Basins of attraction of mechanically stable packings on the density landscape

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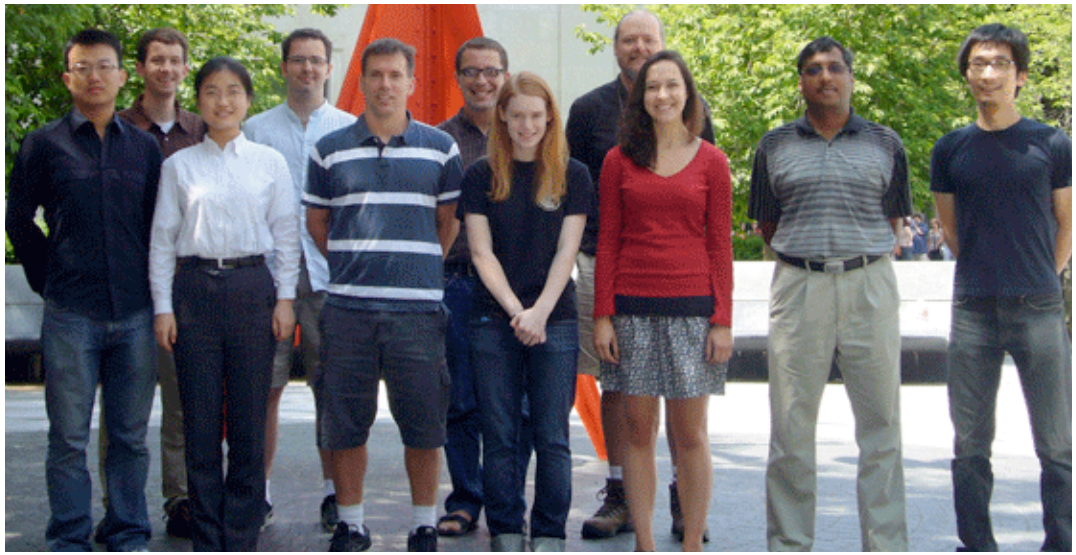
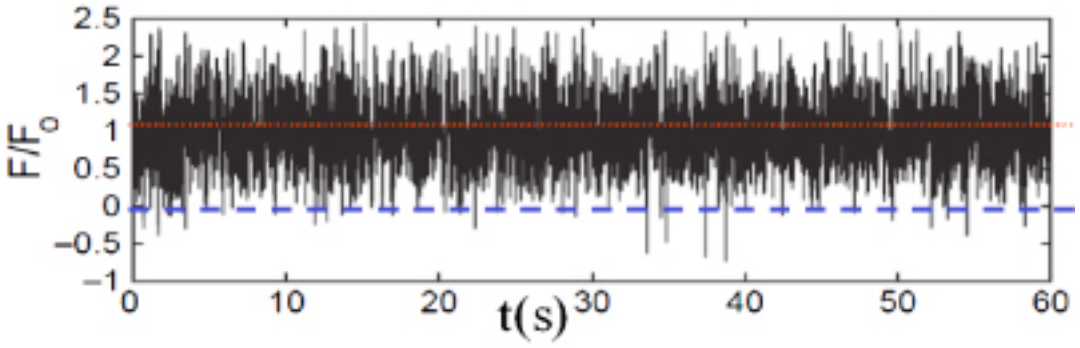
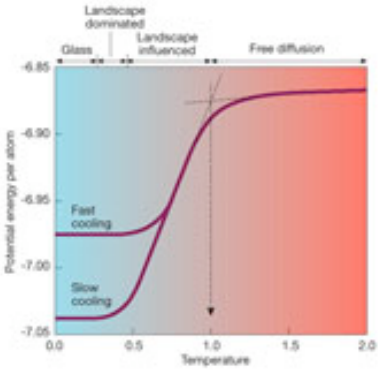


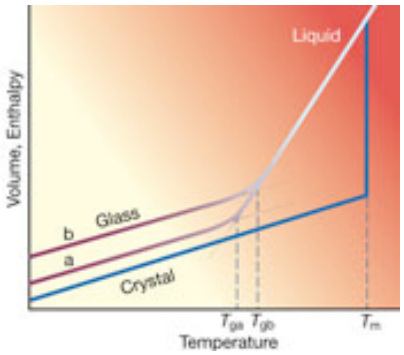
The O'Hern group: (back row from left to right) Carl Schreck, Thibault Bertrand, Robert Hoy, and Mark Shattuck; (front row from left to right) Tianqi Shen, Alice Zhou, Corey O'Hern, Sarah Penrose, Amy Werner-Allen, S. S. Ashwin, and Guo-Jie Gao.

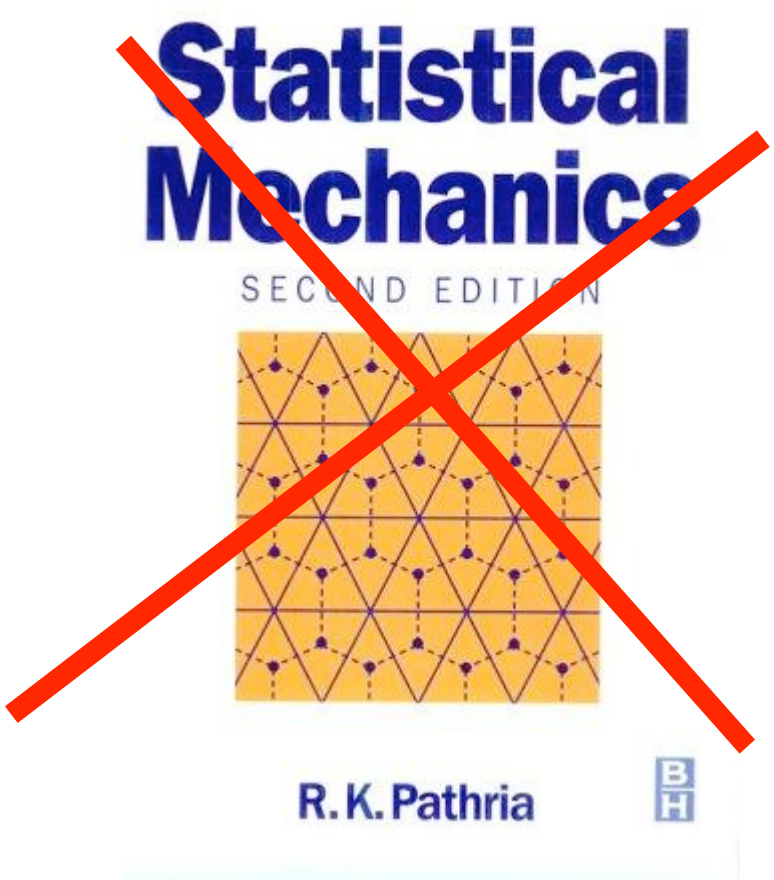
Stress Fluctuations



Glass Formation



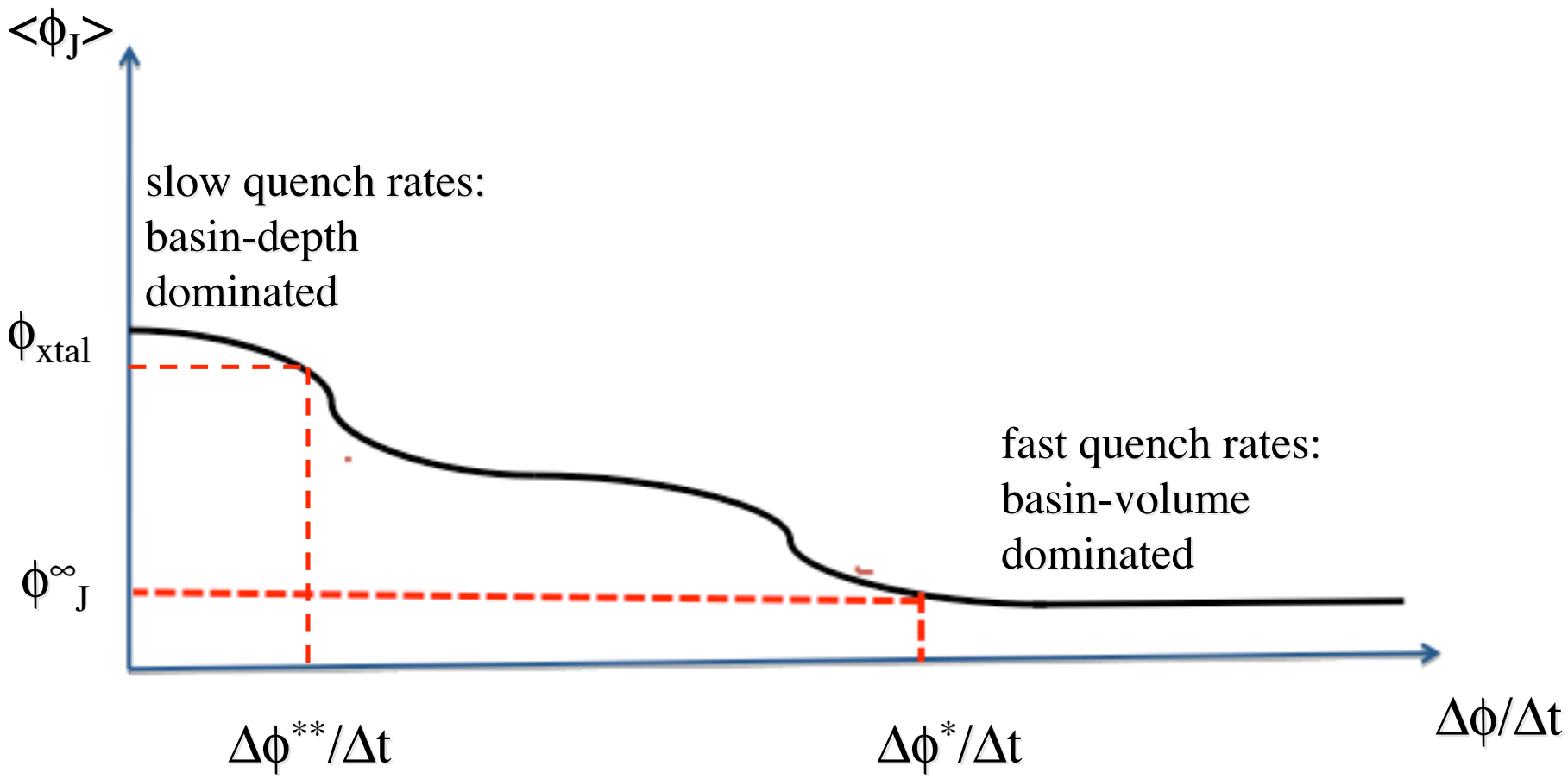


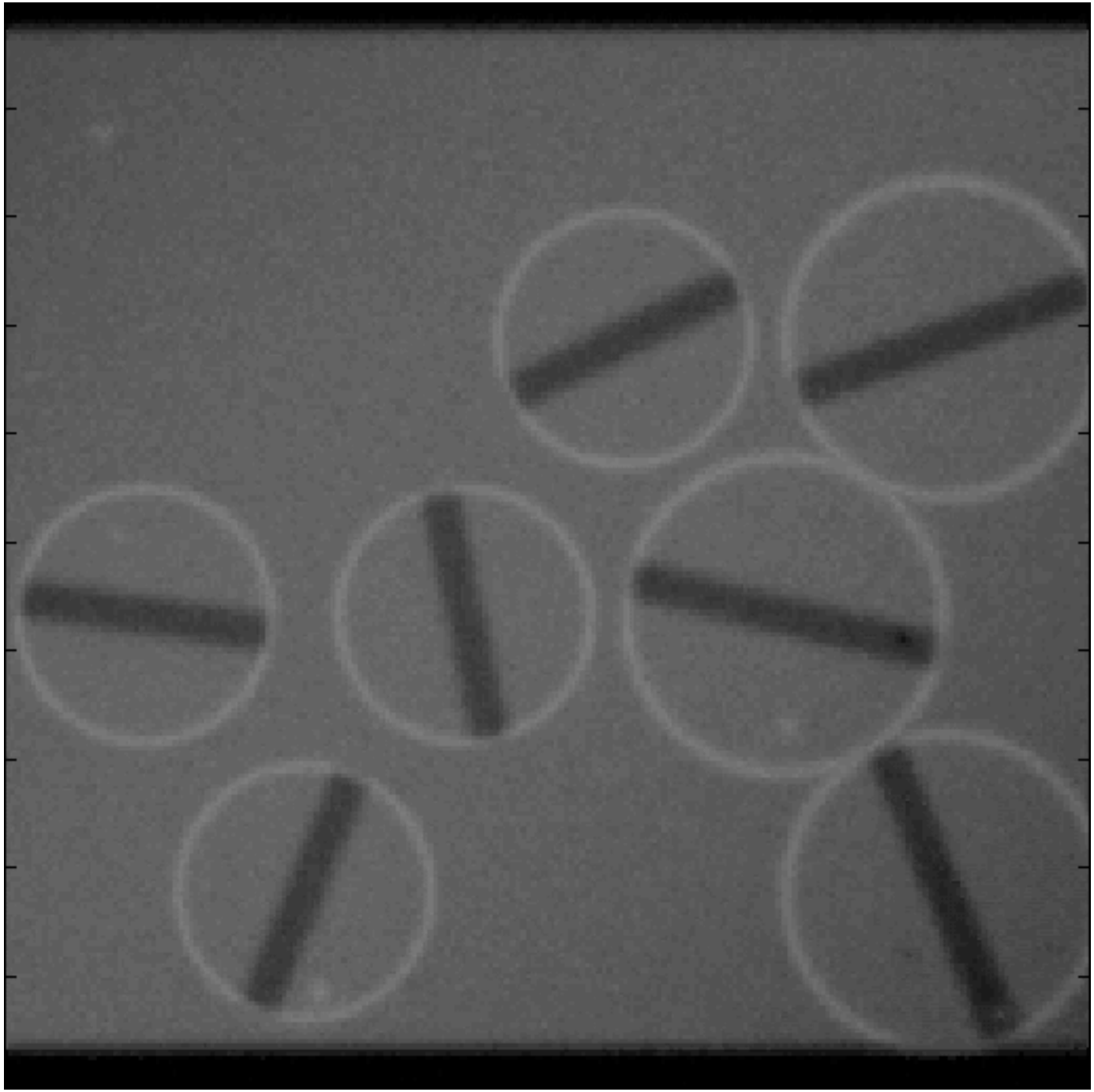


- Athermal
- Driven
- Dissipative
- Finite system size

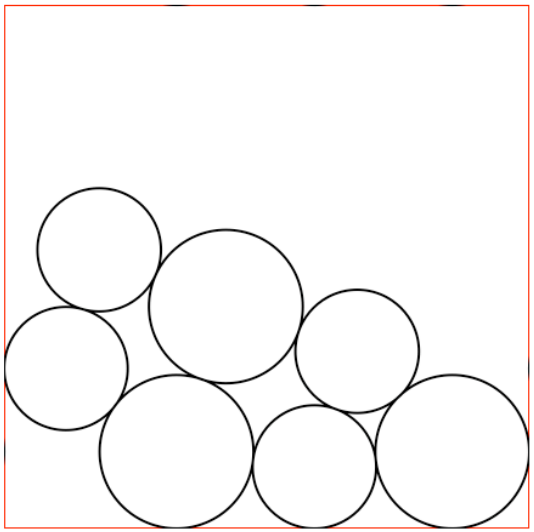
- What is probability with which granular packings occur?
- Edwards' hypothesis

Protocol Dependence of Granular Packings

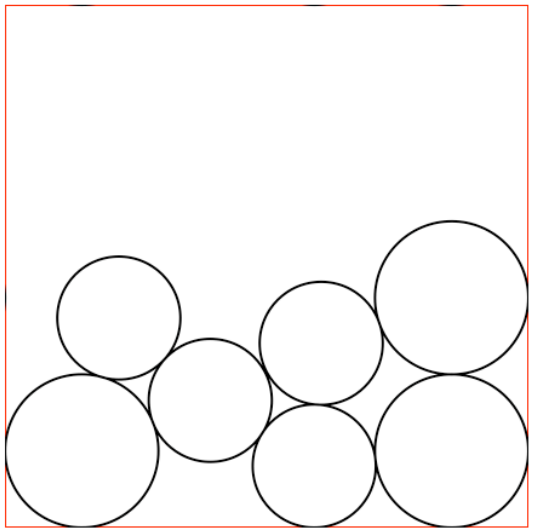




Mechanically Stable Frictionless Packings



1



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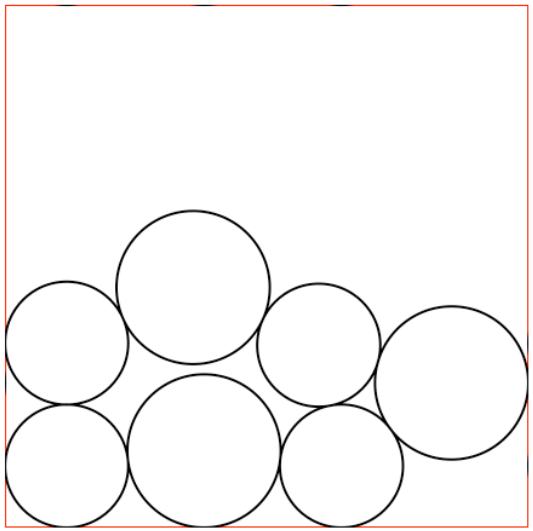
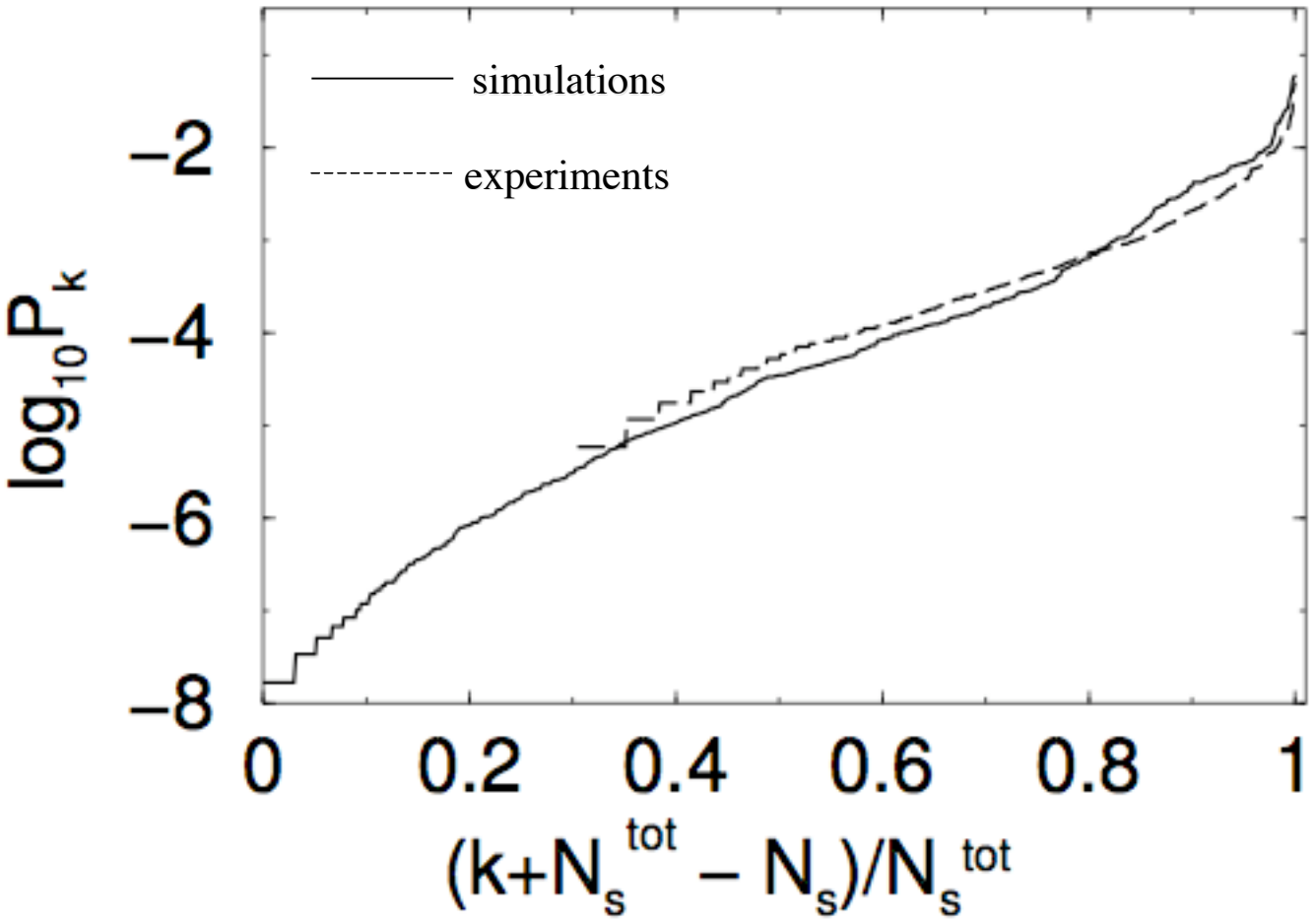


3

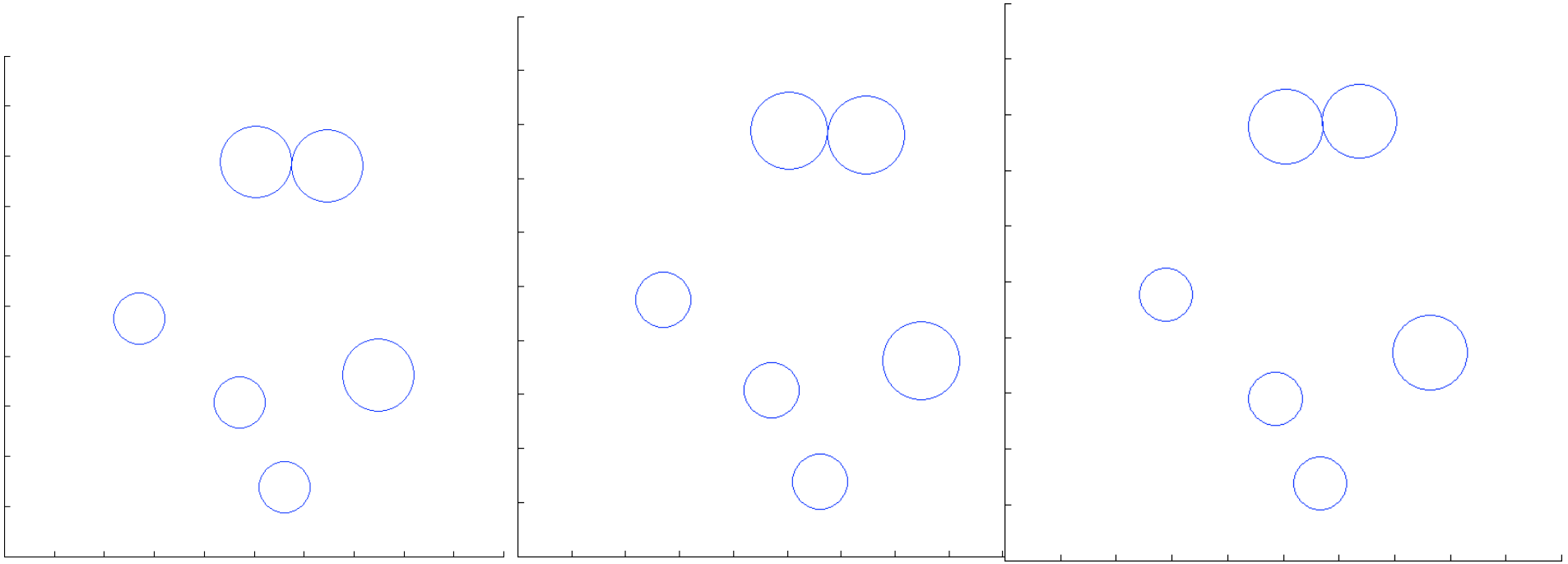
- Distinct MS packings distinguished by particle positions $\{\vec{r}_i\}$
- # of constraints ≥ # of degrees of freedom

Sorted Probabilities



- 7 (4) orders of magnitude variation in probabilities in simulations (experiments)

Rate dependence and basin volume



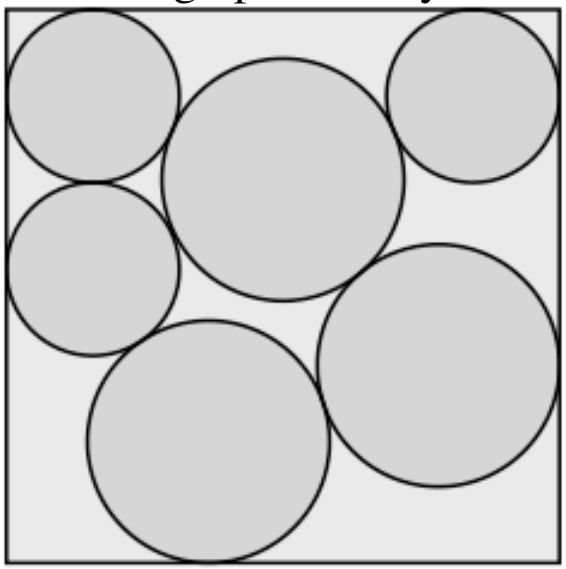
fast rate; ϕ_f=0.622

slow rate; ϕ_f=0.730

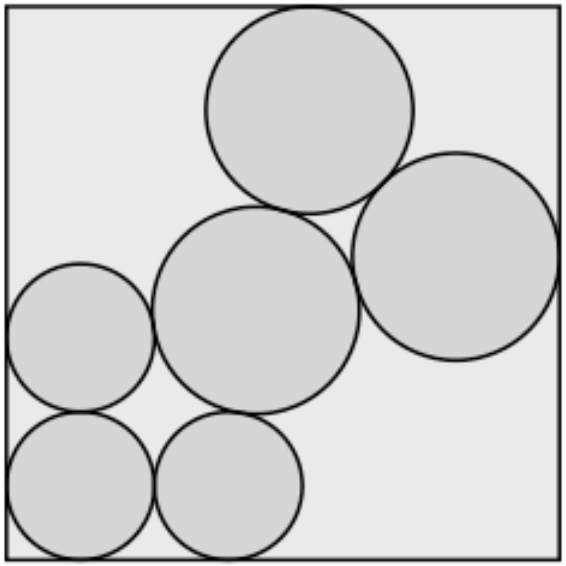
fast rate; different IC; ϕ_f=0.730

N^*	N_s
4	4
6	46
8	500
10	3983
12	16935

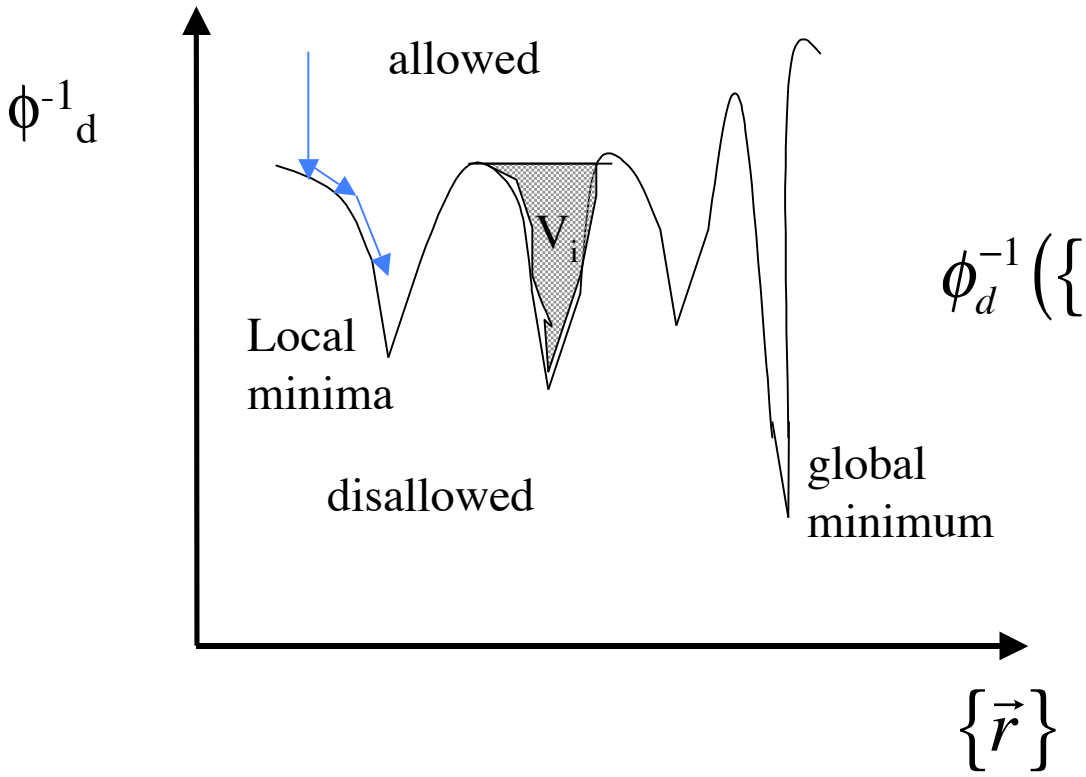
High probability



Low probability



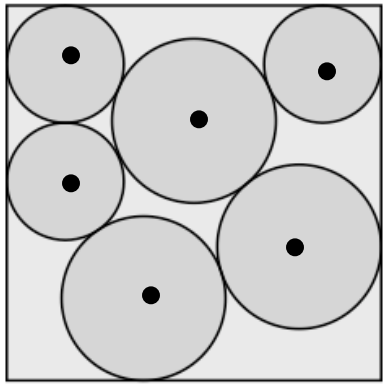
What determines MS packing probabilities: Density landscape for hard spheres



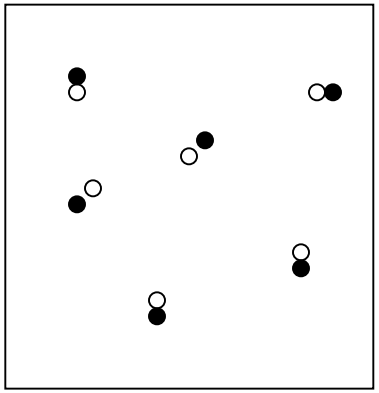
$$\phi_d^{-1}\left(\{\vec{r}\}\right) = \frac{6V}{\pi N \min_{mn} \left|\vec{r}_m - \vec{r}_n\right|^3}$$

N. Xu, D. Frenkel, and A. J. Liu, xxx.lanl.gov/cond-mat1101.5879

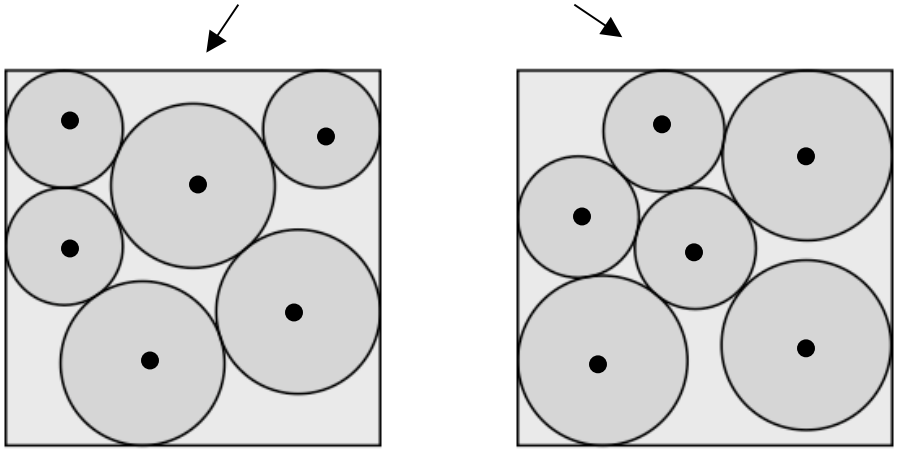
Method 1 (small l): Probability to return to a given MS packing



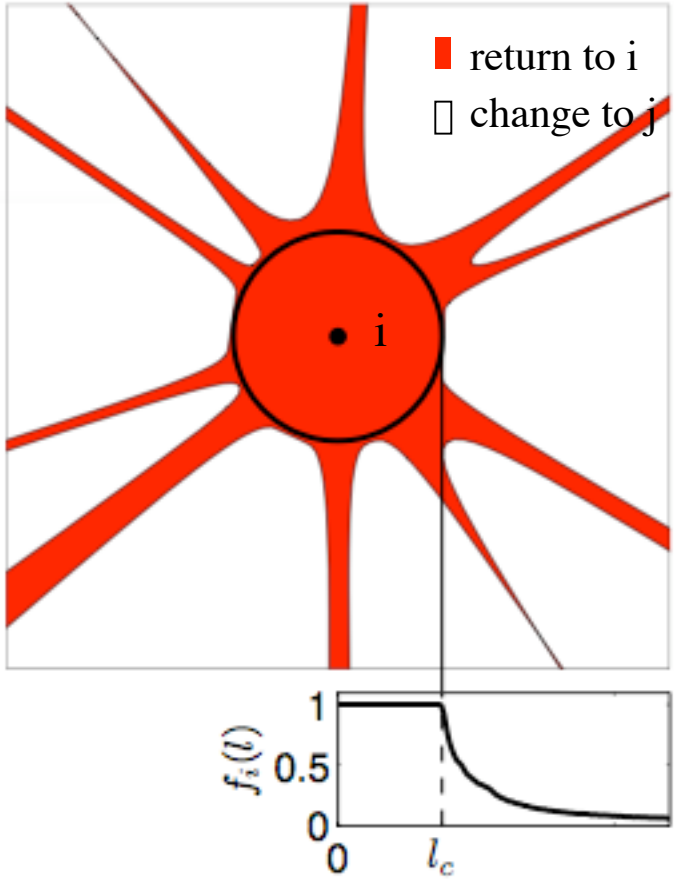
$$\phi_i^{MS}, \{\vec{r}\}_i^{MS}$$



$$\{\vec{r}\} = \{\vec{r}\}_i^{MS} + l\hat{e}_r$$

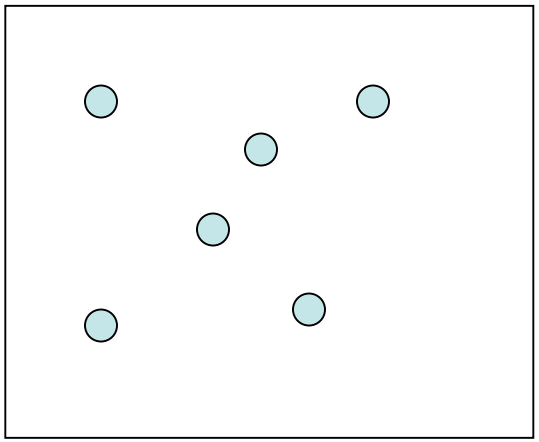


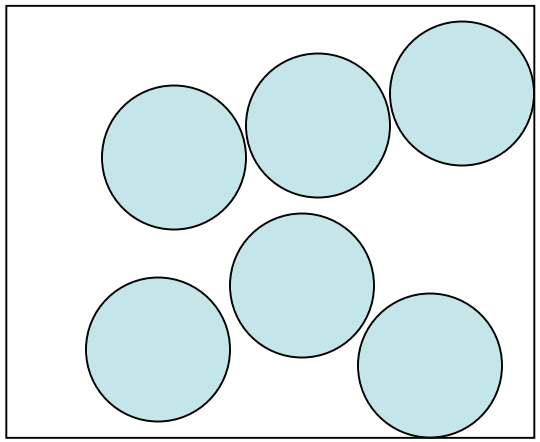
$$f_i(l) = \frac{M_i}{M}$$

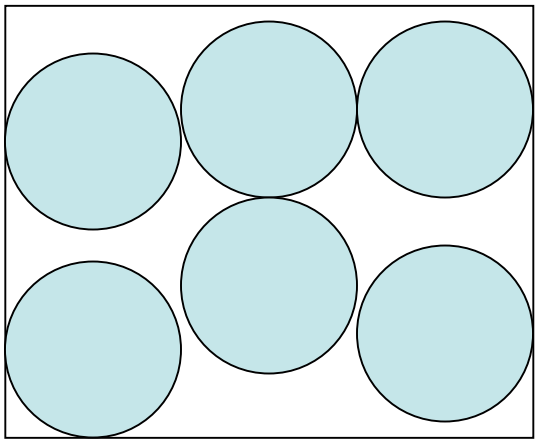


$$l = \sqrt{(x_{1f} - x_{10})^2 + (x_{2f} - x_{20})^2 + \cdots + (x_{Nf} - x_{N0})^2 + (y_{1f} - y_{10})^2 + (y_{2f} - y_{20})^2 + \cdots + (y_{Nf} - y_{N0})^2}$$

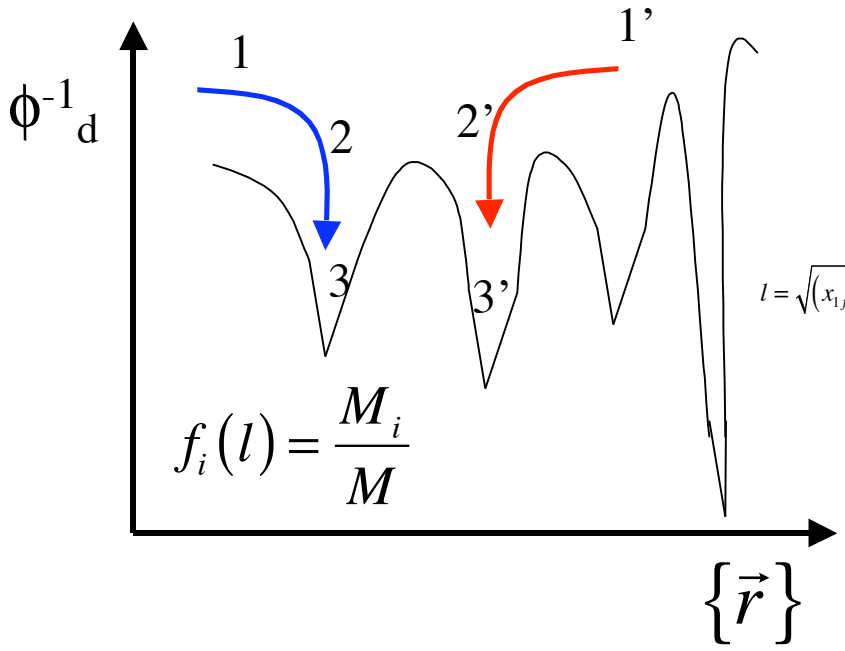
Method 2 (large l): Random initial conditions







$\phi_1, \{\vec{r}\}_1$ $\phi_2, \{\vec{r}\}_2$ $\phi_3, \{\vec{r}\}_3$



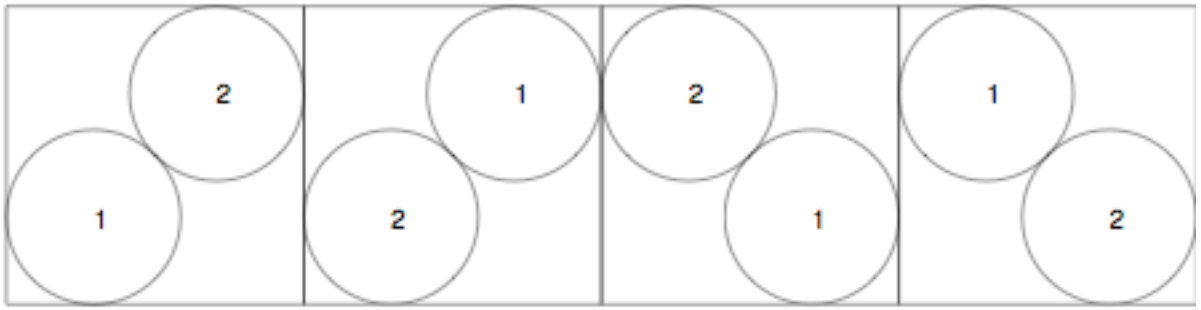
$$l = \sqrt{(x_{1f} - x_{10})^2 + (x_{2f} - x_{20})^2 + \cdots + (x_{Nf} - x_{N0})^2 + (y_{1f} - y_{10})^2 + (y_{2f} - y_{20})^2 + \cdots + (y_{Nf} - y_{N0})^2}$$

Basin Volumes

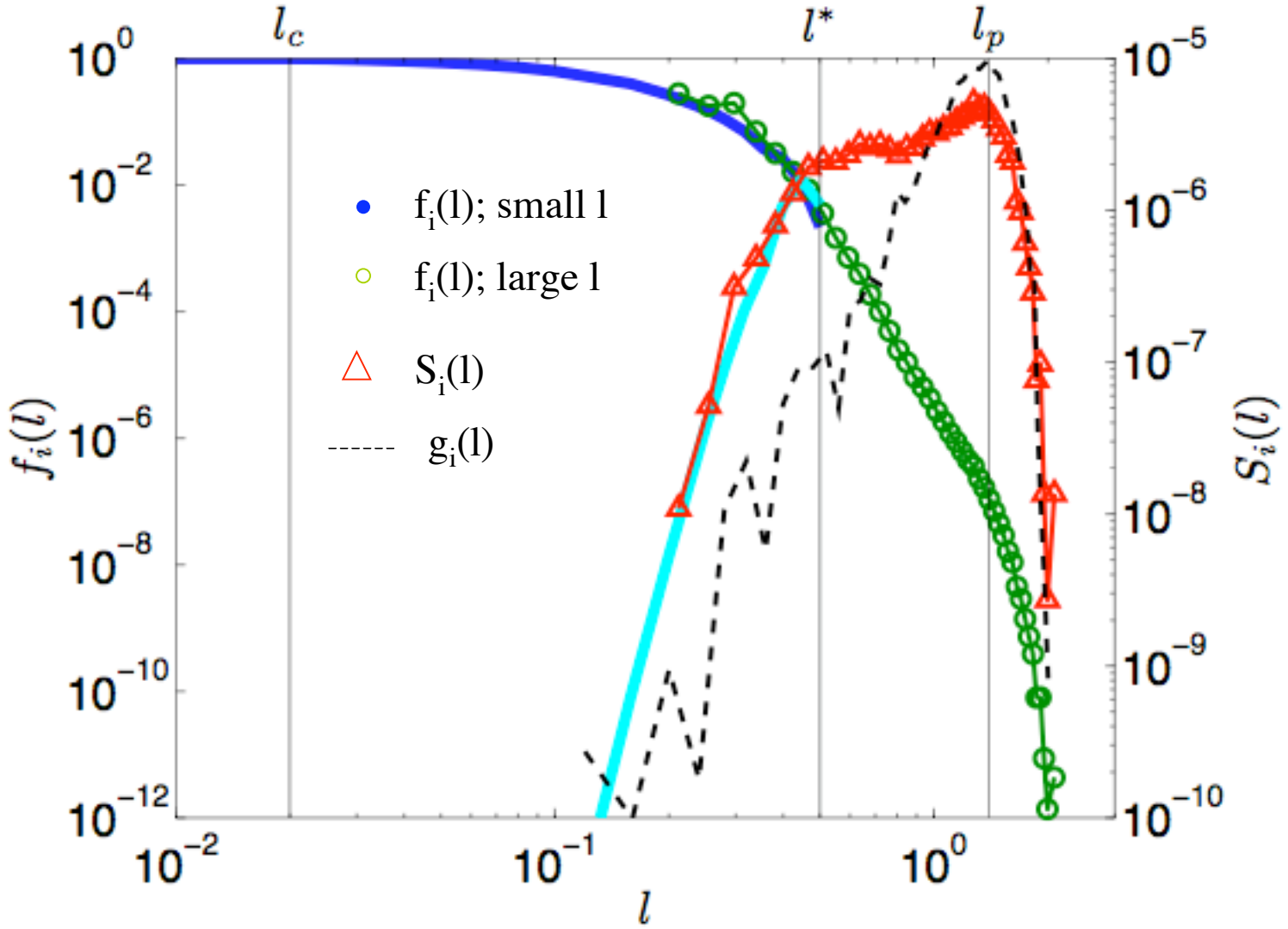
$$P_i = \frac{V_i}{L^{dN}} \qquad V_i = \int_0^{\sqrt{dN}} S_i(l)\,dl$$

$$S_i(l) = A_{dN} f_i(l) l^{dN-1} \mathrm{P}_i N_s! N_l!$$

polarizations and permutations

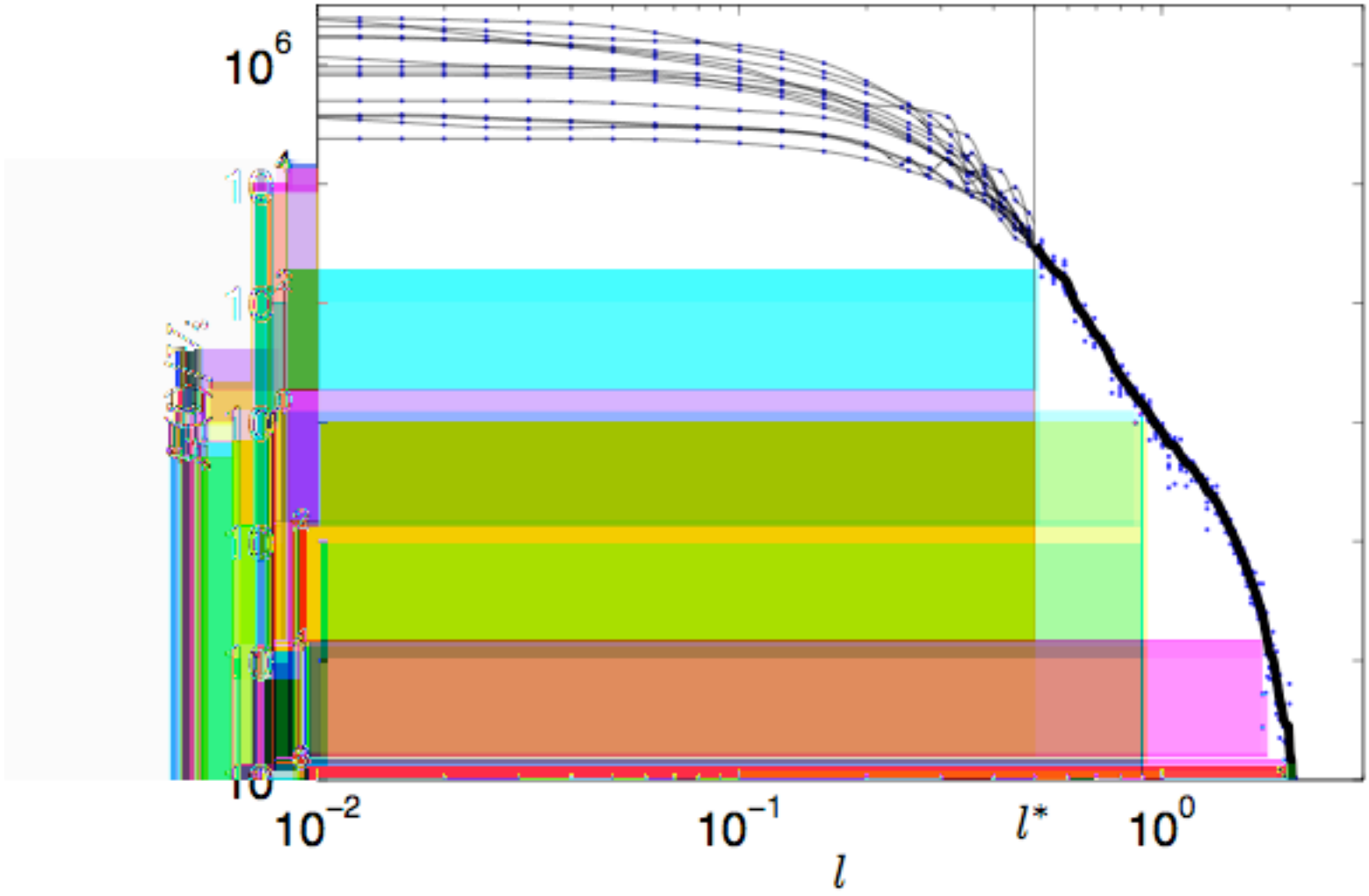


Weighted/Unweighted basin profile functions



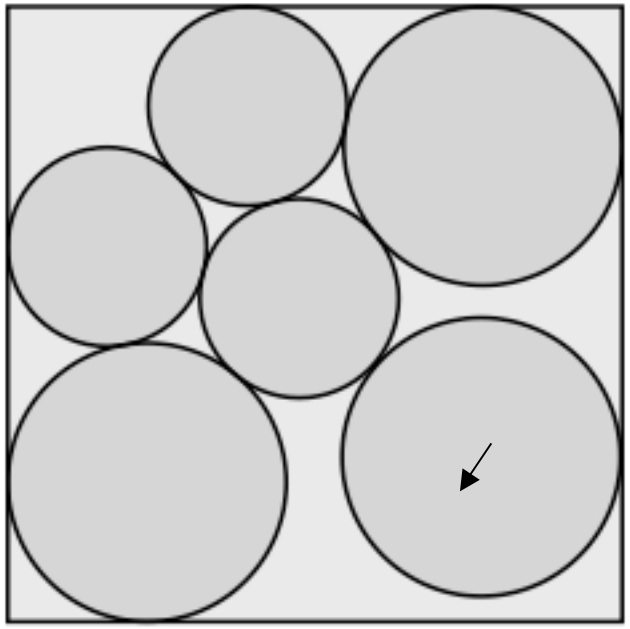
- Probability of MS packing determined by large l, not core region l_c
- Large probability near peak in MS packing separation distribution

Collapse for $l > l^*$

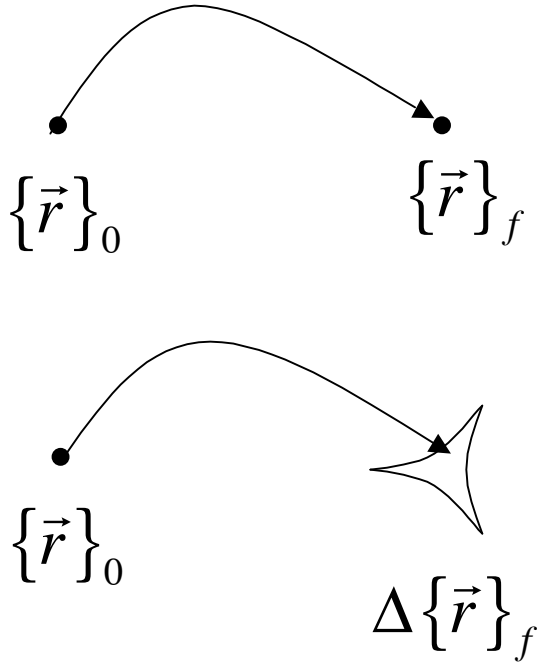


- Complete enumeration not necessary to determine P_i

Floaters



Particles with fewer than 3 contacts



Conclusions and Future Directions

- Probability for MS packings determined by large l, not nearby regions of configuration space
- Study ϕ_i and quench rate dependence of probabilities