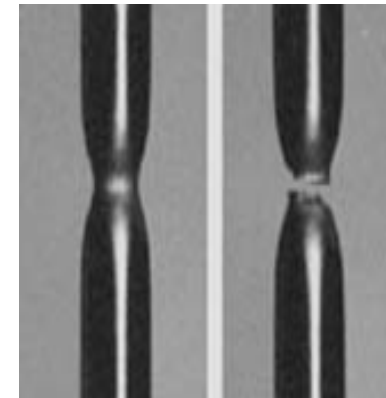
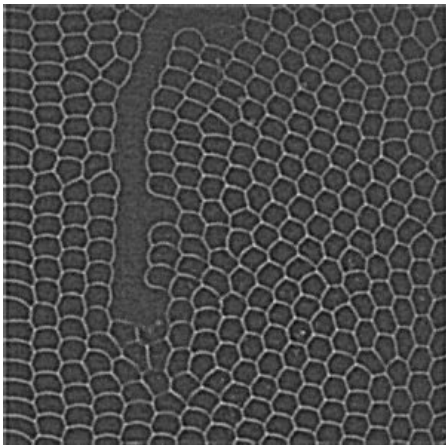
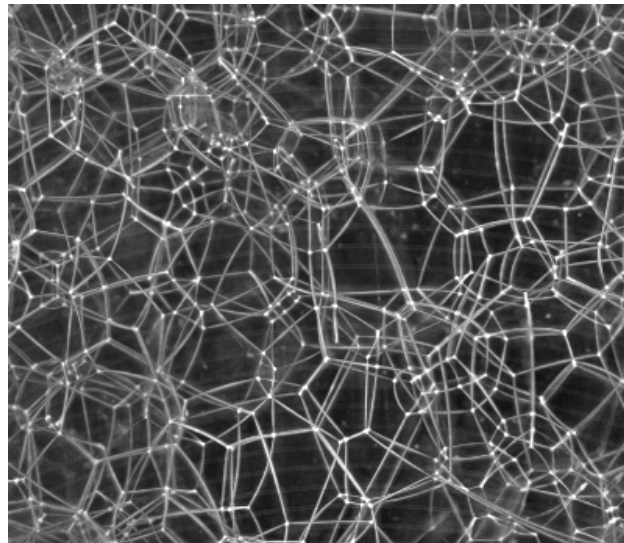
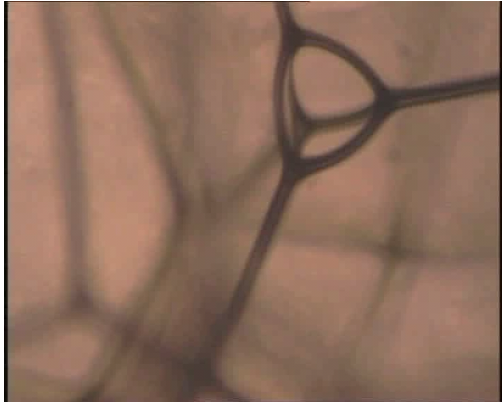


# Fracture in Aqueous Foam



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Shehla Arif (now McGill), J.-C. Tsai (now Academia Sinica, Taipei)

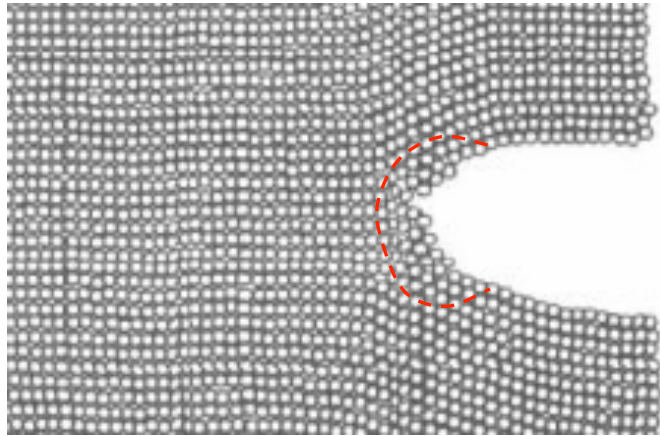
# Outline

- I. Bubbles as atoms: Foam
- II. Understanding brittle and ductile failure
- III. Understanding fracture speeds
- IV. A new Brittle-to Ductile Transition

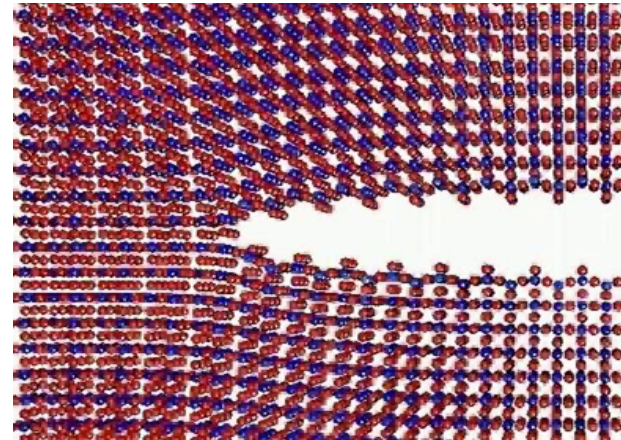
# Fracture: Ductile and Brittle cracks

$\alpha$ -Fe

[Cheung & Yip 1993]



[Michael Marder 2004]

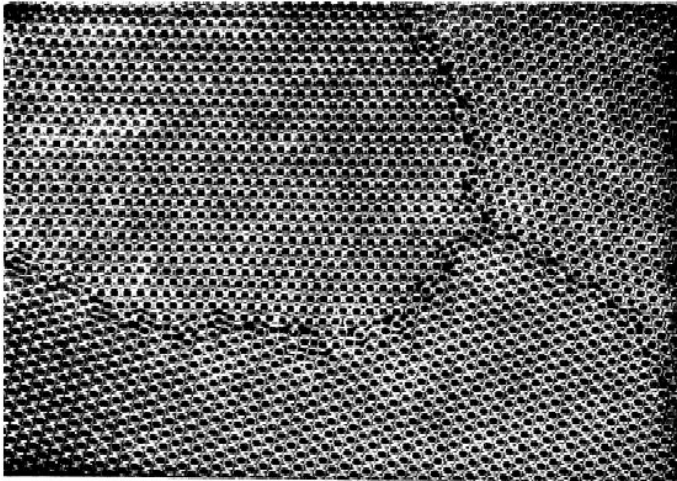


- plastic deformation (**process zone**)
- blunt tip
- defects/dislocations
- generally relatively slow
- occur at **low rate of applied stress**
- elastic (small) deformations
- sharp tip
- no defects
- fast ( $\leq c_R$ ), **velocity gap!**
- **high rate of applied stress**

length scales nm, time scales ps!

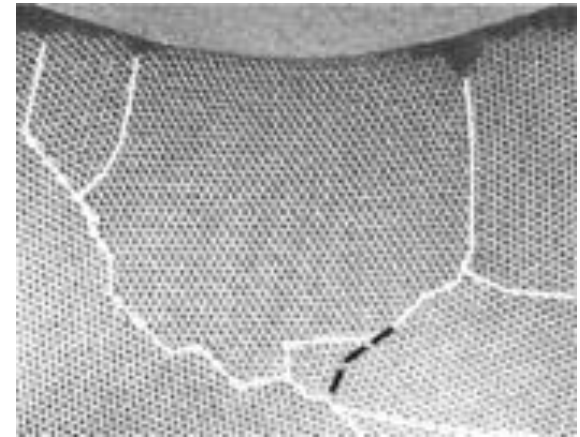


# 2-D foams: A model for atomic solids



**Grains**

L. Bragg,  
(1947)



**Plasticity**

S. Suresh,  
MIT (2004)

Can we fracture a layer of liquid foam?

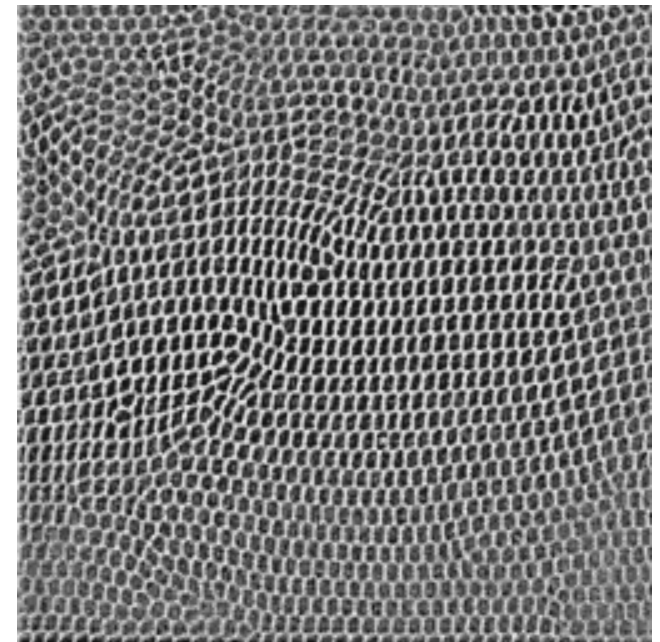
Length scales: mm

Physical properties dominated by

surface tension  $\gamma$  and

viscosity of water  $\mu$

Geometry is important!

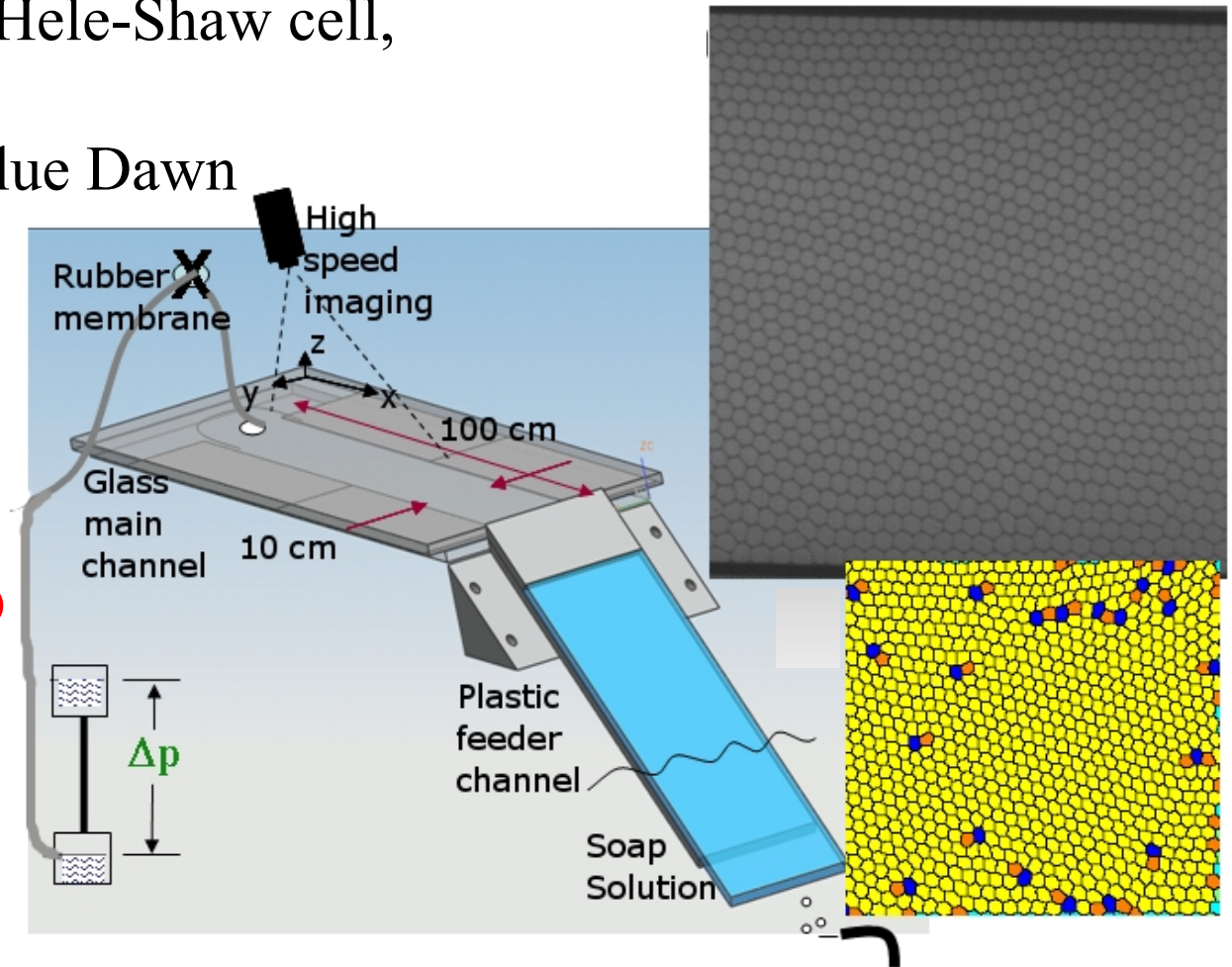


# Experimental set-up

- Rectangular-channel Hele-Shaw cell, gap  $b \approx 1\text{ mm}$
- Dry aqueous foam, Blue Dawn

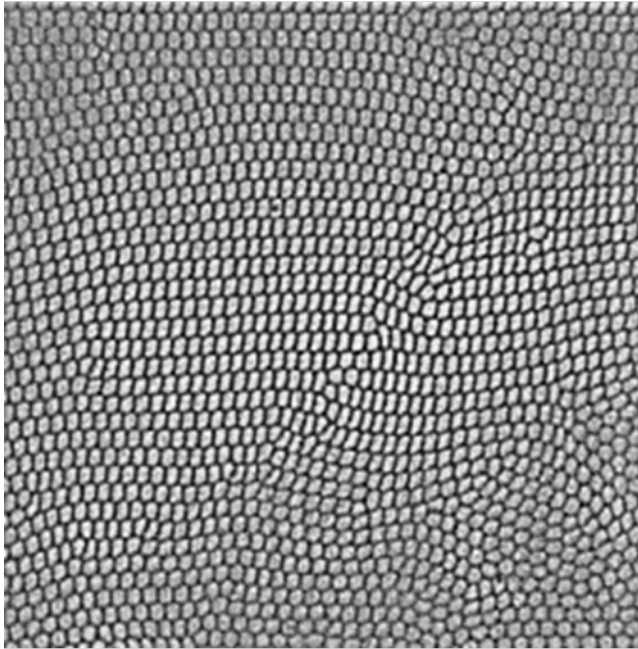
$$\gamma \approx 0.025 \text{ N/m,}$$
$$\mu \approx 10^{-3} \text{ Pa s}$$

- Bubble distance  $D=3 \text{ mm} \rightarrow$  quasi-2D
- $\Delta P=1000\dots 4000\text{Pa}$

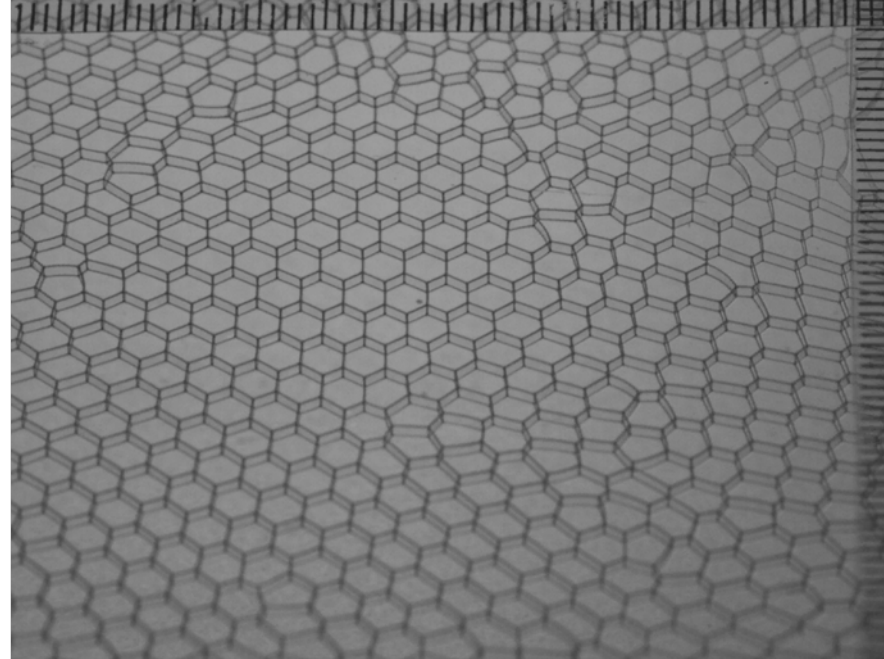


rise time of  $\Delta P$  (rate of applied stress) controllable!

# Ductile Cracks



[500fps]

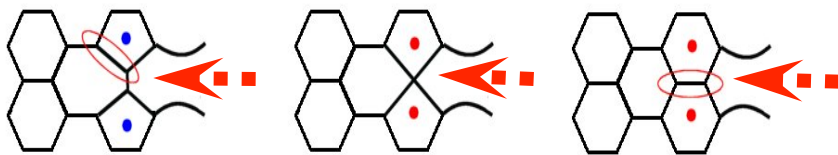


[100fps]

Pressure rise time  $\sim 1$  s

Fingering, plastic behavior!

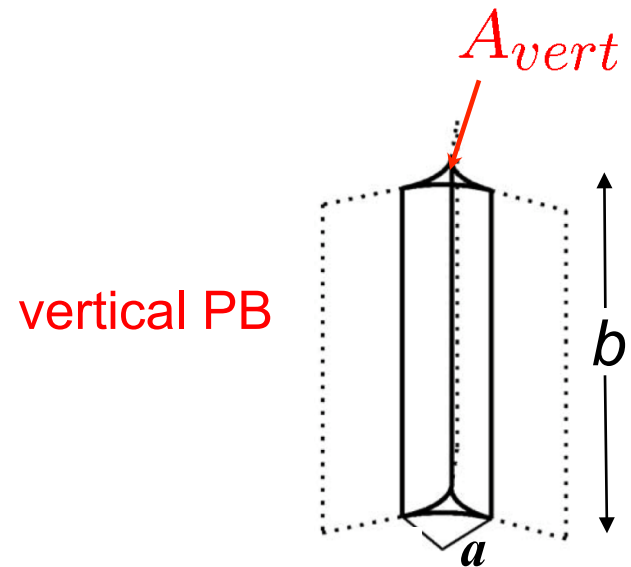
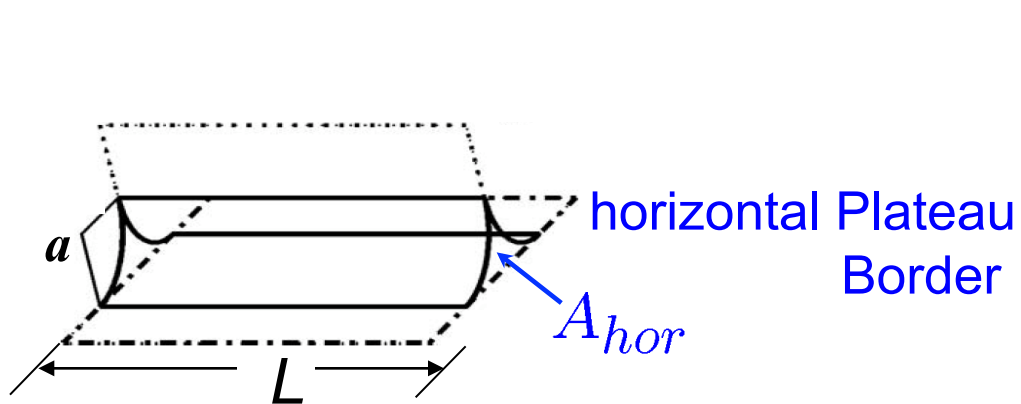
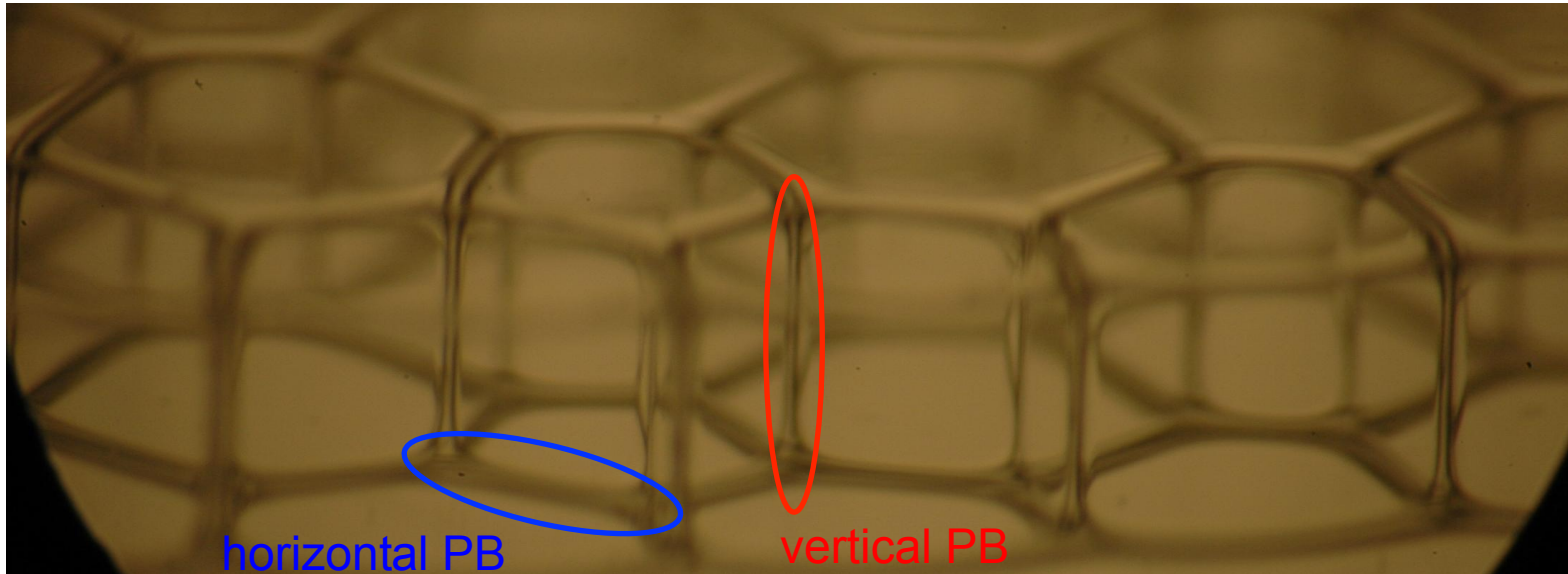
T1 process



Observation: **never** propagates faster than  $\sim 0.4$  m/s!

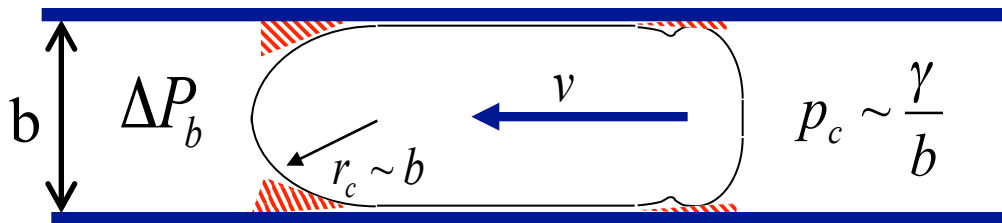


# Quasi-2D foam geometry



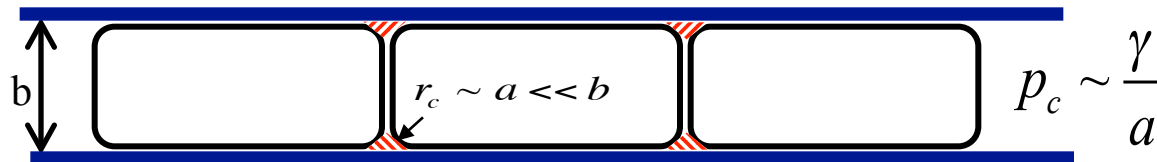
# Quasi-2D foam rheology

Bretherton's (1961) single bubble



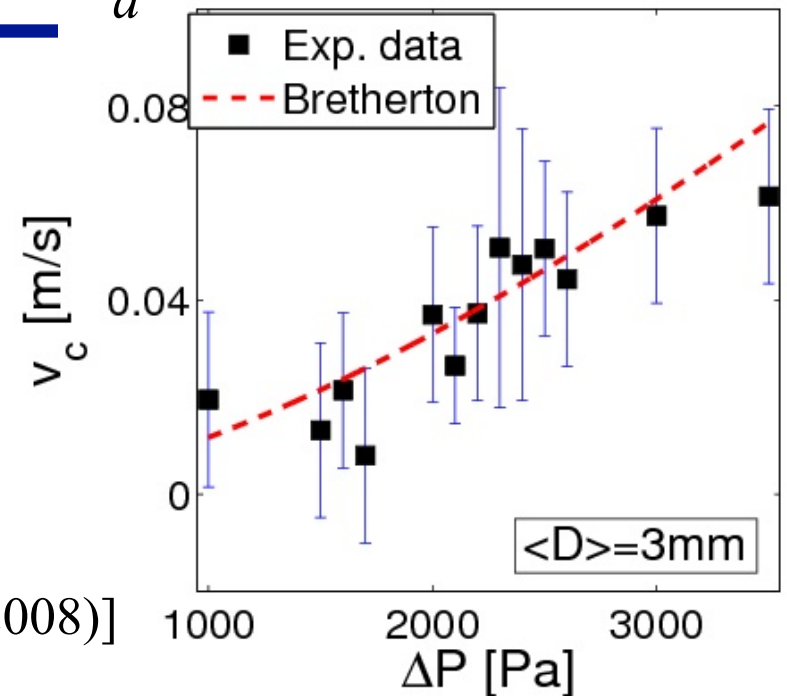
$$Ca = \frac{\mu v}{\gamma}$$

Monolayer of dry foam



$$\Delta P_b = 4.70 p_c Ca^{2/3}$$

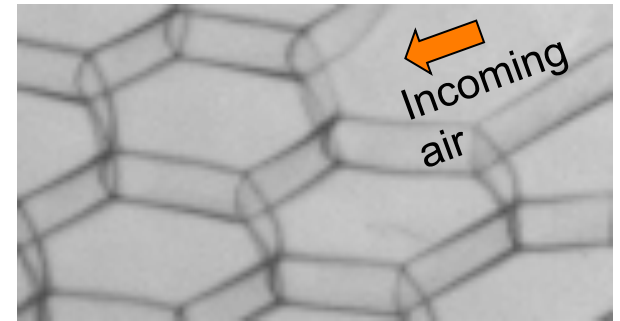
Bretherton theory  
quantitatively valid!



[SH, S. Arif, JC Tsai, Phil. Trans. Roy. Soc. A (2008)]



# Ductile threshold velocity



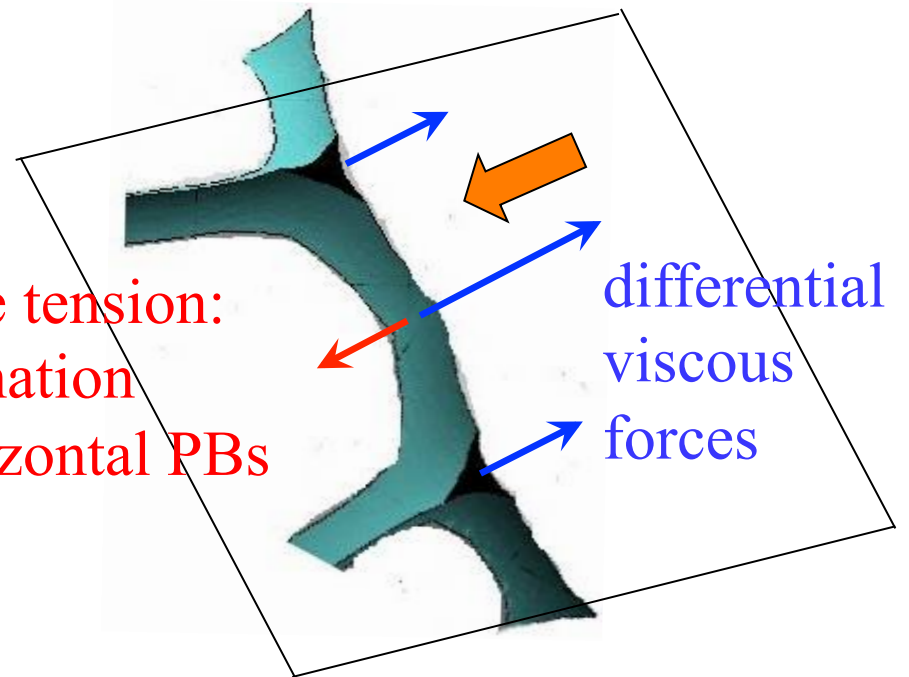
$$\frac{2\gamma}{L} = 4.70 \frac{\gamma}{2a} Ca^{2/3}$$

$$Ca_c = \left( \frac{4a}{4.70L} \right)^{3/2}$$
$$v_c = 0.36 \text{ m/s}$$

confirmed  
experimentally

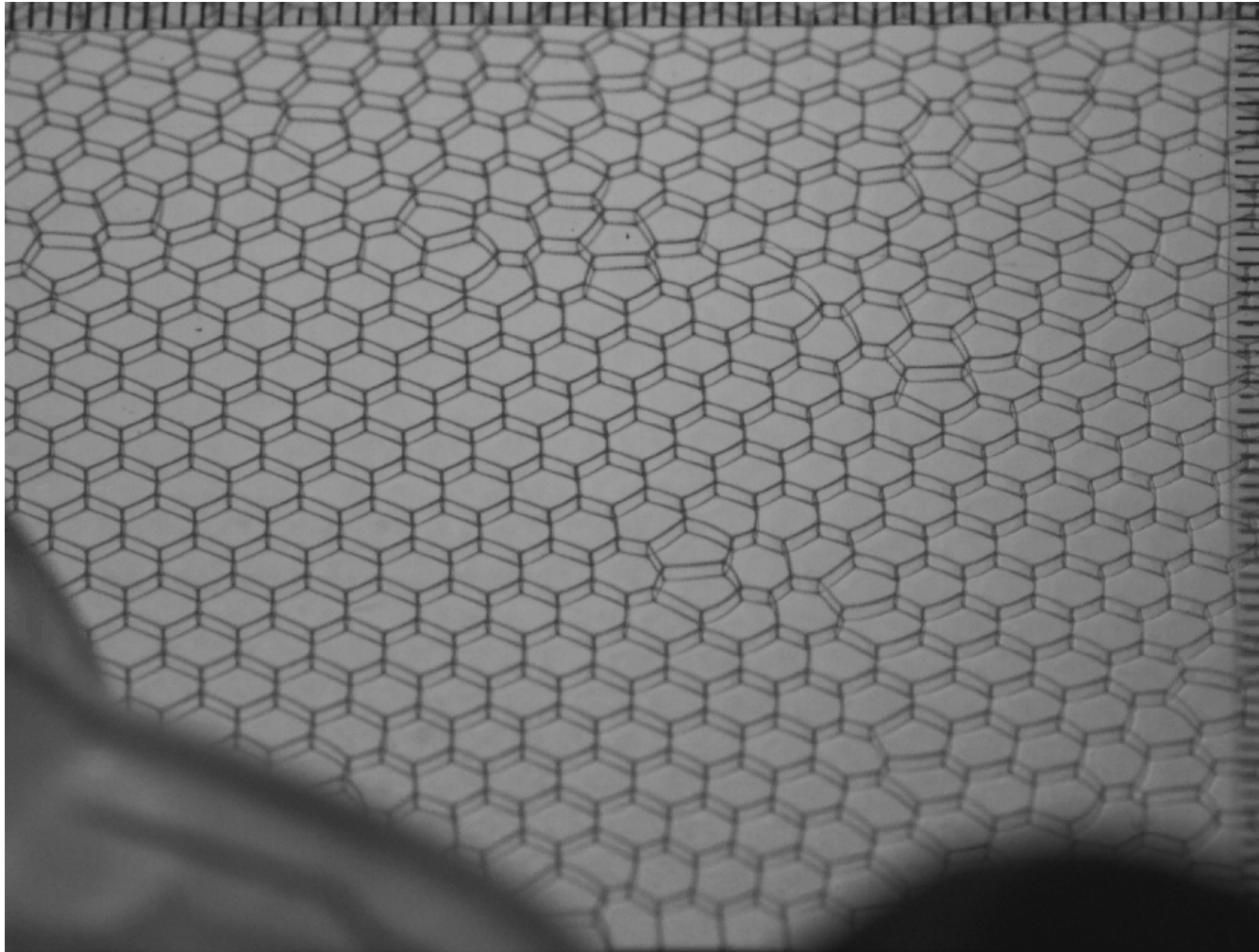
surface tension:  
deformation  
of horizontal PBs

differential  
viscous  
forces



Establishes upper speed limit for ductile propagation mode

# Brittle Cracks

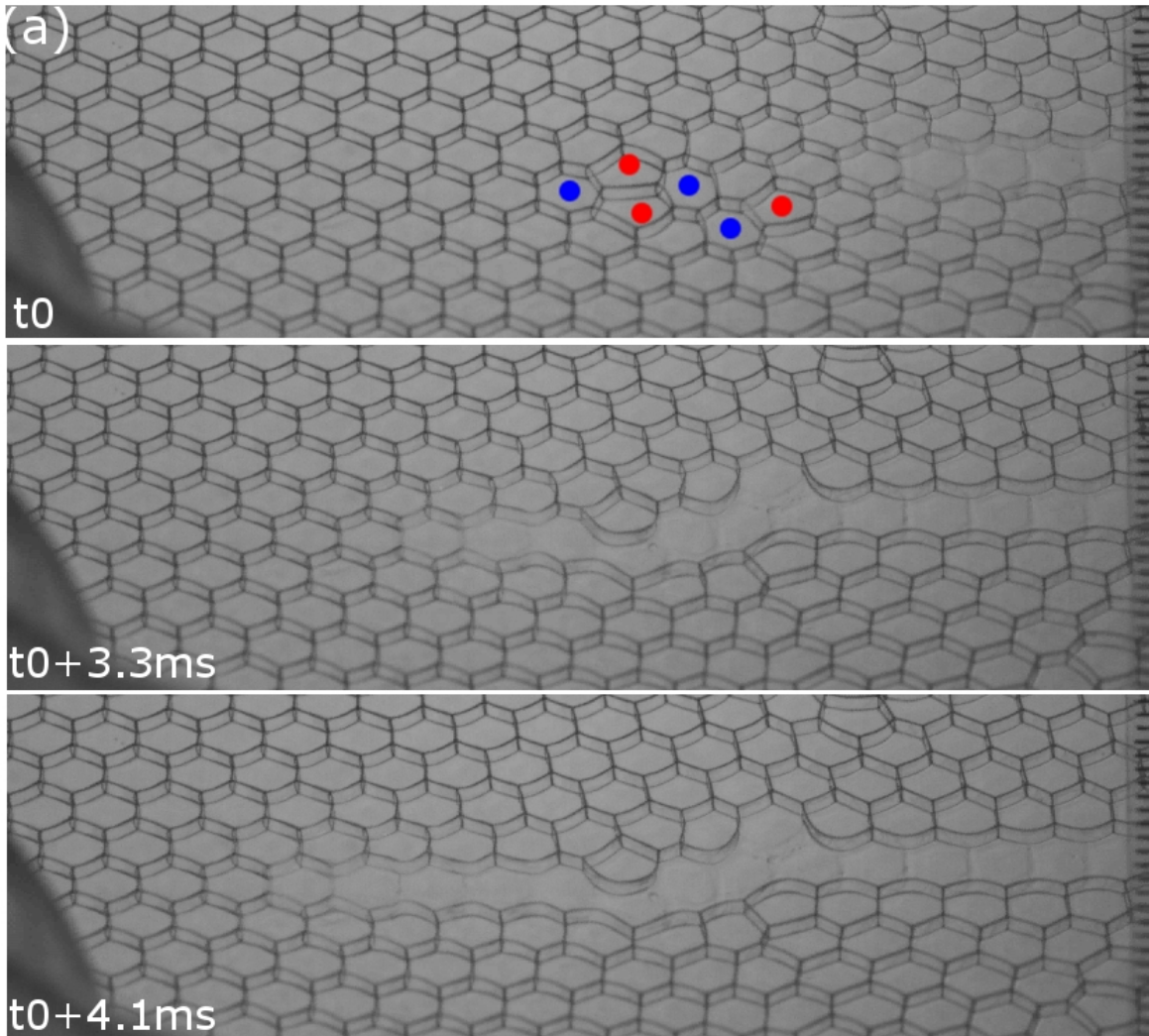


Pressure rise time  $< 1\text{ms}$

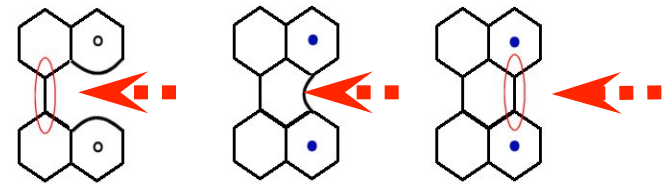
[2400fps]

**Straight cleavage, elastic behavior!**

# Brittle Cracks

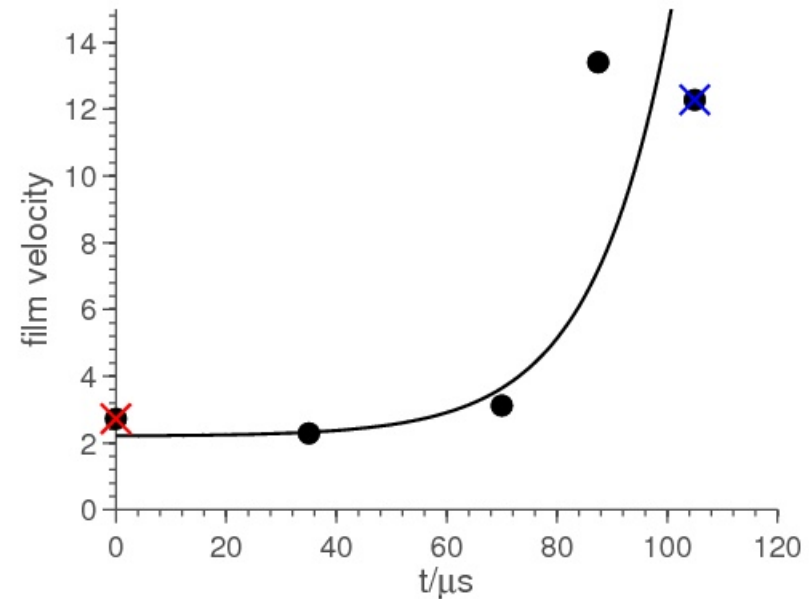
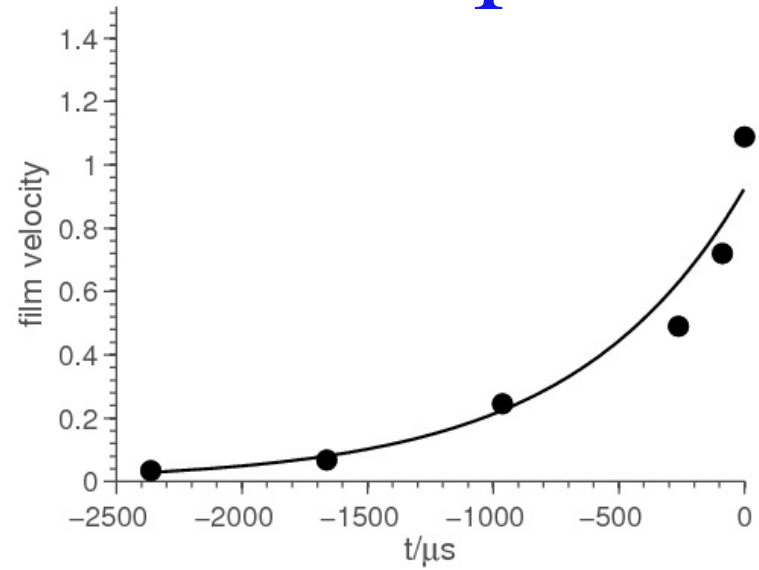
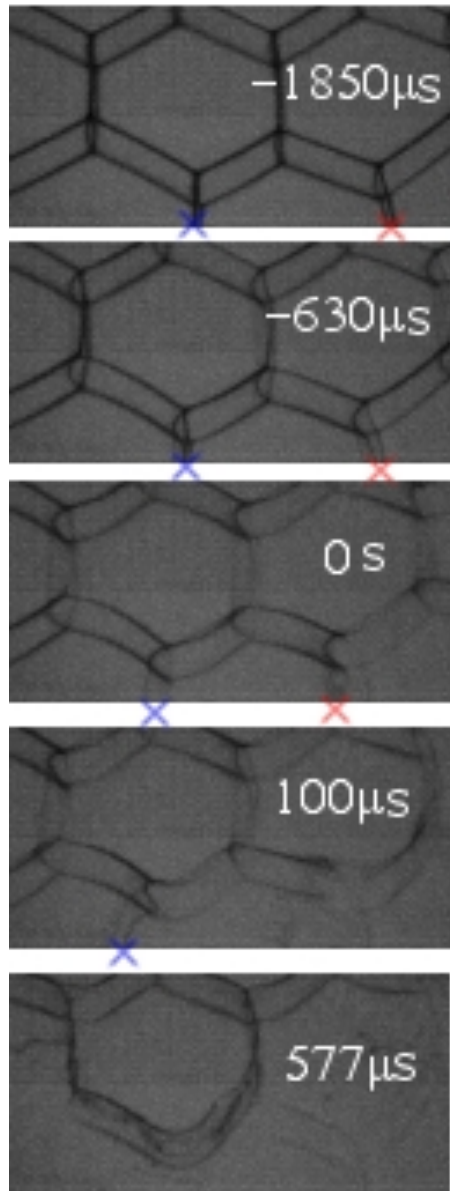


Film rupture



Observation: never  
propagates slower  
than  $\approx 10$  m/s:  
velocity gap!

# A close look at film rupture



acceleration up to  $10^5$  m/s<sup>2</sup>!



# Film stability



$$\tilde{\omega}_{im} = \left( -\tilde{k}^3 \coth(\tilde{k}) \left\{ 1 - \left[ 1 - \left( 1 - \frac{\tilde{k}_c^4}{\tilde{k}^4} \right) \tanh^2(\tilde{k}) \right]^{1/2} \right\} \right)^{1/2}$$

$$\tilde{k} = kh \quad \tilde{k}_c = (\rho a_f / \gamma)^{1/2} h^2$$

$$\tilde{k}, \tilde{k}_c \ll 1 \Rightarrow \omega_{im}(k_m) \approx (\rho/2\gamma)^{1/2} h^{1/2} a_f$$

film acceleration:

$$a_f = \Delta P / (\rho h) \propto h^{-1} \Rightarrow \omega_{im} \propto 1/h^{1/2}$$

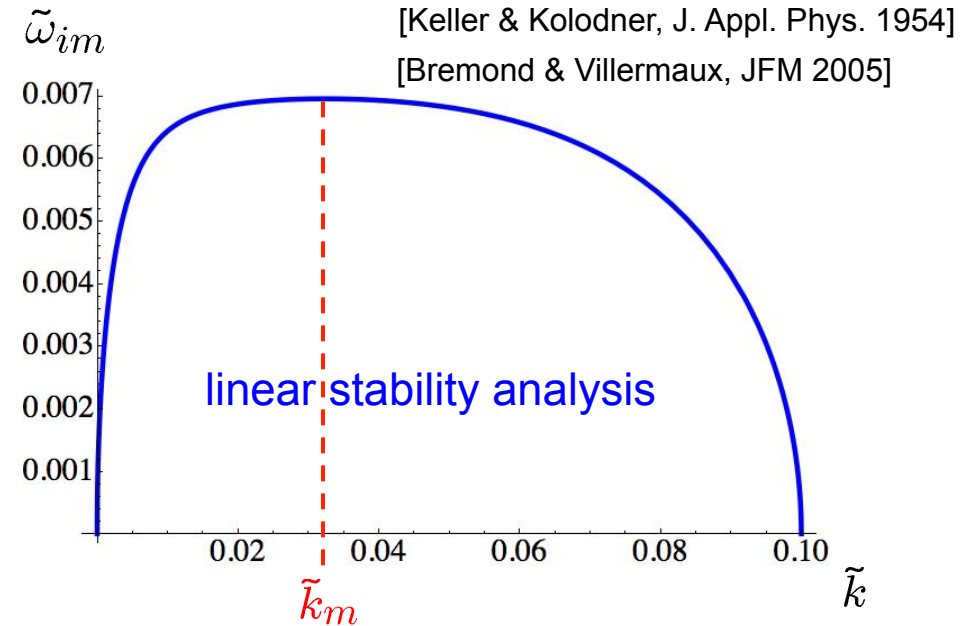
thinner films are more unstable!

breakage time scale:

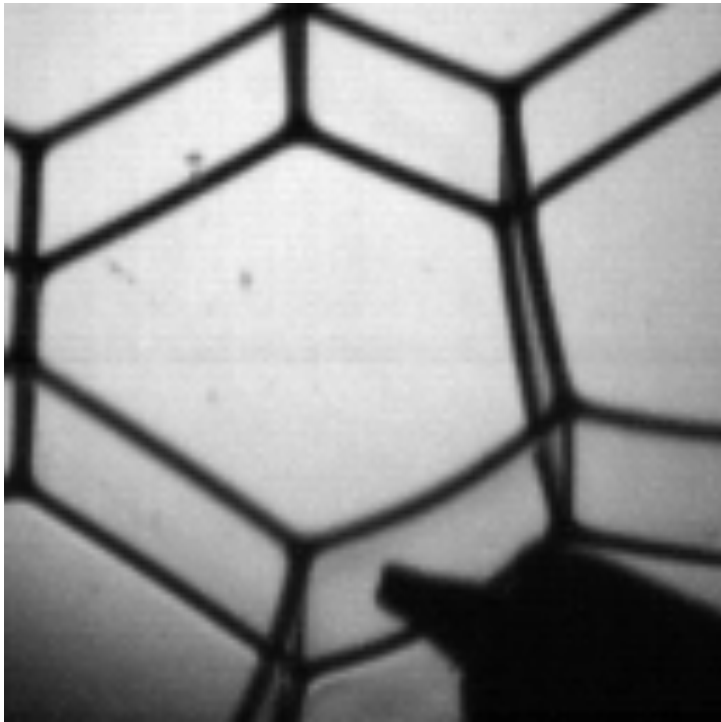
$$\tau_{break} = \ln(h/\zeta_0) \frac{(2\rho\gamma h)^{1/2}}{\Delta P}$$

For typical  $h \sim 1 \mu\text{m}$ ,  $\Delta P = 2000 \text{ Pa}$ :  $\tau_{break} \sim 100 \mu\text{s}$

Predicts growth of crack speed with  $D$ ,  $\Delta P$ ,  $1/h^{1/2}$  (confirmed experimentally)

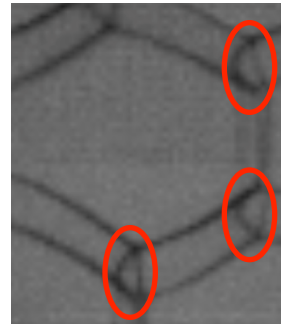


# Film breakage and waves



**Wave** propagates around bubbles through unbroken films

**Measured:**  $c \approx 10.4\text{m/s}$



**Oscillating mass** in vertical PBs!

[57000fps]

**2D theory:**

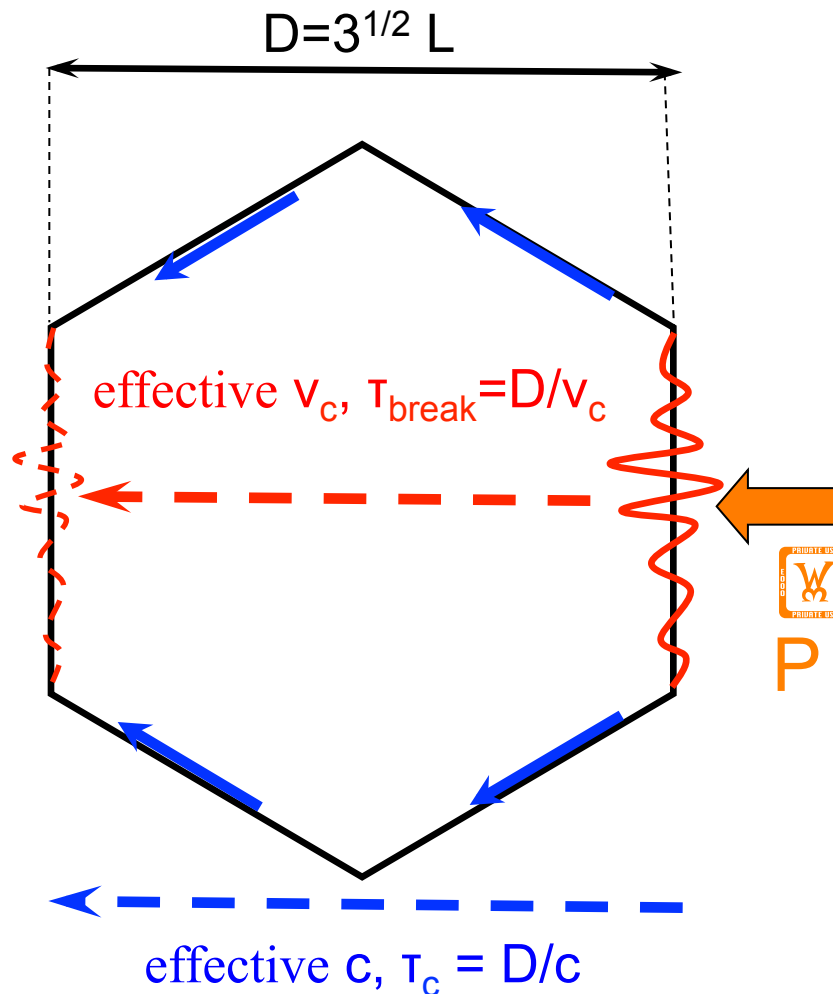
$$c \approx \left( \frac{6\gamma}{\varepsilon_2 \rho L} \right)^{1/2}$$

For our  $a \approx 140\mu\text{m}$ ,  $b=1\text{mm}$ ,  
 $L=2\text{mm}$ :

$\varepsilon_2 \approx 0.001$ ,  $c \approx 9.8\text{ m/s}$  !

[Schwartz & Princen, JCIS 1987]

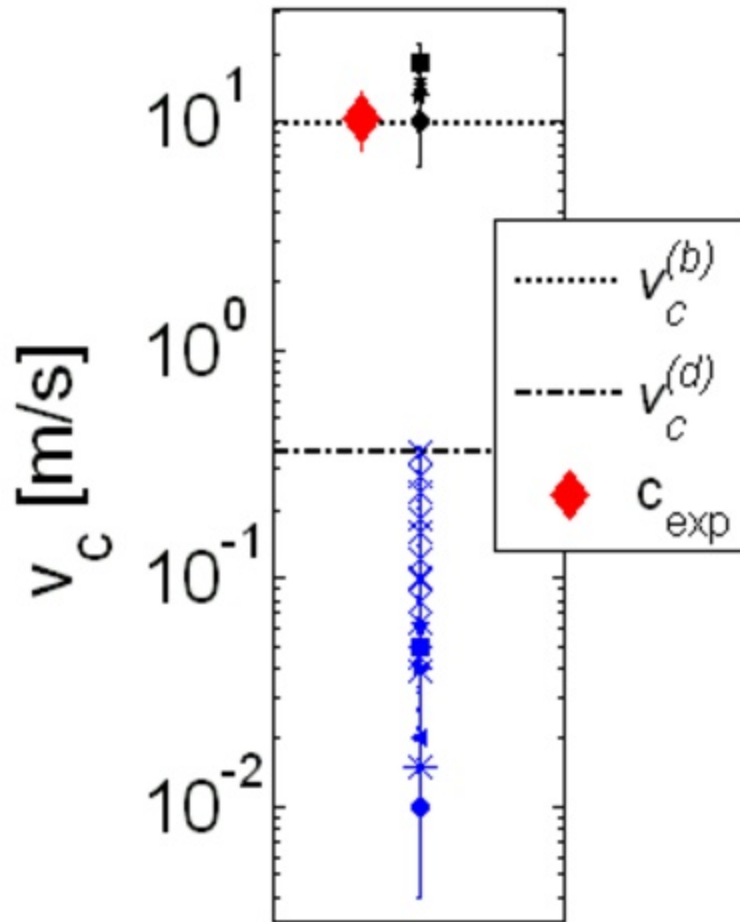
# Brittle threshold velocity



if  $\tau_{\text{break}} > \tau_c$ ,  $\Delta P$  cannot be maintained: no brittle crack !

minimum (lower bound) speed:  $v_c^{(b)} = c$ , brittle cracks are supersonic

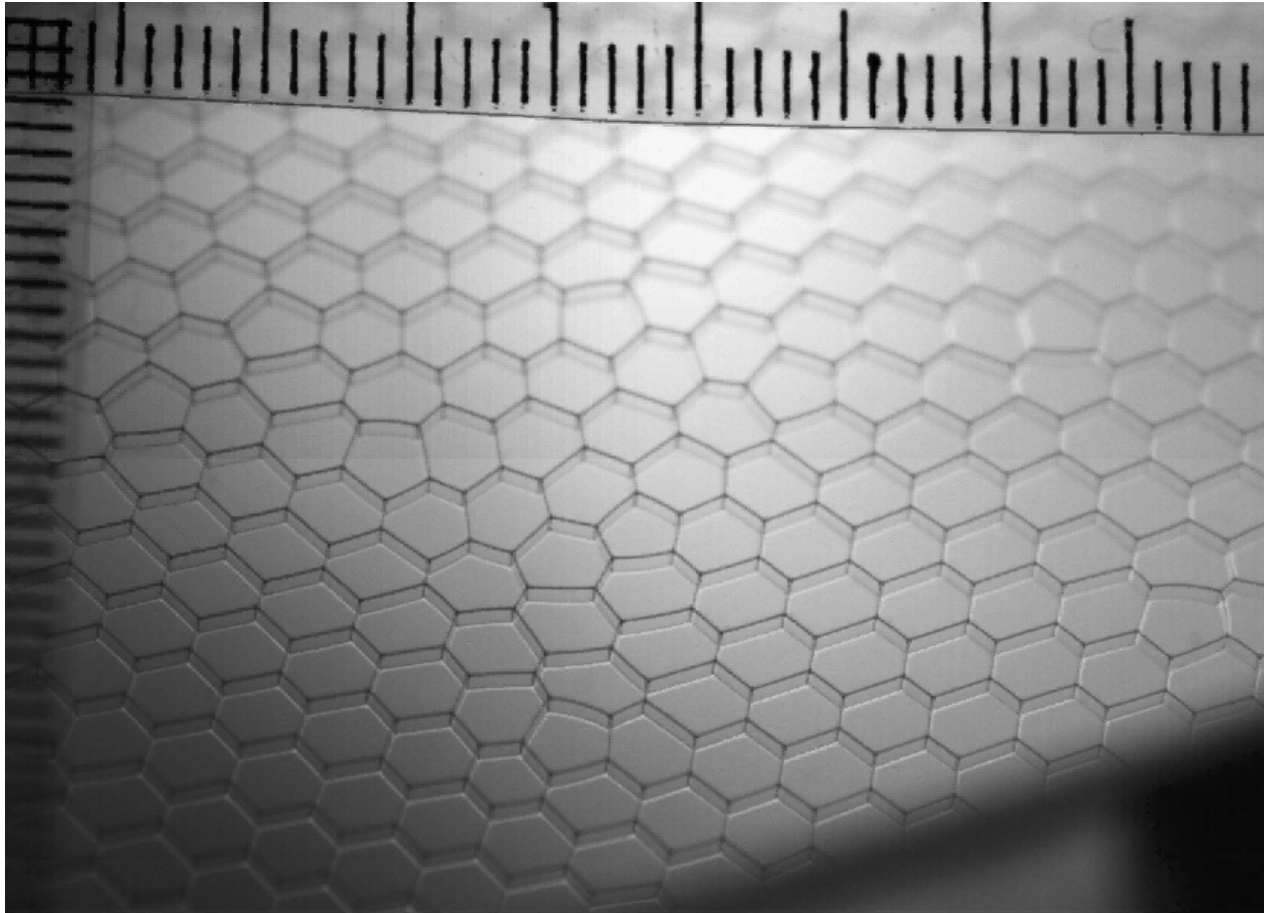
# The velocity gap



- All ductile velocities below  $v_c^{(d)}$
- All brittle velocities above  $v_c^{(b)}$
- All brittle velocities supersonic



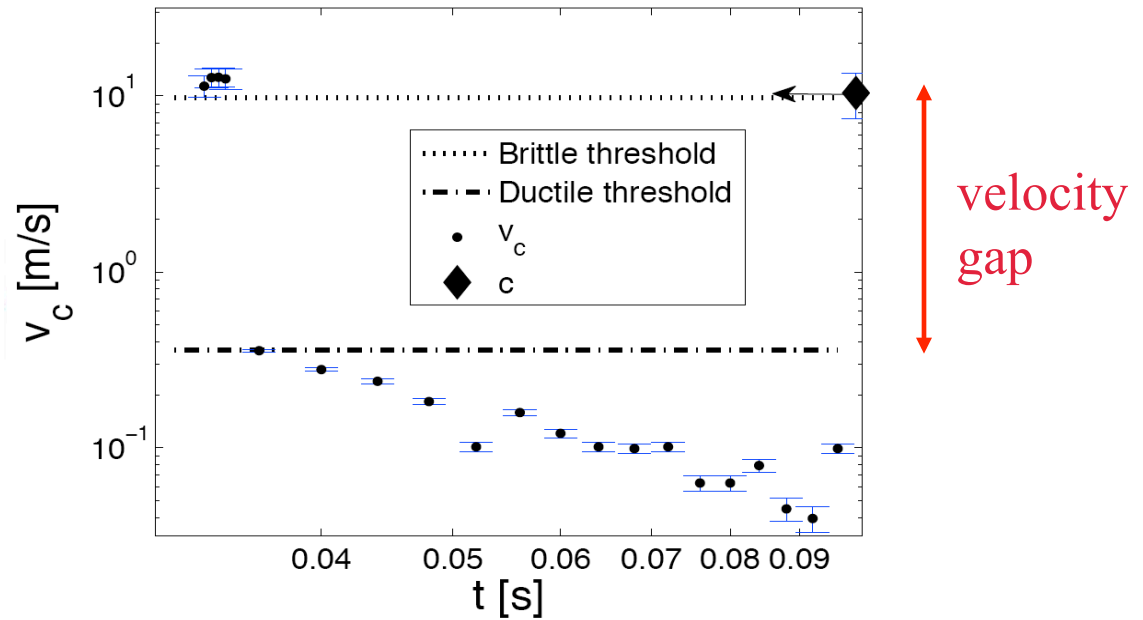
# Spontaneous Brittle-to-Ductile Transition



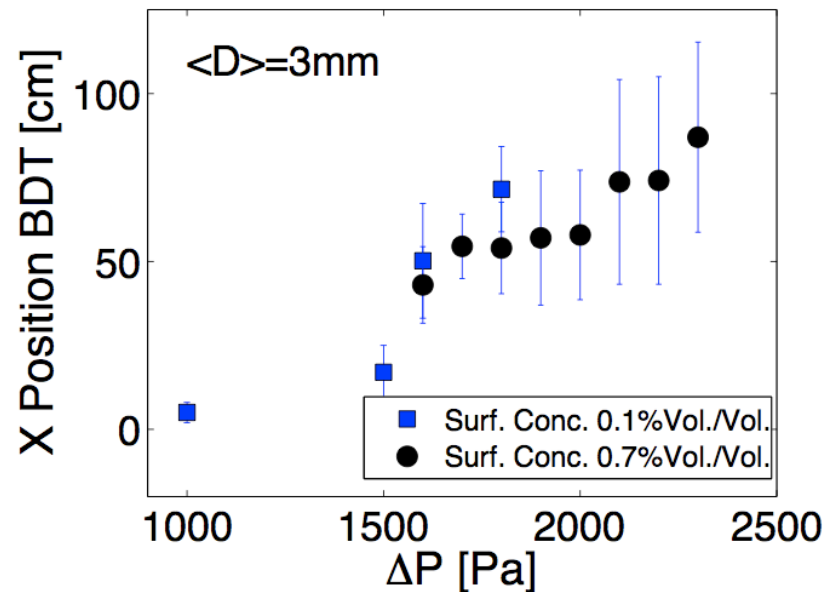
[2400fps]

# Spontaneous Brittle-to-Ductile Transition

Dynamics obeys velocity gap!



Location of BDT varies with  $\Delta P$



# Conclusions

- Quasi-2D foam displays two distinct failure modes – brittle and ductile
- Fracture velocities are explained from first principles
- Velocity gap: a consequence of limiting velocities
- Spontaneous Brittle-to-Ductile transition because of dissipative effects

Future work: Effects of defects, microstructure