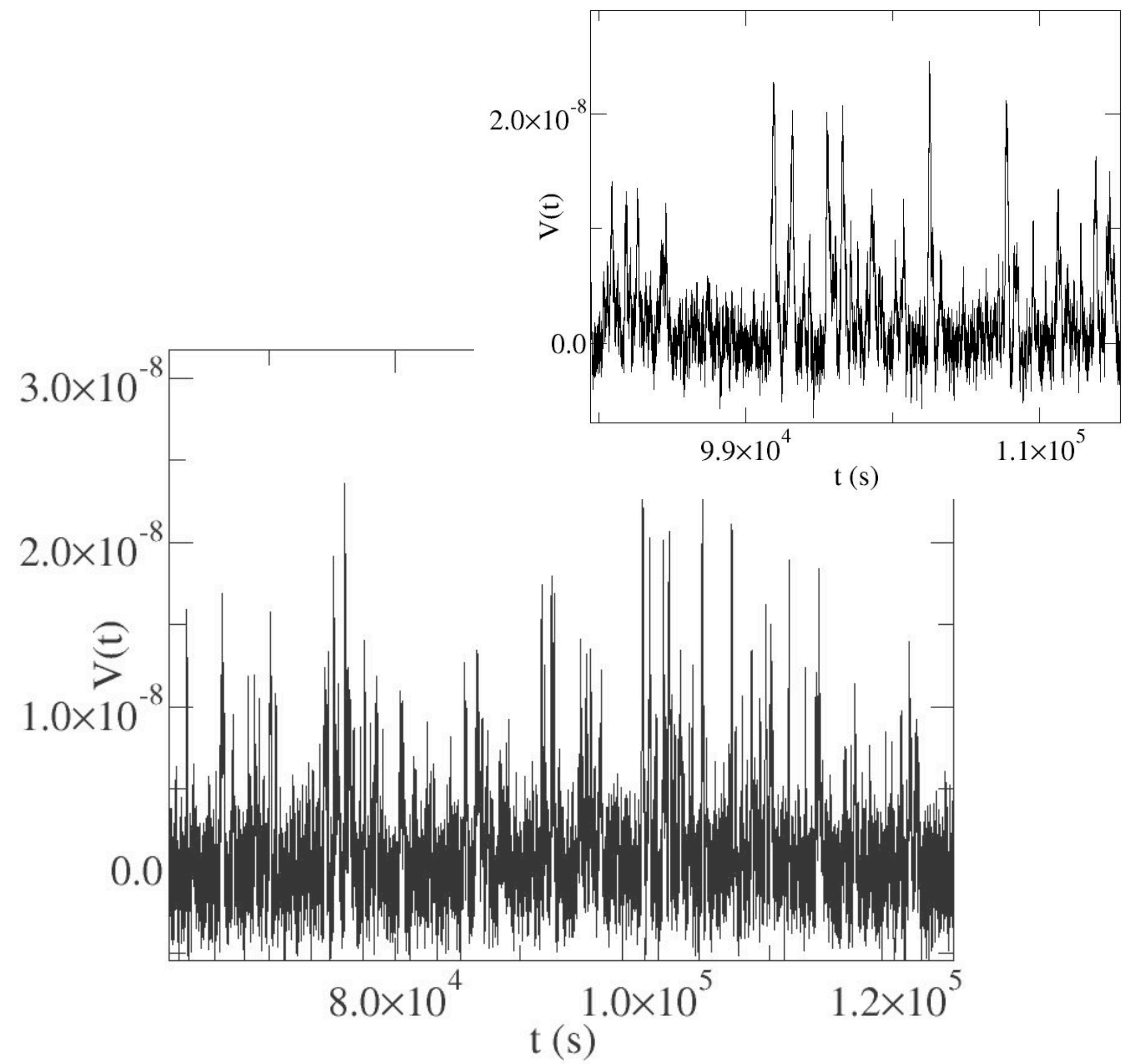
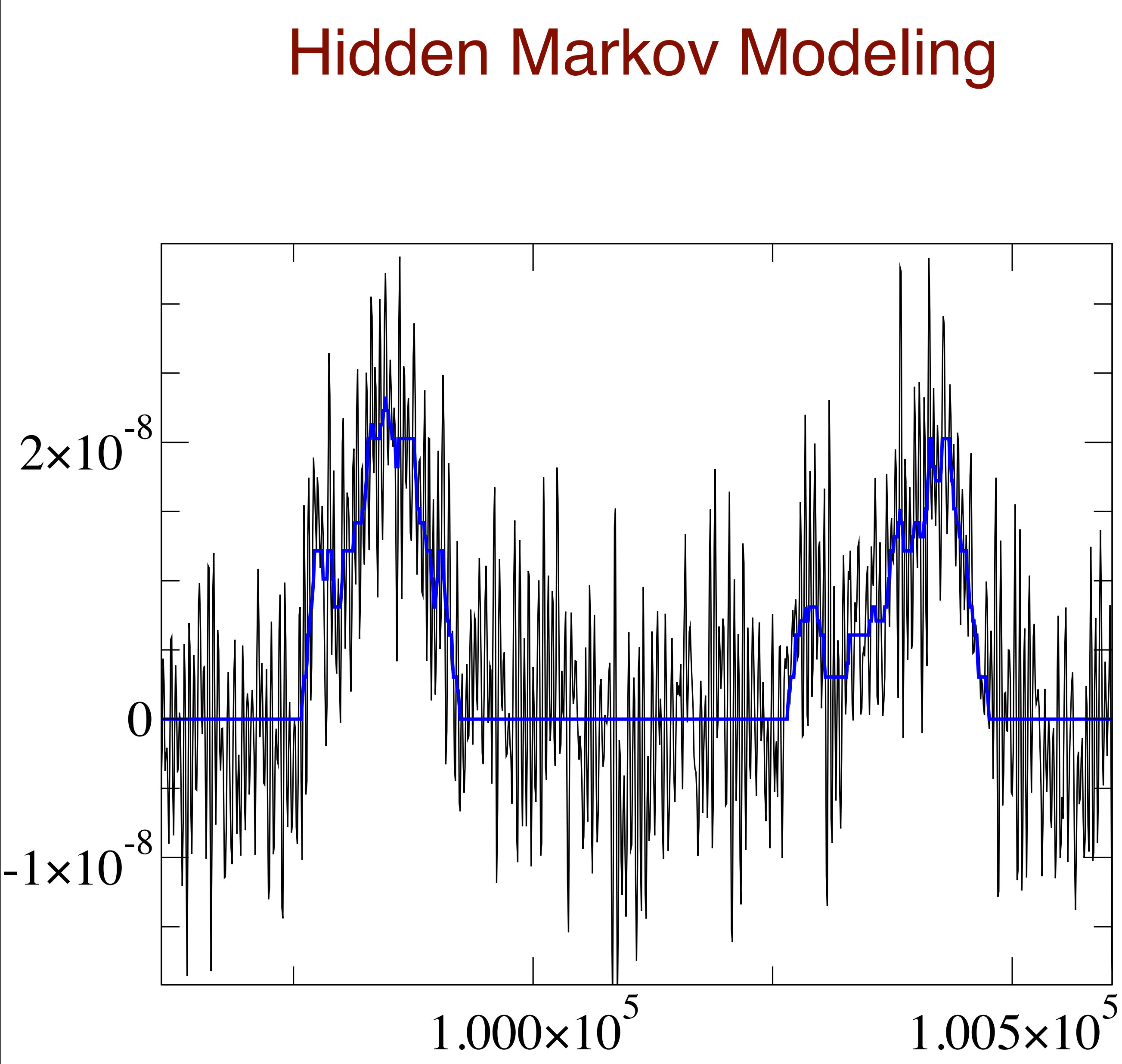
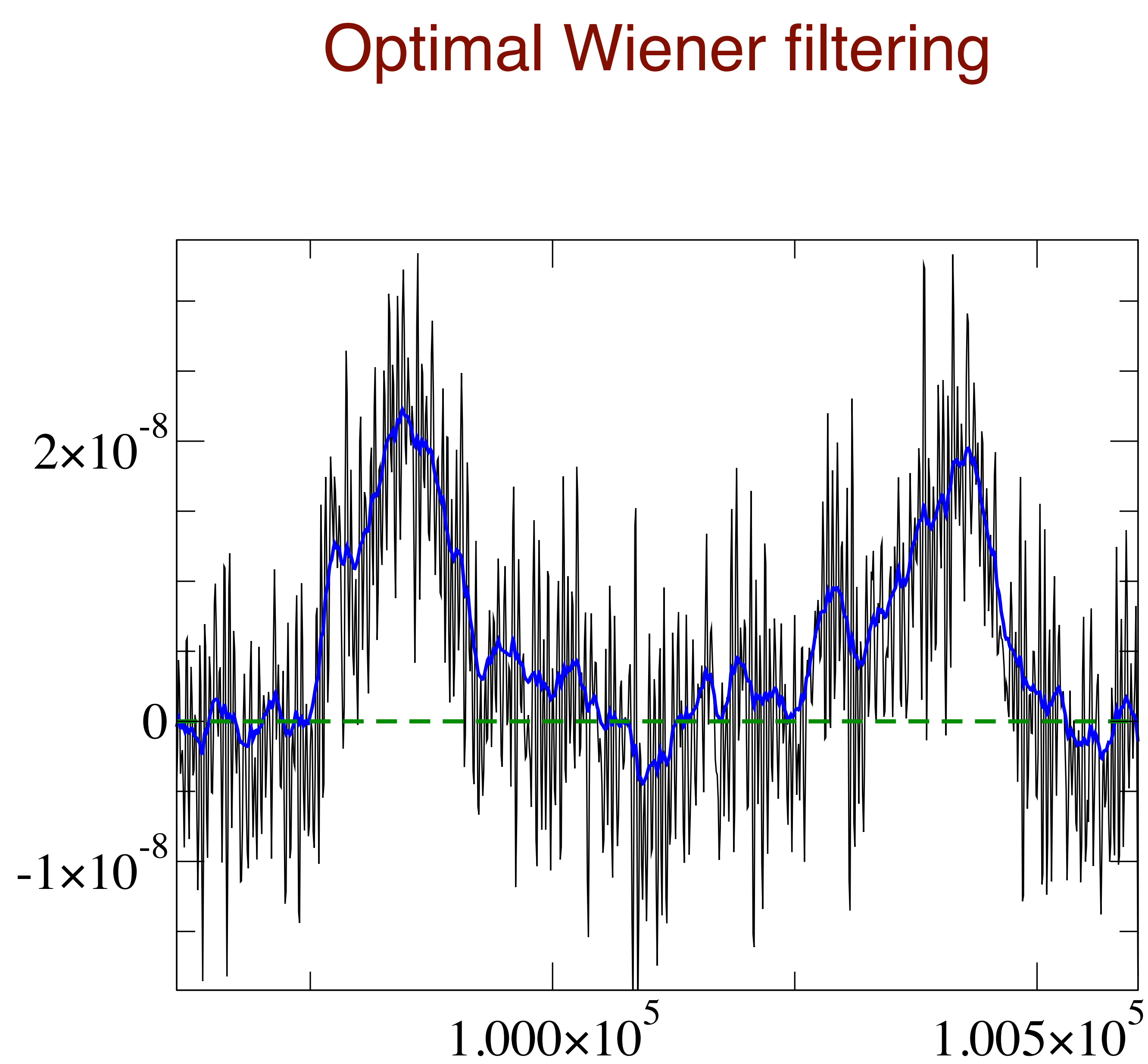


- Challenge:

Get scaling out of the “noisy” noise!

Data analysis: “Fixing” a timeseries





- Multiple methods to efficiently filter experimental data...

Real Space vs. Fourier Space modeling

Hidden Markov Modeling

Optimal Wiener filtering

Real Space vs. Fourier Space modeling

Hidden Markov Modeling

Optimal Wiener filtering

- Markov property:

$$P(q(t) = S_j | q(t-1) = S_i, q(t-2) = S_k, \dots) = P(q(t) = S_j | q(t-1) = S_i)$$

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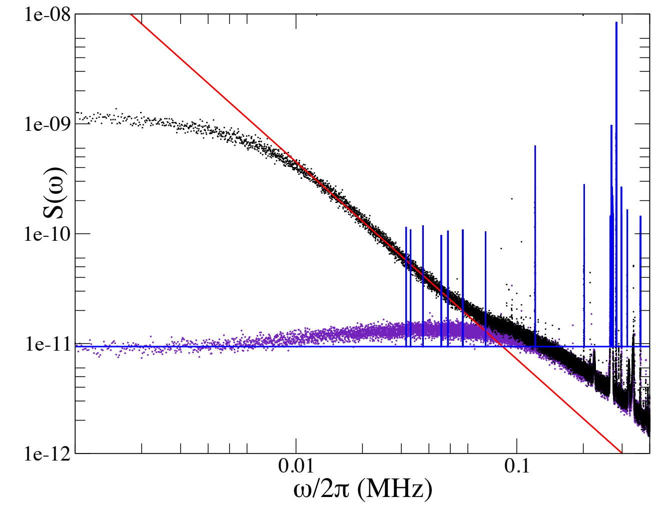

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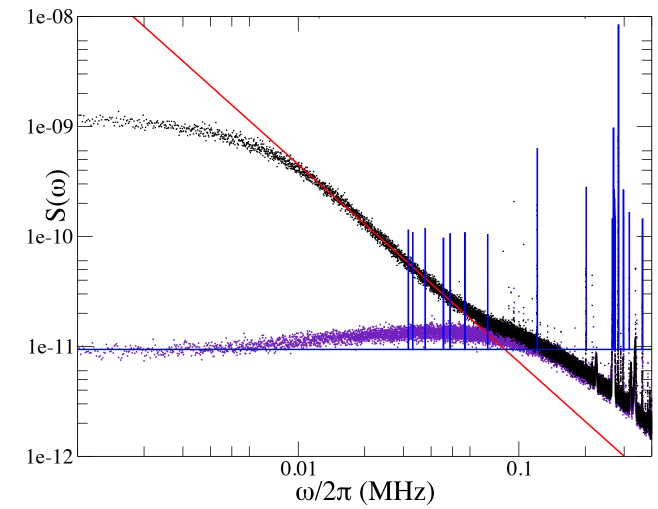

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$$v(t) \rightarrow |\tilde{v}(f)|^2 \sim f^{-\gamma}$$

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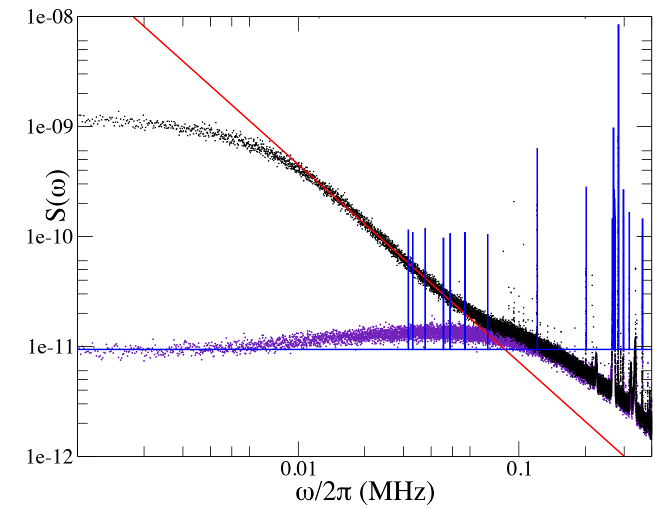

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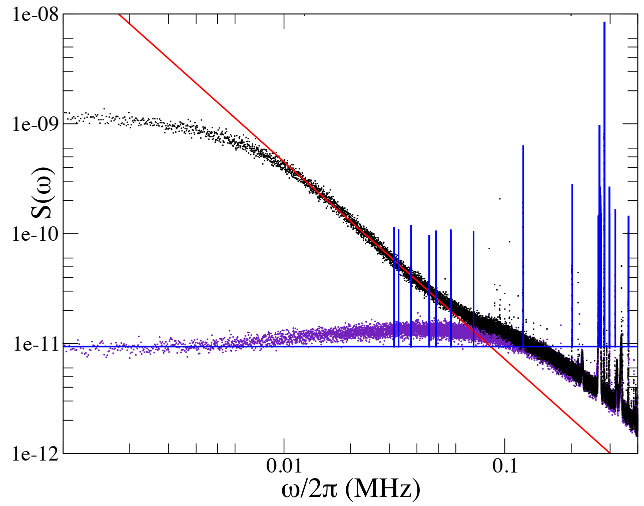

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$$\tilde{V}(f) = \frac{\tilde{V}_{out}(f)}{\tilde{h}(f)} \frac{|\tilde{v}(f)|^2}{|\tilde{n}(f)|^2 + |\tilde{v}(f)|^2}$$

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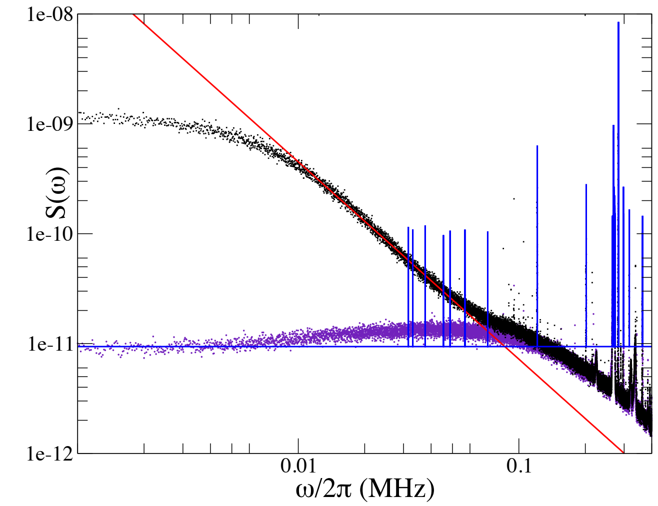
novel methods of identifying scaling properties?

Optimal Wiener filtering



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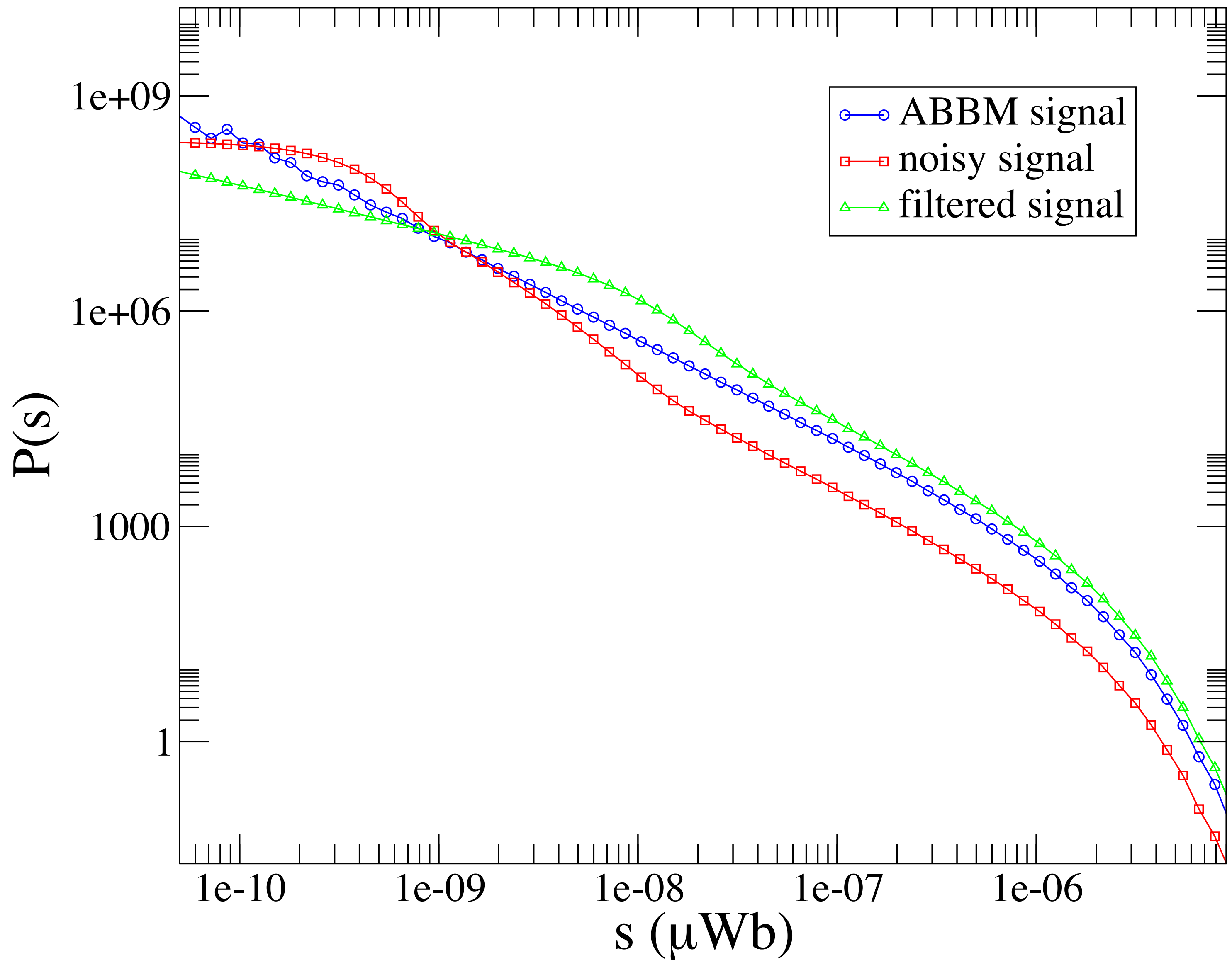
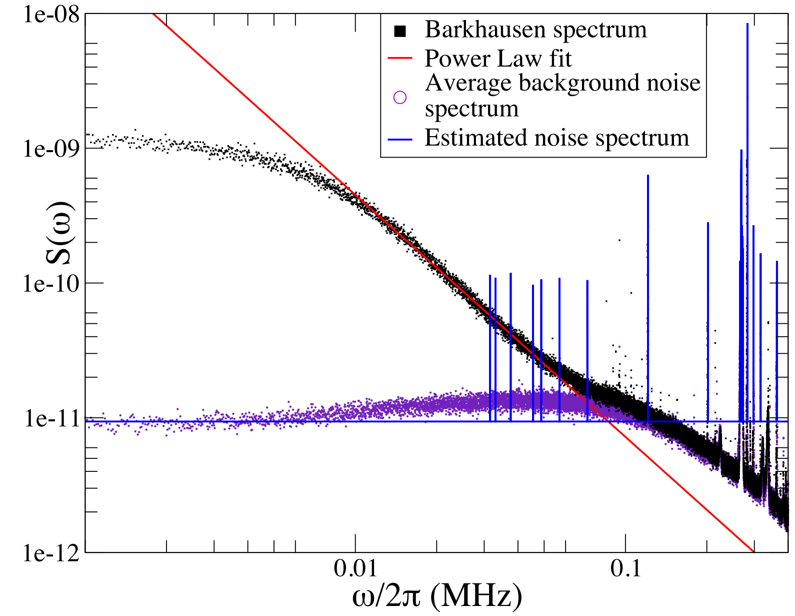
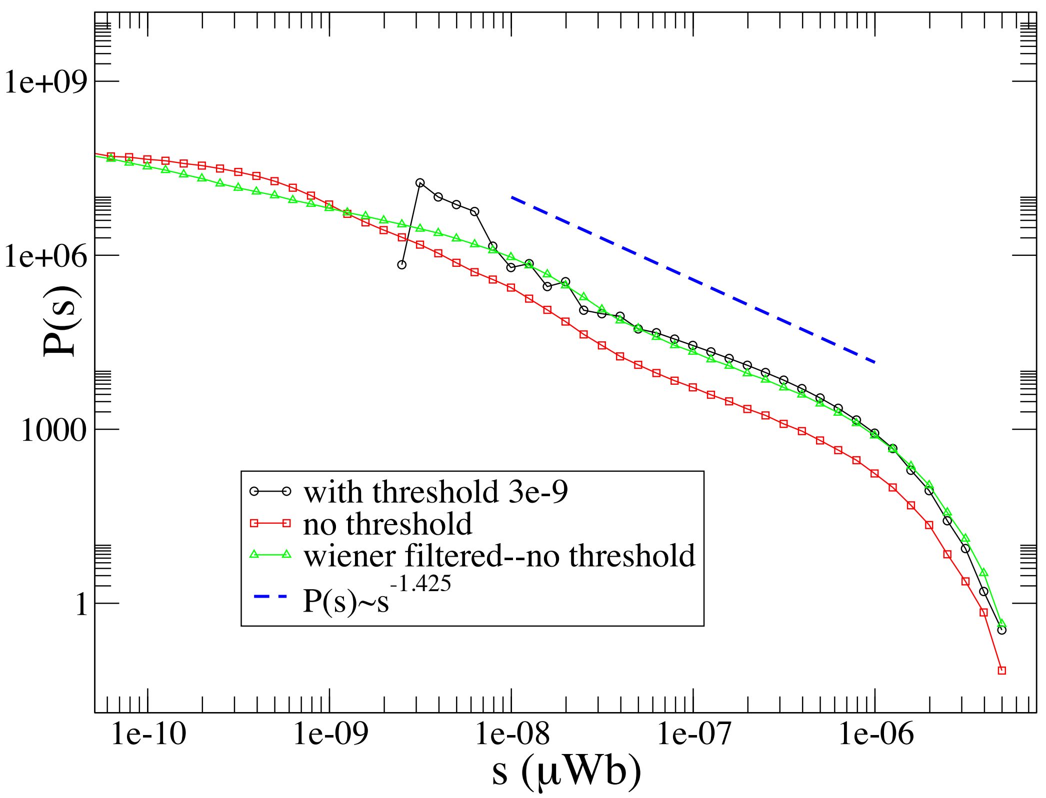
Effects of filtering on distributions

theory with added gaussian noise





experiments

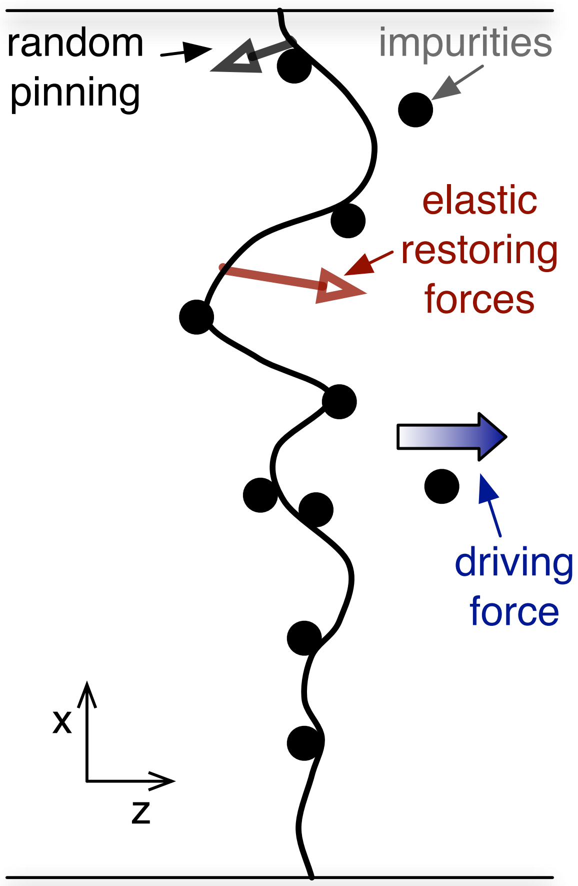


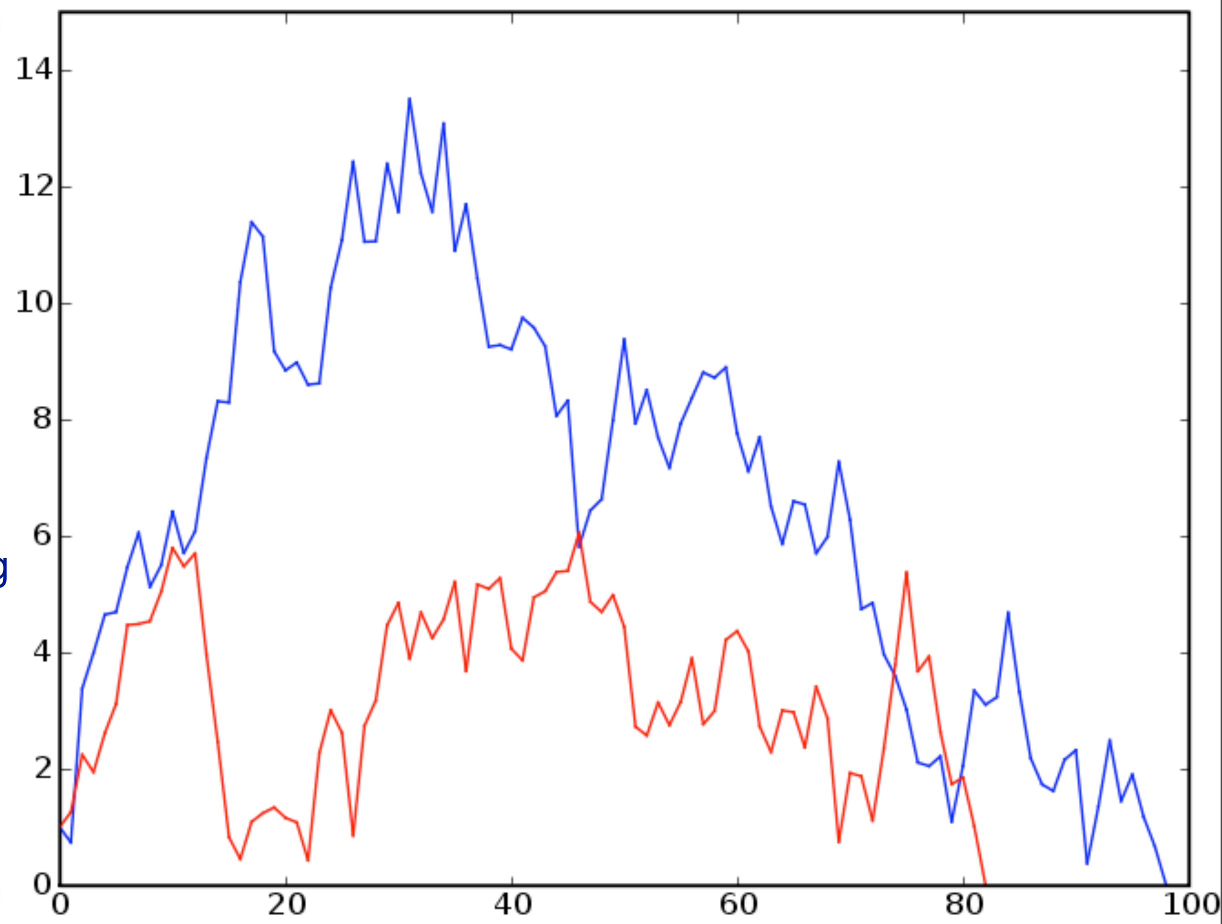
- The predictability of the power law remains similar before and after filtering.

Mean - Field Avalanches: The Shell model

- Infinite-range interface depinning model --

at each step a shell of random fields is triggered:





$$V_{n+1} = P(2JV_n)$$

[Sethna et al. PRL, 1993]

$$V_{n+1} - V_n \simeq -\tilde{k}V_n + \sqrt{JV_n}\xi(t)$$

continuum

$$\frac{dV}{dt} = c - kV + \sqrt{2V}\xi(t)$$

Interpretation required...

c=0 should be consistent: τ=3/2 etc...

Analytical progress and interpretations

$$V_{t+1} = V_t + c - kV_t + \sqrt{2V_?}\xi(t)$$

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- Ito interpretation: noise does not know what the observable is...

$$\langle\sqrt{V(t)}\xi(t)\rangle = 0 \quad \Rightarrow \quad \frac{d\langle V\rangle}{dt} = \langle c - kV\rangle$$

$$\partial_t P(V,t) = -\partial_V (c - kV - 1) P(V,t) + \frac{1}{2}\partial_V \sqrt{2V}\partial_V \sqrt{2V} P(V,t)$$

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- Stratonovich: treat the noise variable similarly to anything else (regular calculus holds)

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- Different interpretation corresponds to a different drift term. For our case, it is a “-1” difference in the right-hand side (Exponent sensitivity)

Avalanche → A first passage problem

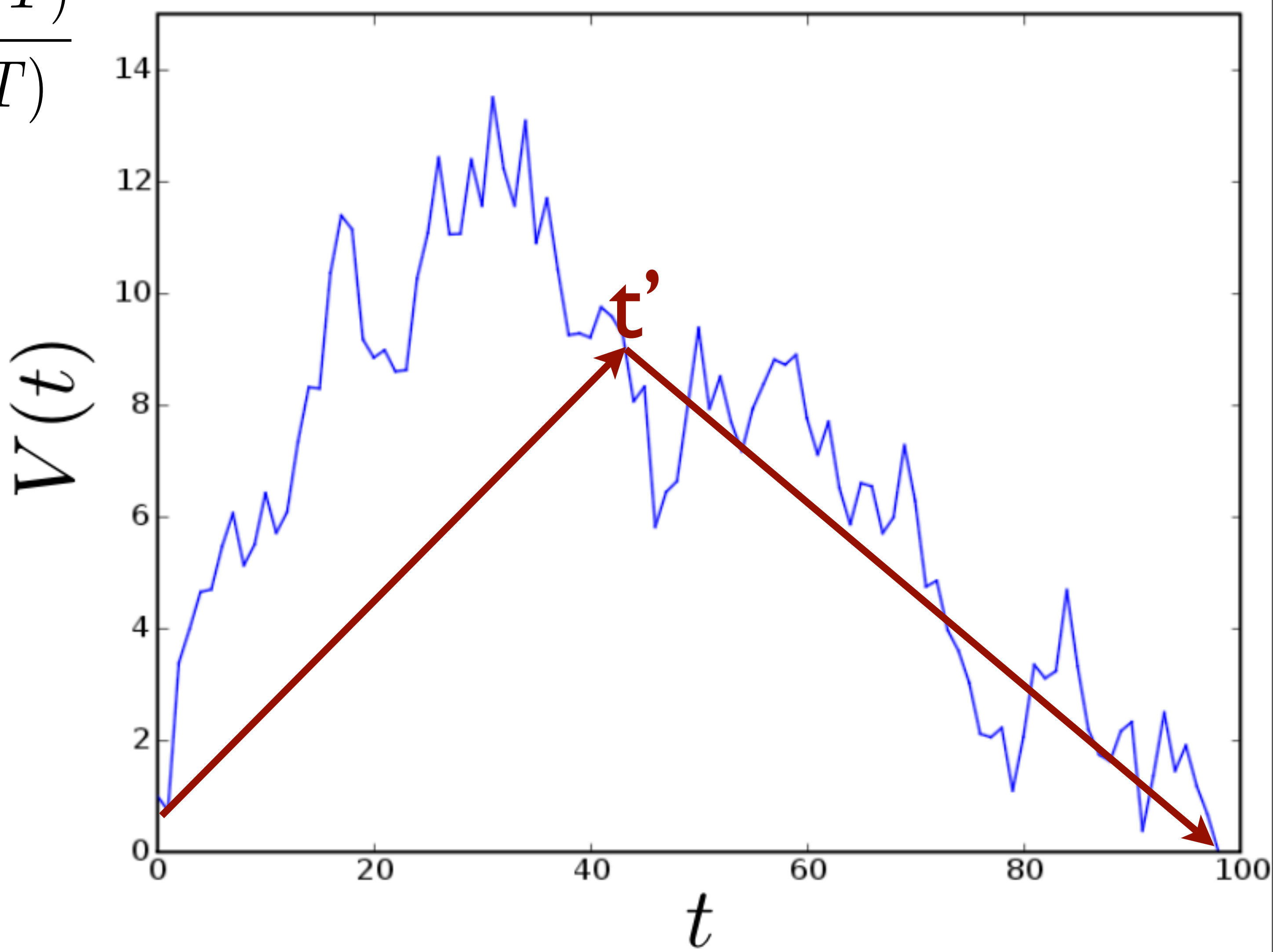
- Avalanche: first return to the origin

- Time to the first return → Duration of the avalanche

- The shape of the avalanche → $VP(V=\epsilon, t=0; V=V, t=t')P(V=V, t=t'; V=0, t=T)$

$$\langle V(t,T)\rangle = \lim_{\epsilon\to 0}\frac{\int_0^\infty dV\, V\, P(\epsilon,0;V,t)P(V,t;0,T)}{\int_0^\infty dV\, P(\epsilon,0;V,t)P(V,t;0,T)}$$

- Target: calculate $P(\epsilon, 0; V, t)$
  analytical formulas, simulations

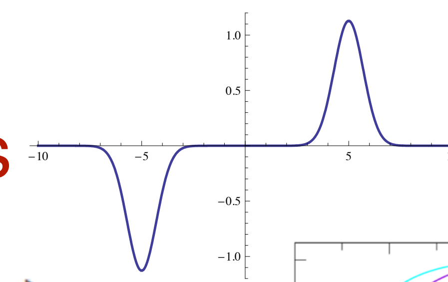


Analytical formulas in different limits...

$$\frac{d\sqrt{V}}{dt} = \frac{c/2}{\sqrt{V}} - \frac{k}{2}\sqrt{V} + \frac{1}{\sqrt{2}}\xi(t)$$

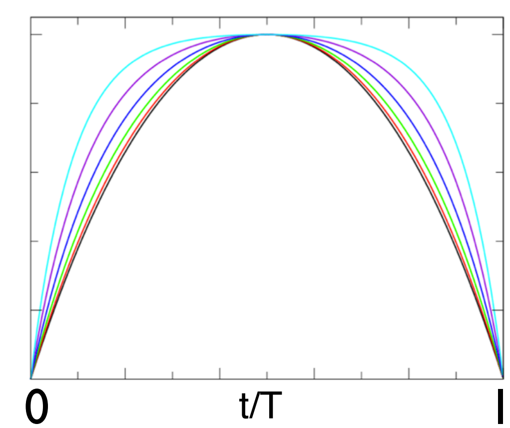
- Motion in a logarithmic potential with a drift...

- $c = 0$: Free random walk -- Absorbing BCs with images



$$\langle V_{c=0,k}(t|T)\rangle = \frac{1}{2k}\frac{(e^{2k(T-t)}-1)(e^{2kt}-1)}{e^{2kT}-1}$$

Also, [P. Le Doussal and C. Wiese, arxiv 2011]

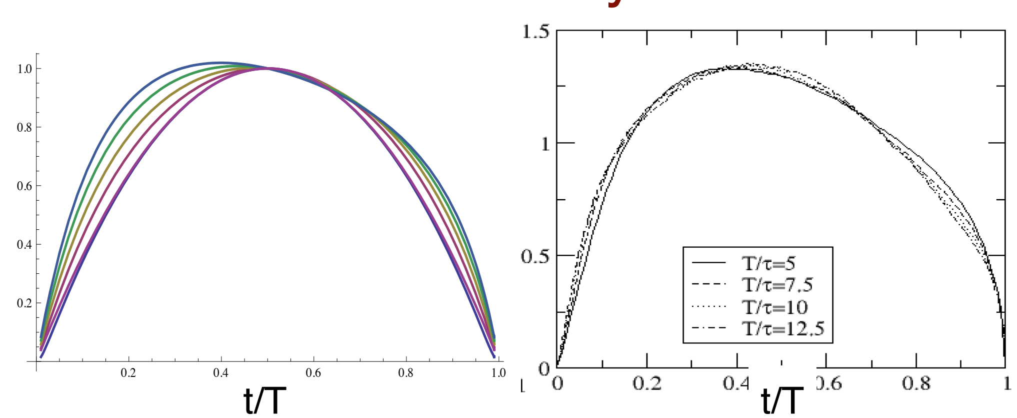


- $k = 0$: Shape integral over Bessel functions -- invariant at long durations

- c, k, k_1 (Eddy current term $\sim \int_0^t v(t')e^{-\frac{t'}{\tau}}dt'$) : Weak-noise perturbation expansion (method of images generically applicable)

$$\left.\begin{array}{c}\dfrac{\partial\langle x^n\rangle}{\partial t} = \dots \\ \sigma(t)^2 = \langle x^2\rangle - \langle x\rangle^2 \\ \mu(t) = \langle x\rangle\end{array}\right\} P(x,t) \simeq \frac{1}{\sqrt{2\pi\sigma(t)^2}}e^{-(x-\mu(t))^2/2\sigma(t)^2}$$

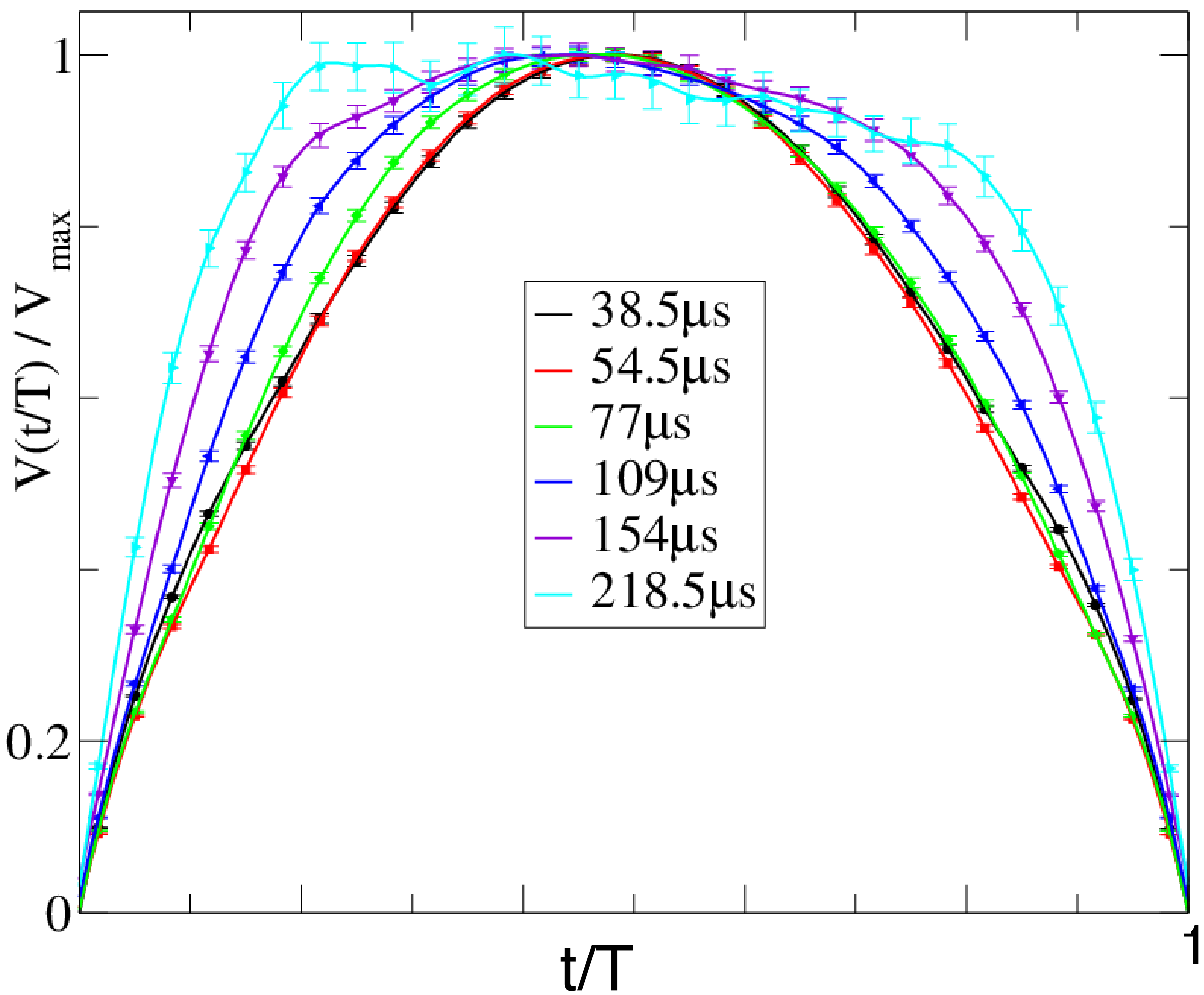
effects from Eddy currents



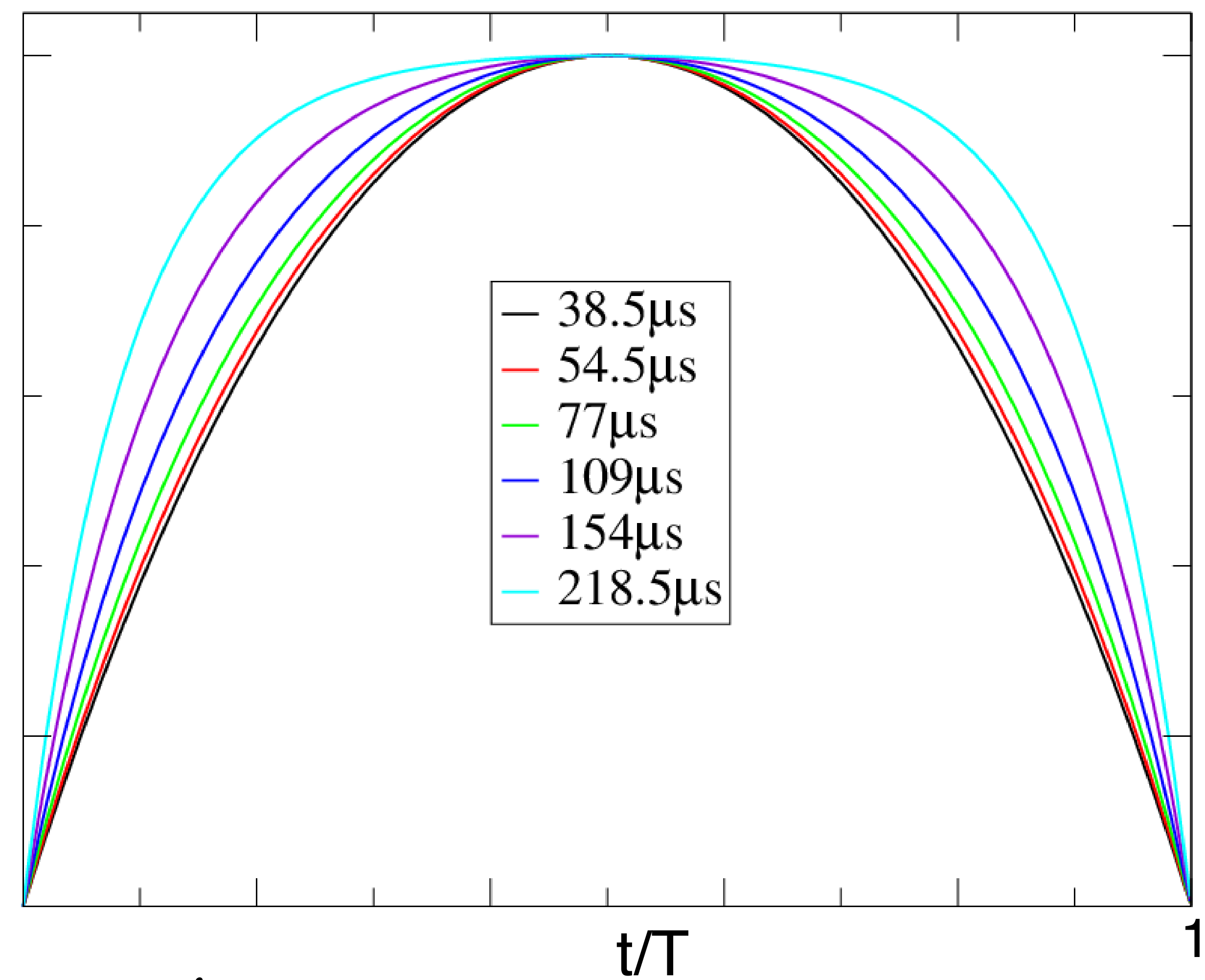
[S. Zapperi *et al.*, Nat. Phys.'05]

The average temporal avalanche shape

experiment



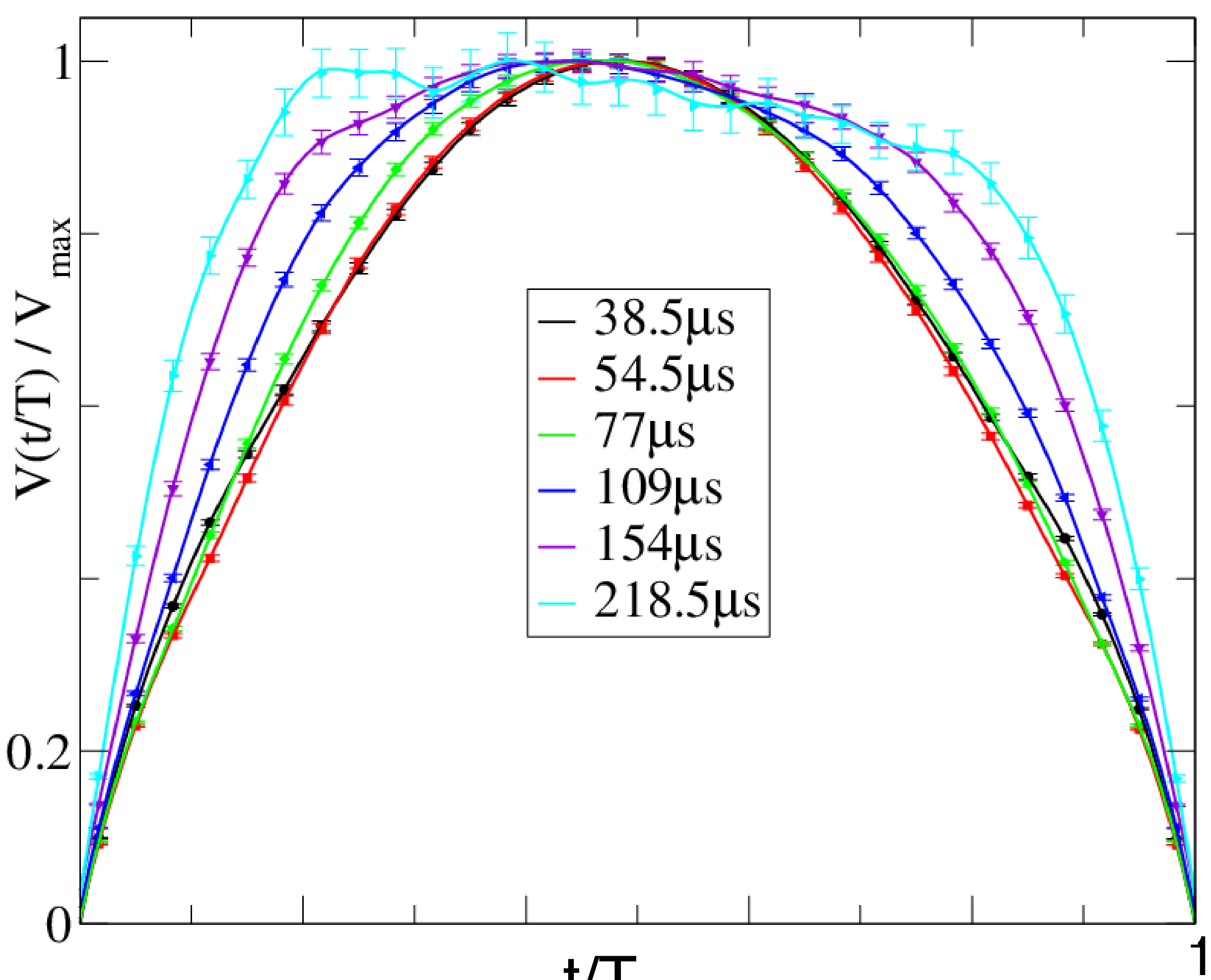
theory



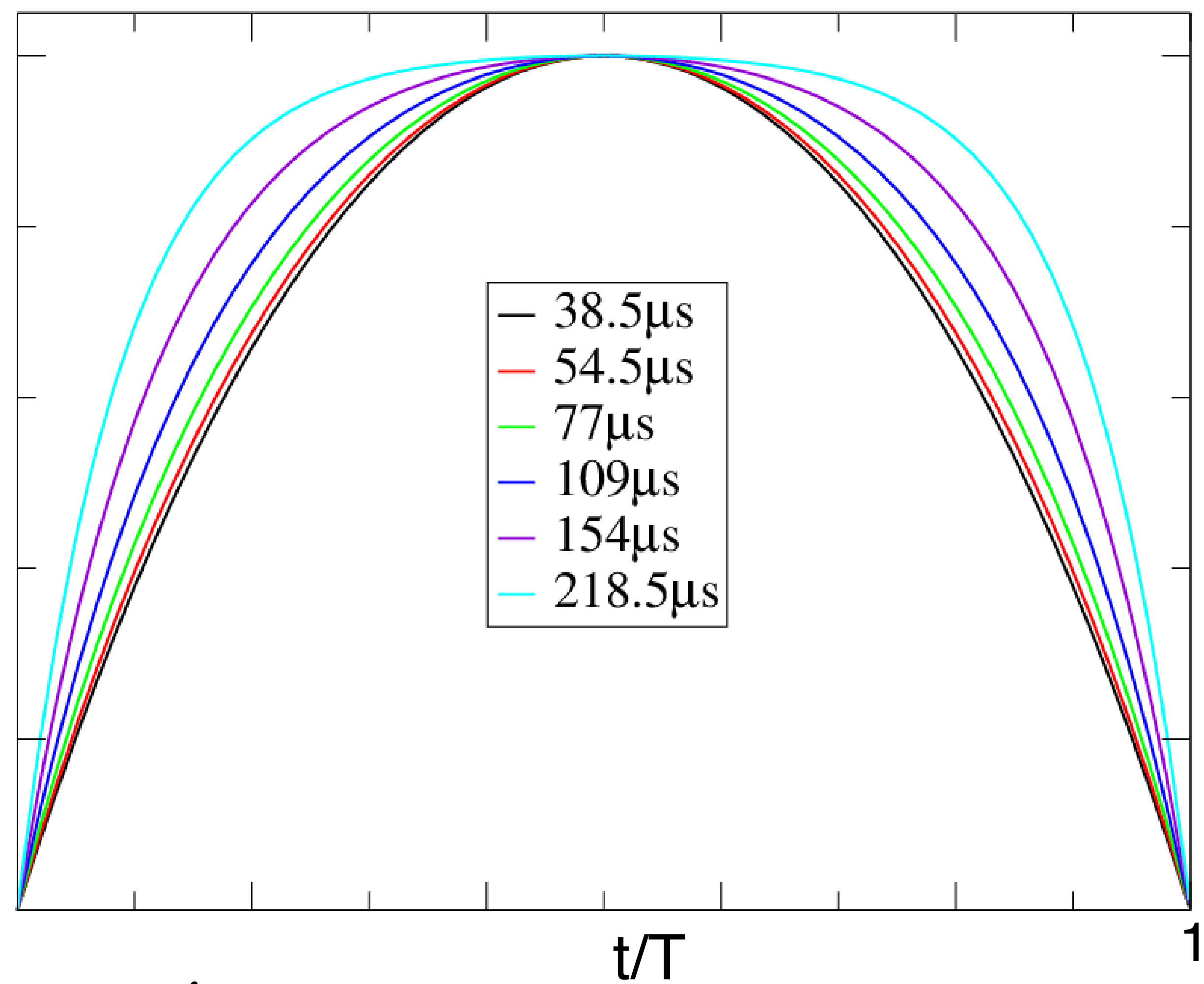
$$V(t,T,k,c) = T^{\frac{1}{\sigma\nu z}-1}\mathcal{V}\left(\frac{t}{T}, Tk^{\nu_k z}, \frac{c}{c_0}\right)$$

The average temporal avalanche shape

experiment



theory

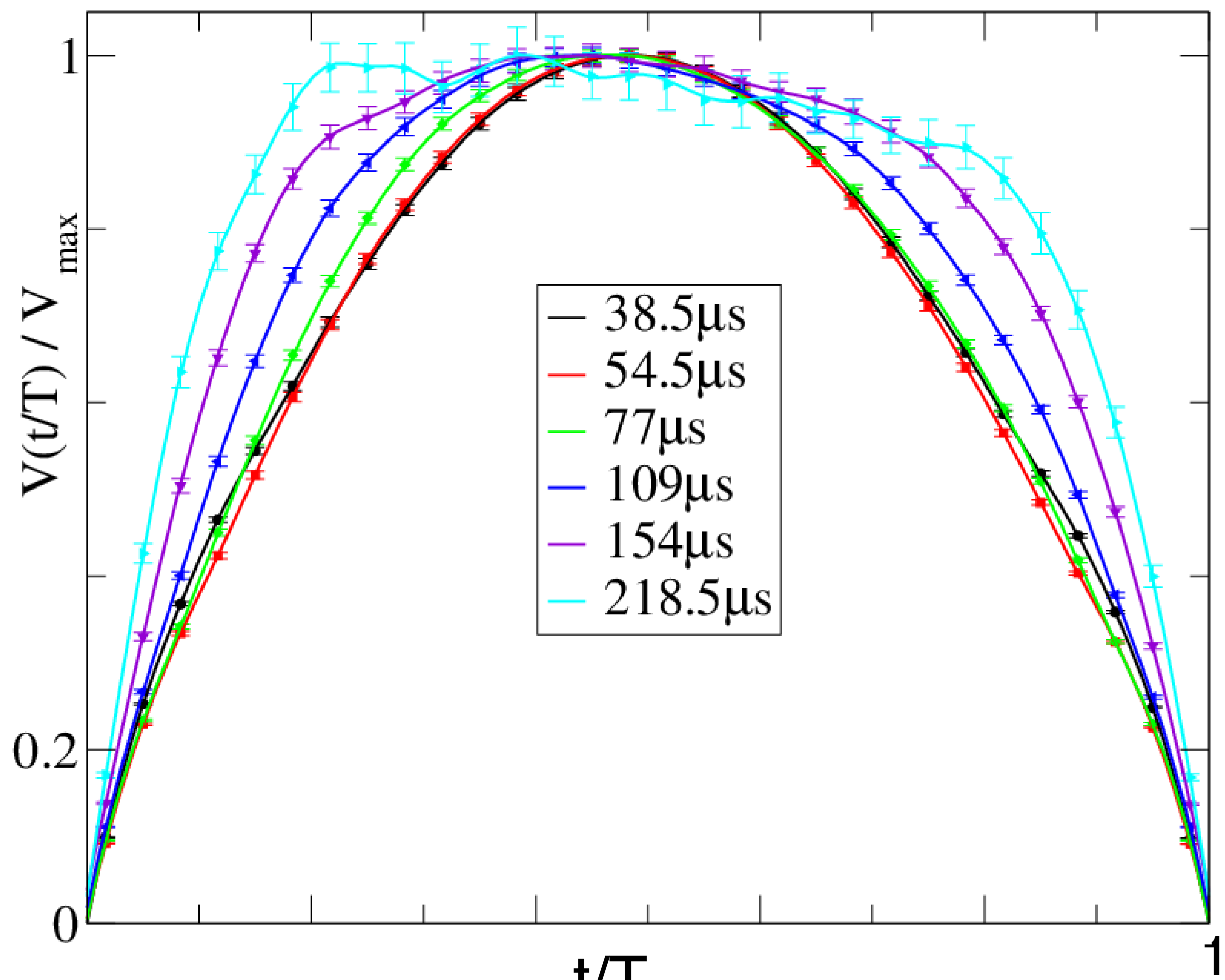


$$V(t,T,k,c) = T^{\frac{1}{\sigma\nu z}-1}\mathcal{V}\left(\frac{t}{T}, Tk^{\nu_k z}, \frac{c}{c_0}\right)$$

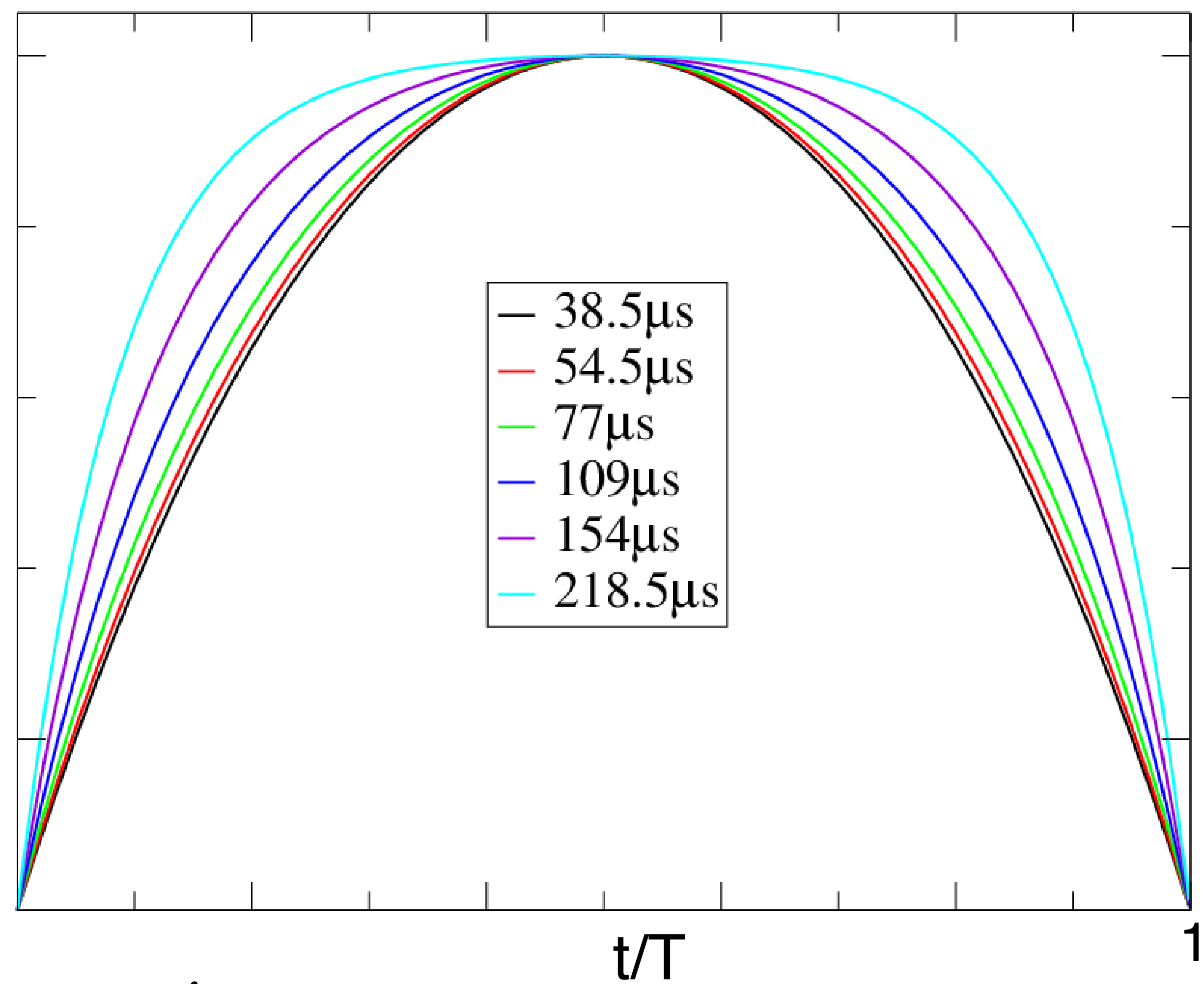
- Quantitative agreement between theory and experiments

The average temporal avalanche shape

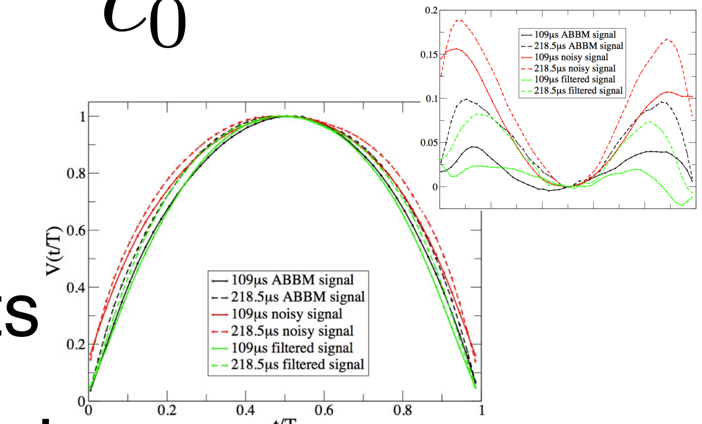
experiment



theory



$$V(t,T,k,c) = T^{\frac{1}{\sigma\nu z}-1}\mathcal{V}\left(\frac{t}{T}, Tk^{\nu_k z}, \frac{c}{c_0}\right)$$

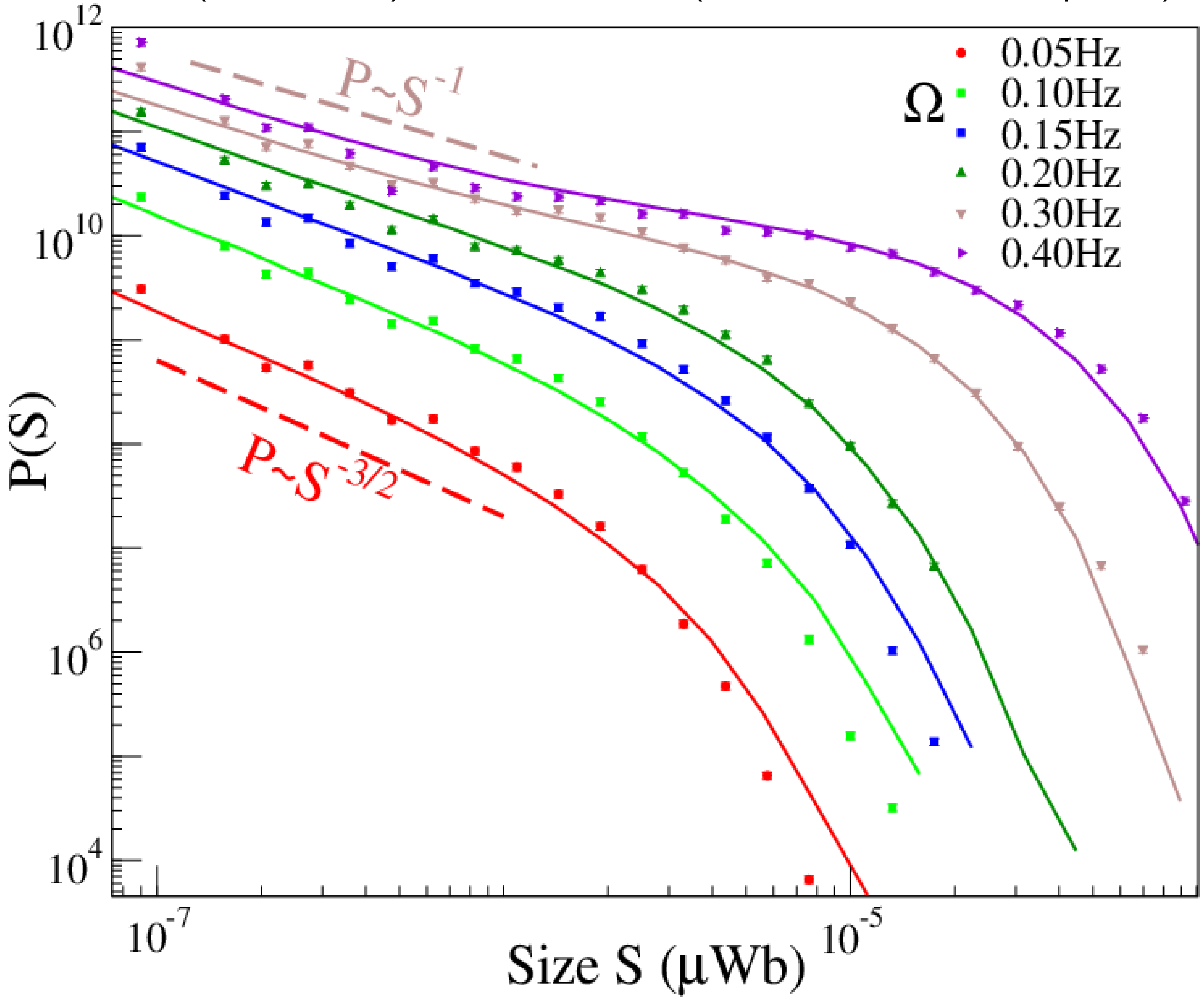


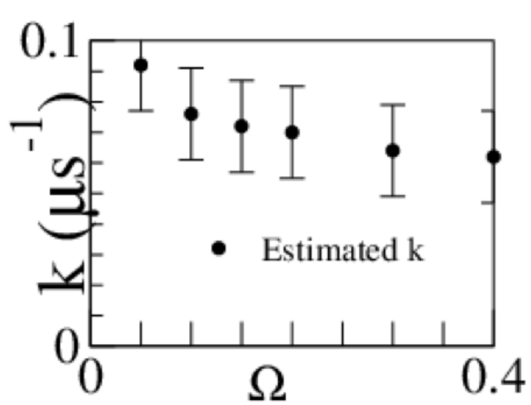
- Quantitative agreement between theory and experiments
- Asymmetry may be attributed to the experimental apparatus response

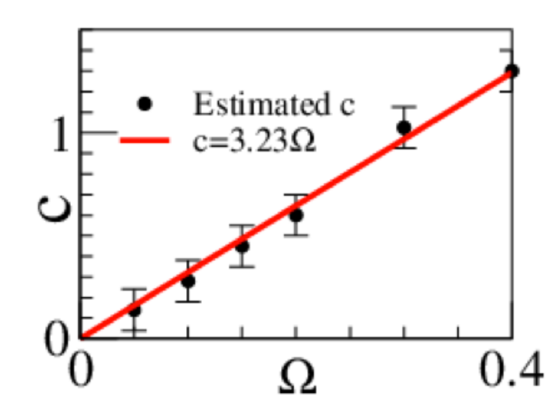
Simulations & comparisons with experiments

Size Histograms

$$P(S,k,c) = S^{-\tau}\mathcal{S}(Sk^{\nu_k(1+\zeta)}, c/c_0)$$



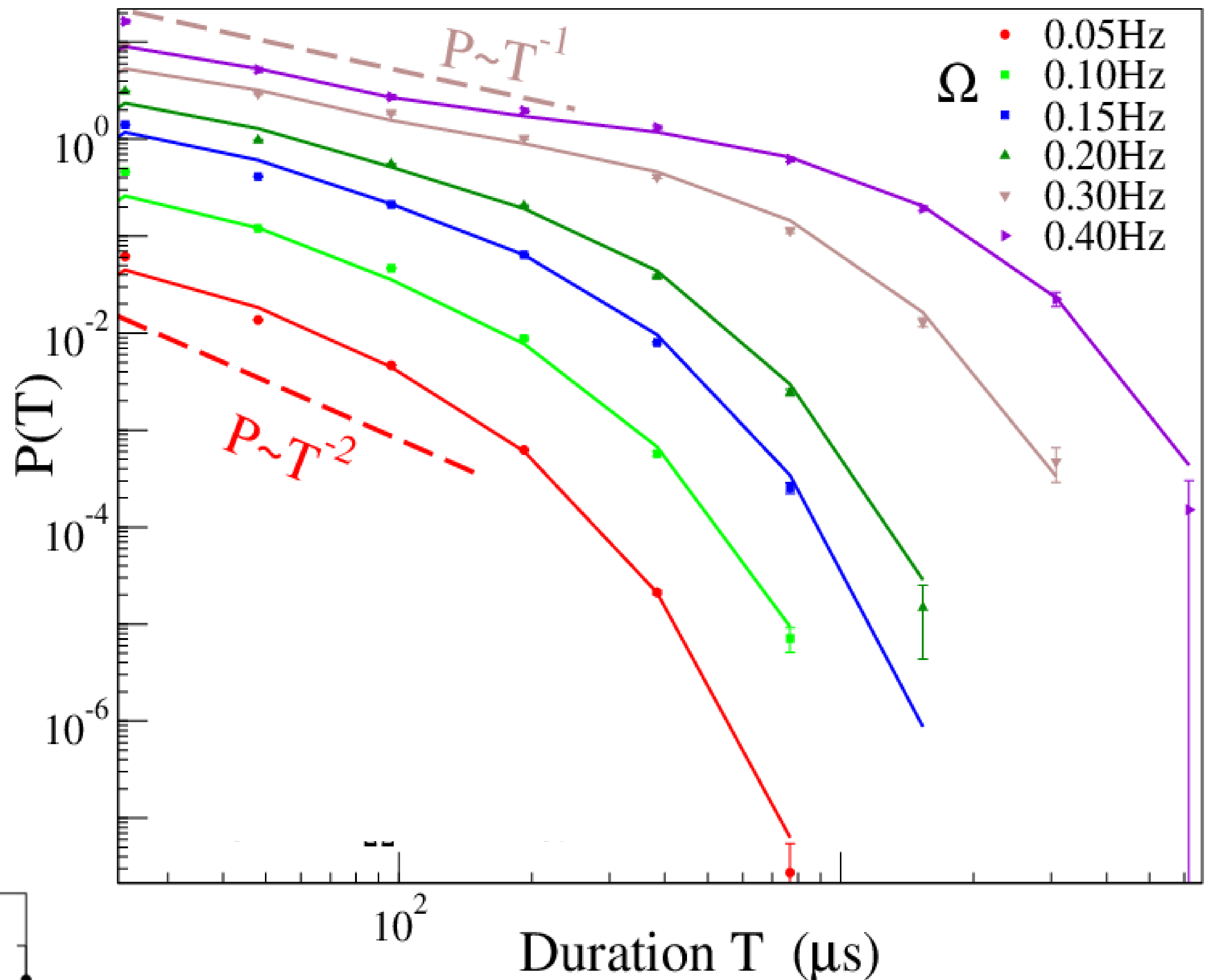


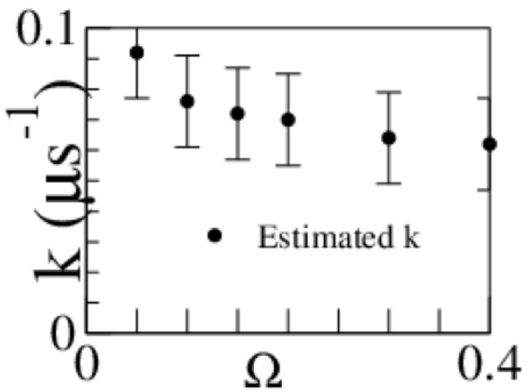


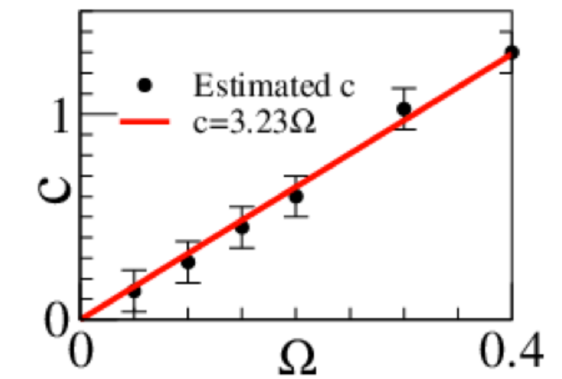
Simulations & comparisons with experiments

Duration Histograms

$$P(T,k,c) = T^{-\alpha}\mathcal{T}(Tk^{\nu_k z}, c/c_0)$$



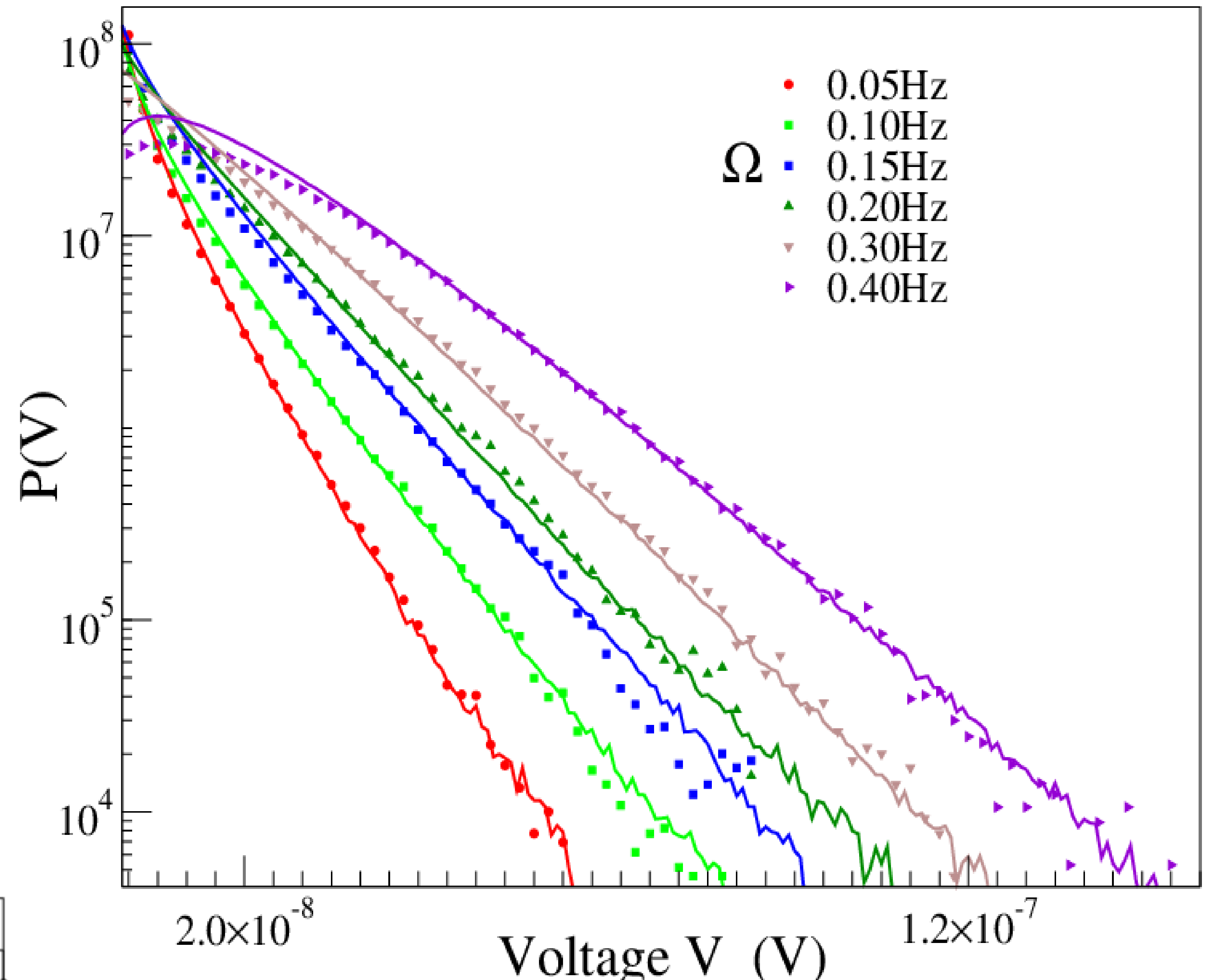


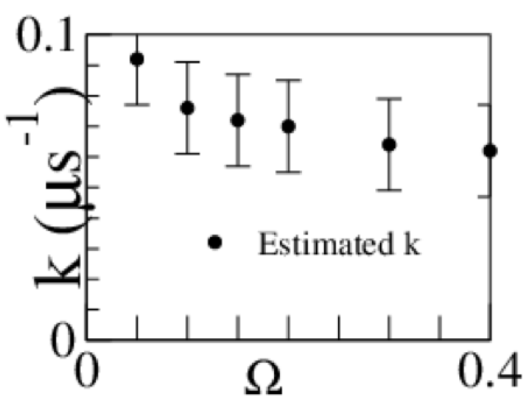


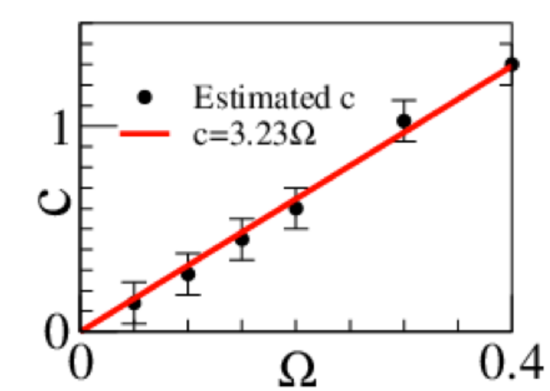
Simulations & comparisons with experiments

Velocity Histograms

$$P(V,k,c) = V^{c-1}\mathcal{Q}(Vk^{\nu_k(z-1)}, c/c_0)$$



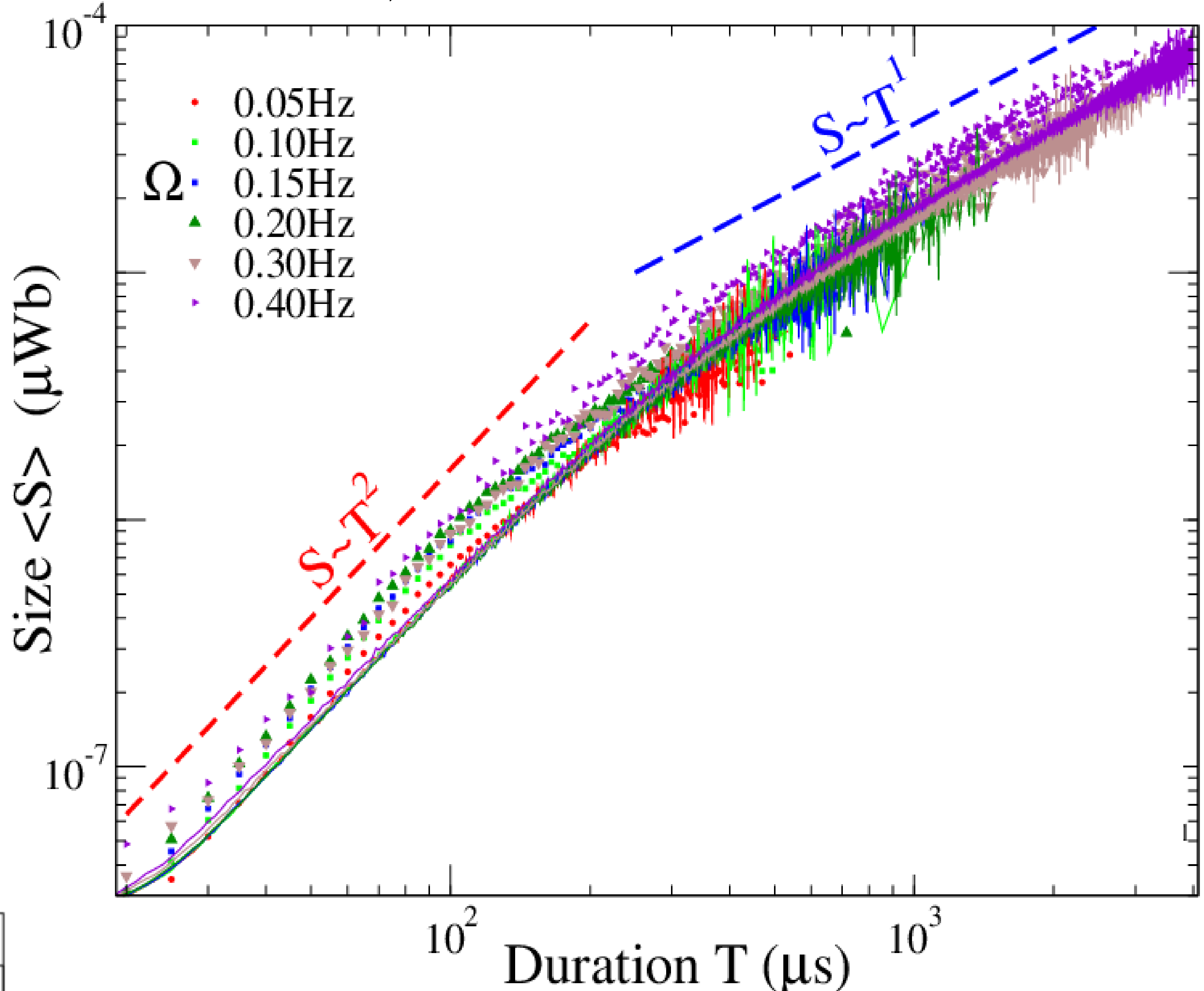


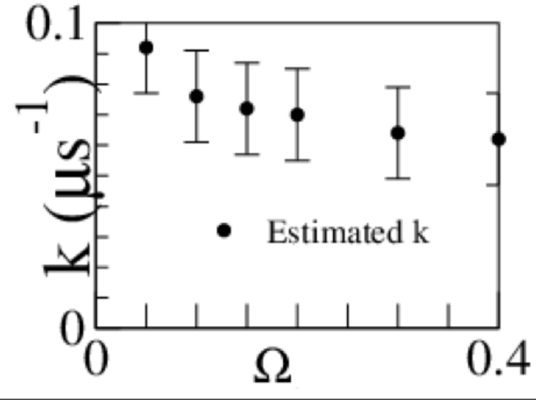


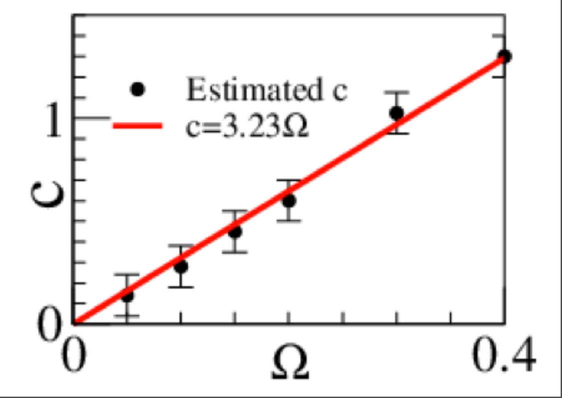
Simulations & comparisons with experiments

<size> vs. duration

$$\langle S\rangle_{k,c} = T^{\frac{1}{\sigma\nu z}}\mathcal{U}(Tk^{\nu_k z}, c/c_0)$$



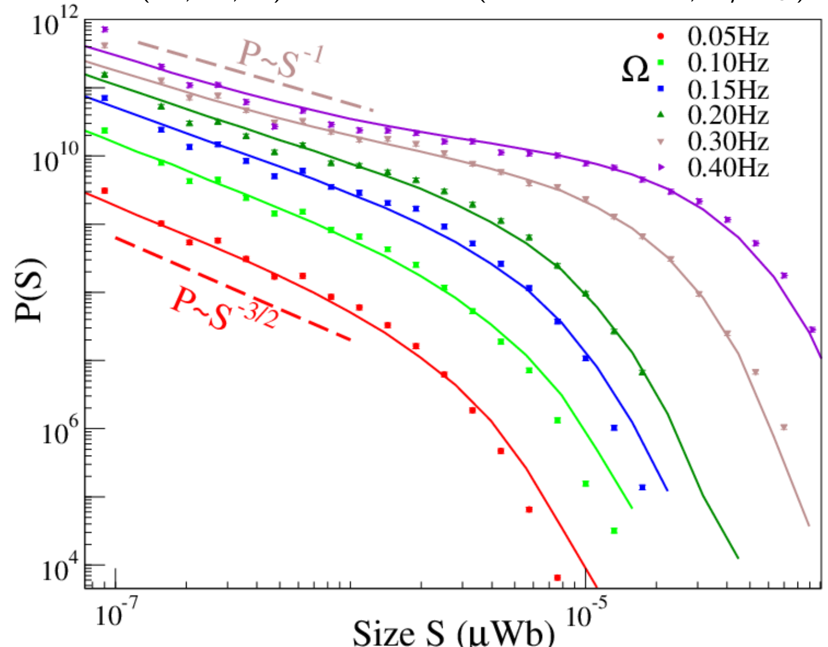




Confirmed mean-field universality

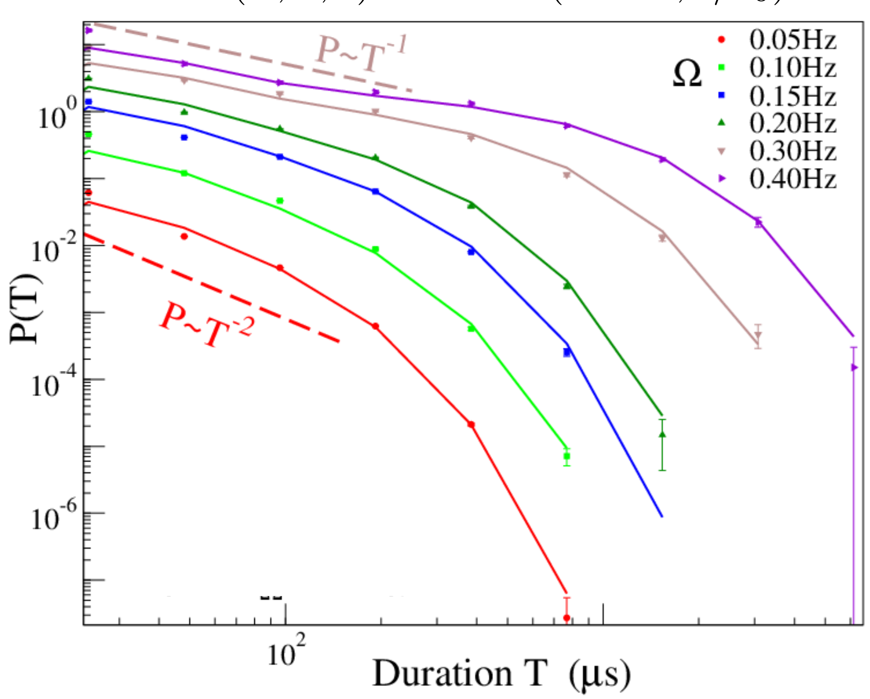
Size Histograms

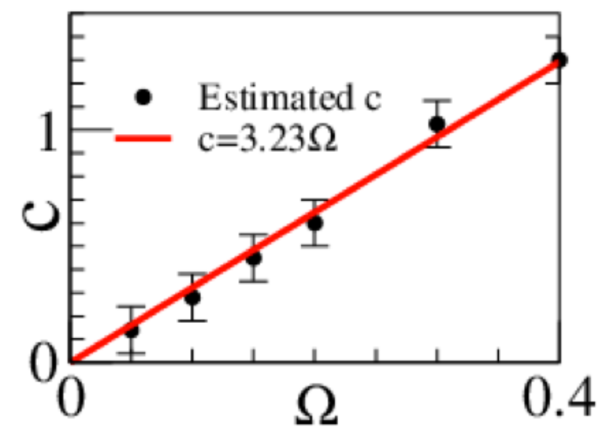
$$P(S,k,c) = S^{-\tau}\mathcal{S}(Sk^{\nu_k(1+\zeta)}, c/c_0)$$

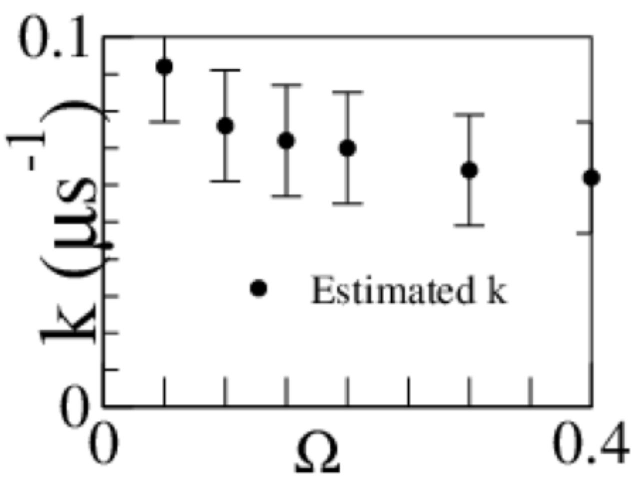


Duration Histograms

$$P(T,k,c) = T^{-\alpha}\mathcal{T}(Tk^{\nu_k z}, c/c_0)$$

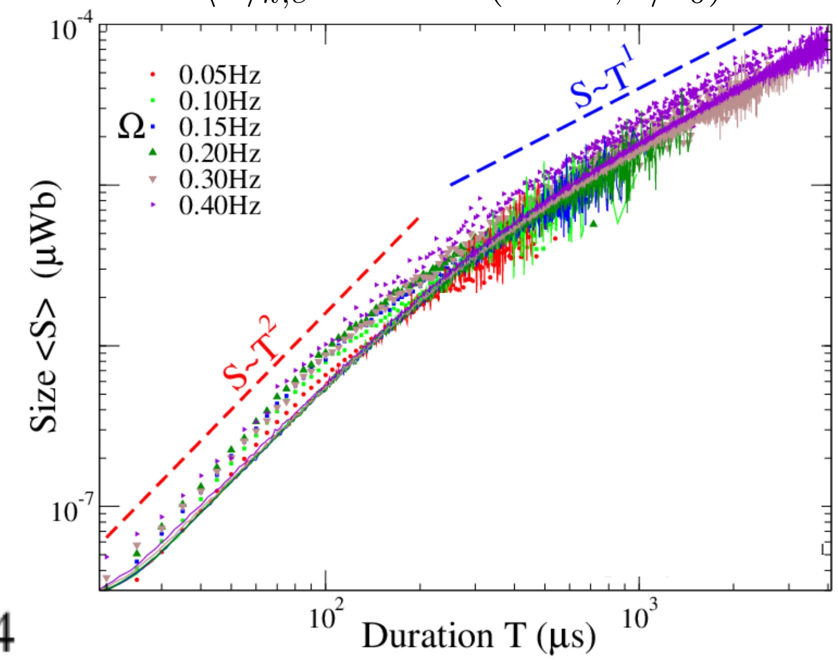






<size> vs. duration

$$\langle S\rangle_{k,c} = T^{\frac{1}{\sigma\nu z}}\mathcal{U}(Tk^{\nu_k z}, c/c_0)$$



Velocity Histograms

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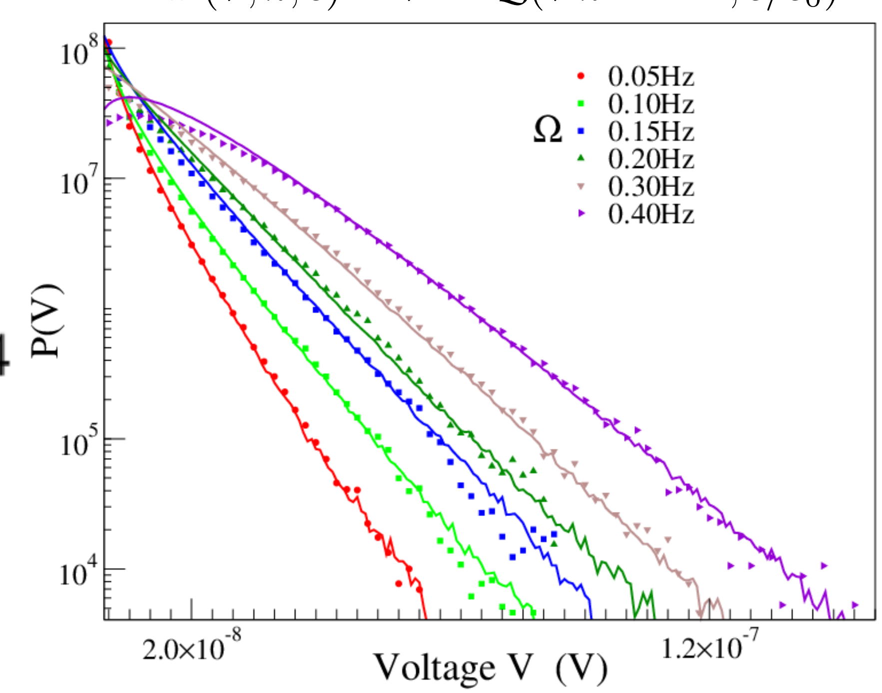

Confirmed mean-field universality

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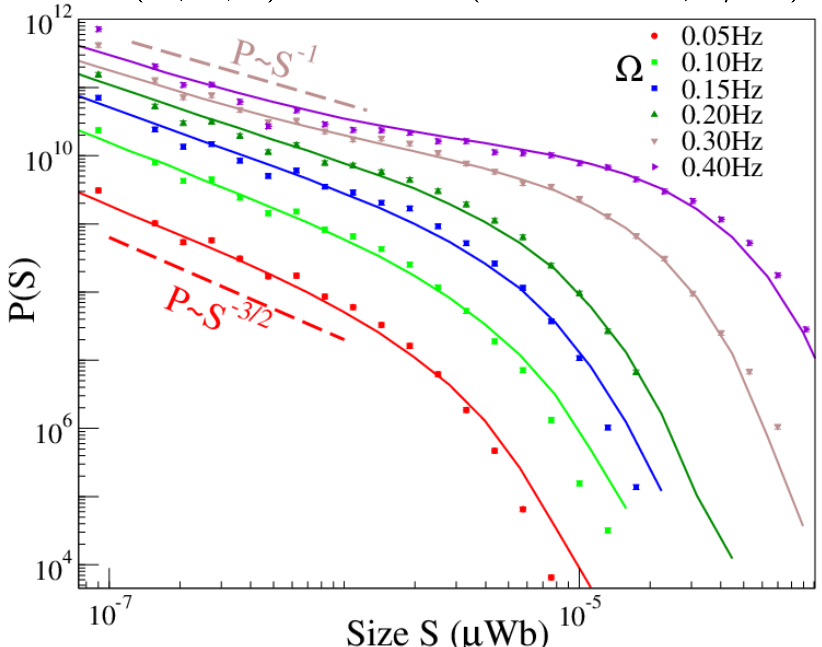
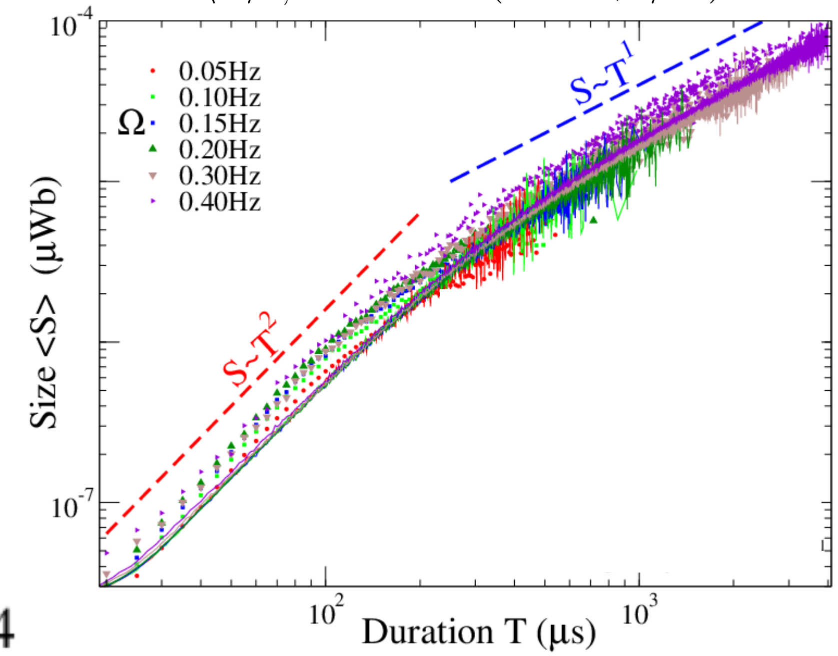
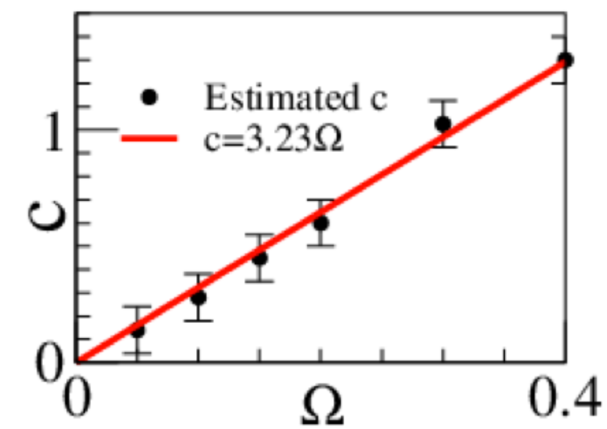
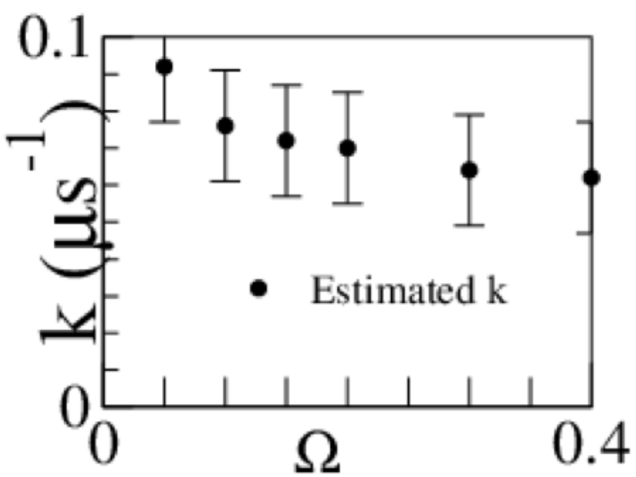


<size> vs. duration

$$\langle S\rangle_{k,c} = T^{\frac{1}{\sigma\nu z}}\mathcal{U}(Tk^{\nu_k z}, c/c_0)$$







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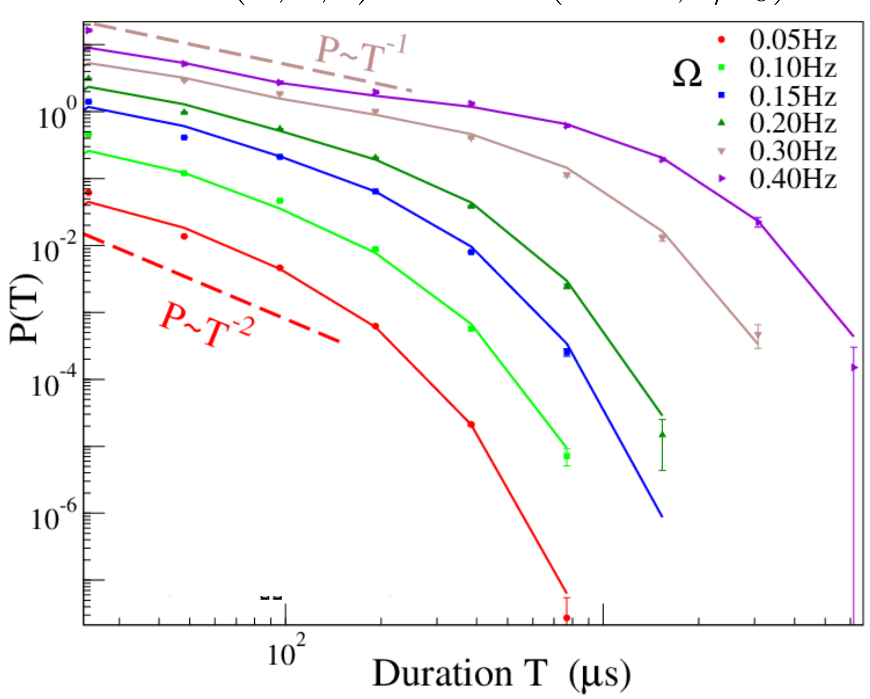

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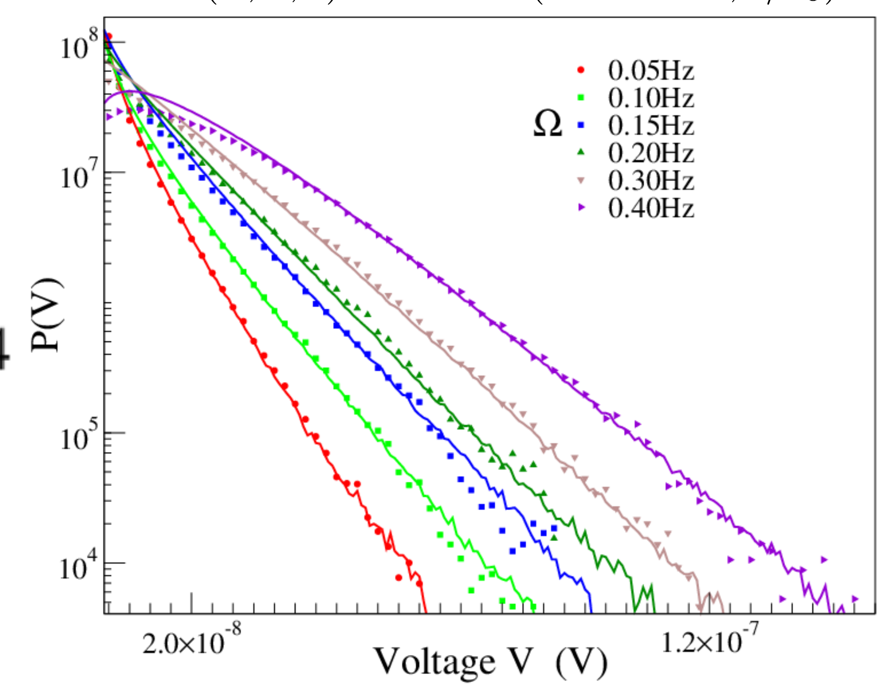

Simultaneous fits of simulations of similar size samples to experimental data

Conclusions

- Successful comparison between theory and experiments of avalanche behavior

- Processing techniques on scale invariant timeseries

- Systematic ways of inferring scaling properties from spatial or temporal raw, distorted data

- Y-J. Chen, SP, G. Durin, S. Zapperi, J. P. Sethna (to be submitted)
- SP and J. P. Sethna (to be submitted)
- SP, F. Bohn, R. L. Sommer, G. Durin, S. Zapperi, J. P. Sethna, arxiv: 0911.2291 (Nat. Phys., 2011)