Electronic Liquid Crystal Phases in Strongly Correlated Systems

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Collaborators

S. Kivelson, V. Emery, E. Berg, D. Barci, E.-A. Kim, M. Lawler, T. Lubensky, V. Oganesyan, K. Sun, C. Wu, S. C. Zhang, J. Eisenstein, A. Kapitulnik, A. Mackenzie, J. Tranquada

•S. Kivelson, E. Fradkin and V. J. Emery, *Electronic Liquid Crystal Phases of a Doped Mott Insulator*, Nature **393**, 550 (1998)

•S. Kivelson, I. Bindloss, E. Fradkin, V. Oganesyan, J. Tranquada, A. Kapitulnik and C.Howald, *How to detect fluctuating stripes in high tempertature superconductors*, Reviews of Modern Physics **75**, 1201 (2003)

•E. Fradkin, Lectures at the Les Houches Summer School on ``Modern theories of correlated electron systems", Les Houches, France (2009), to appear in the Proceedings of Les Houches; arXiv:1004.1104

•E. Fradkin, S. Kivelson, M. Lawler, J. Eisenstein, and A. Mackenzie, *Nematic Fermi Fluids in Condensed Matter Physics*, Annual Reviews of Condensed Matter Physics **1**, 153 (2010)

•E. Berg, E. Fradkin, S. Kivelson, and J. Tranquada, *Striped Superconductors: How the cuprates intertwine spin, charge, and superconducting orders,* New Journal of Physics **11**, 115009 (2009).

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- In lattice systems these symmetries are discrete
- In addition to their charge and spin orders, these phases may also be superconducting

How Liquid Crystals got an $\hbar$
or
Soft Quantum Matter

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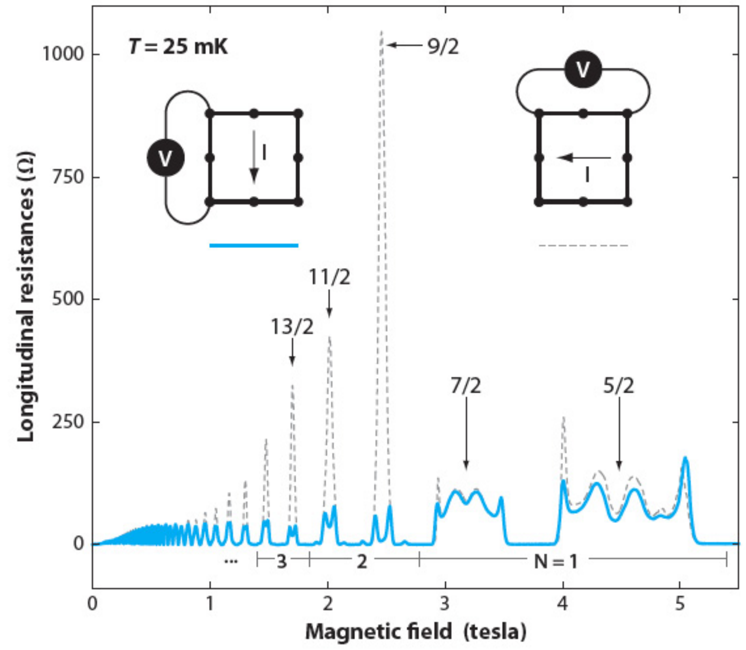
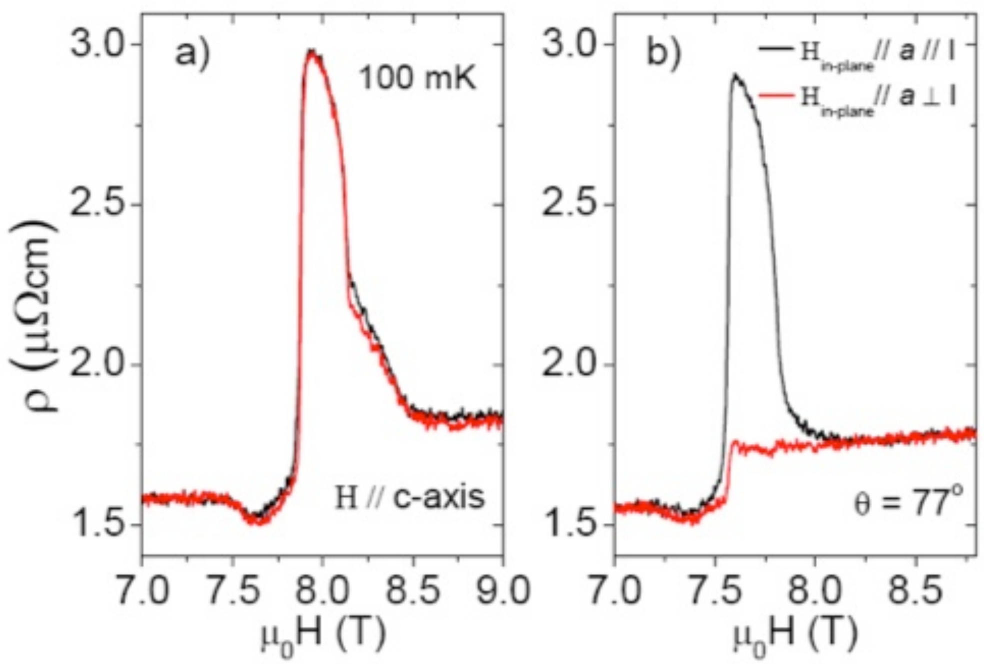
- Is charge order a friend or a foe of high T_c

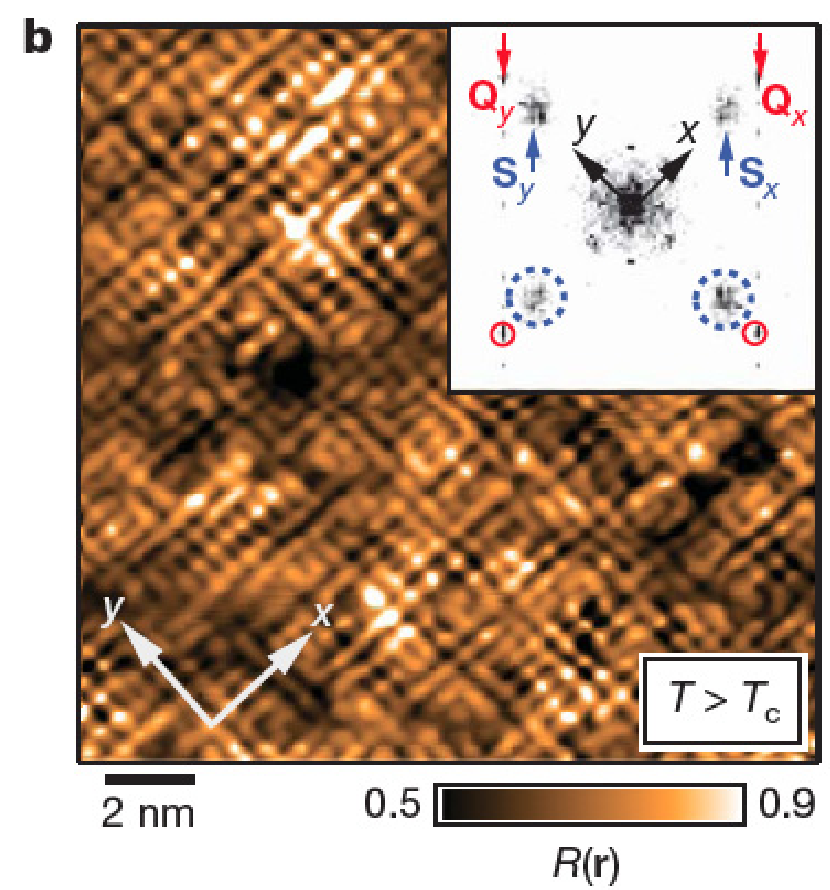
Nematic Order



2DEG in large magnetic fields
J. Eisenstein et al, 1998

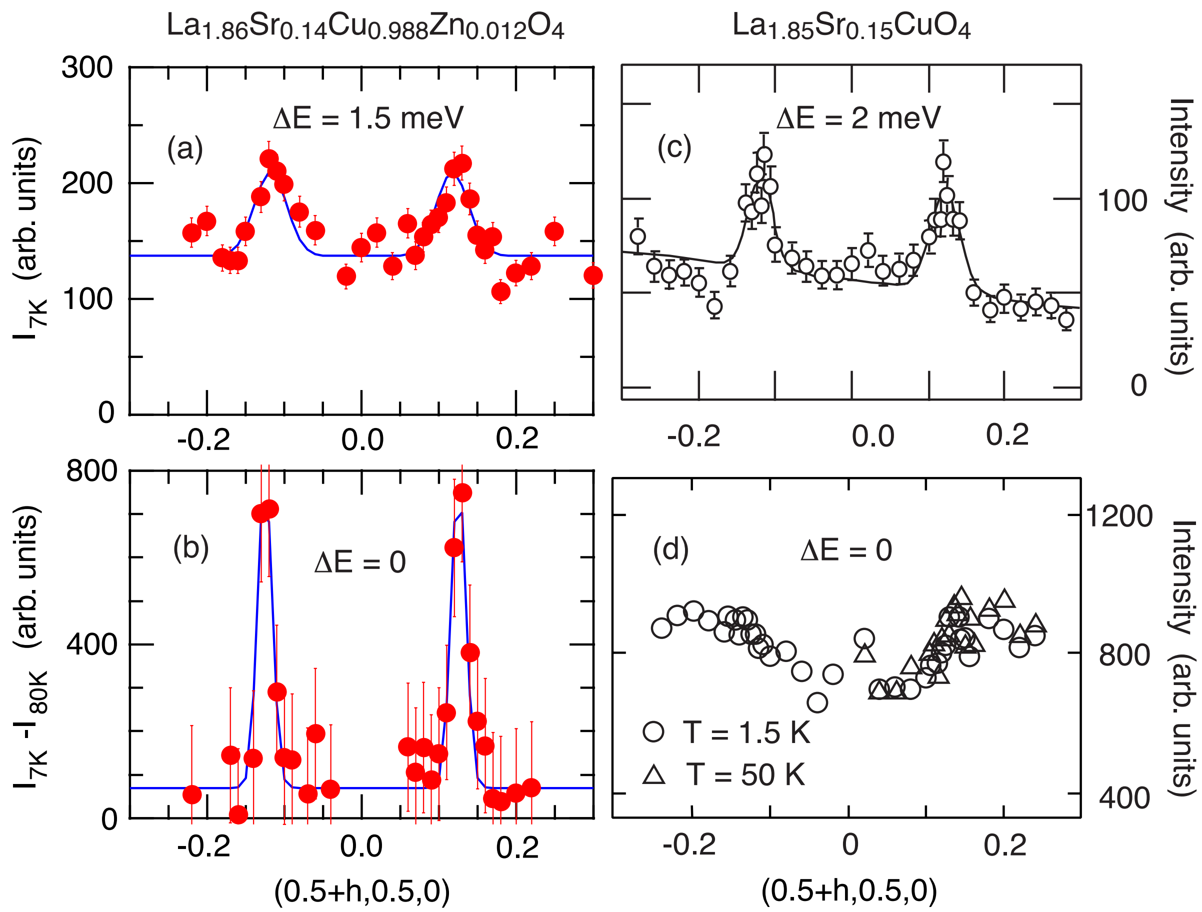


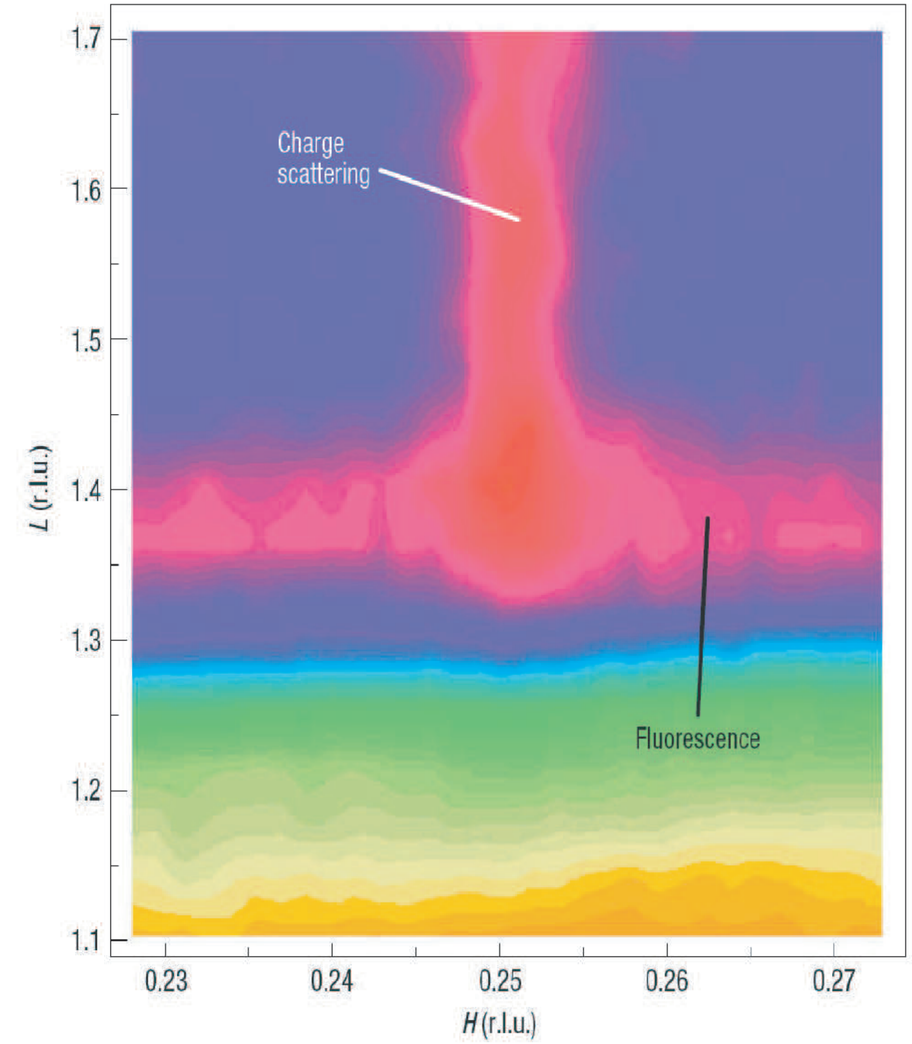
$Sr_3Ru_2O_7$, Mackenzie et al, 2007



BSCCO, Davis et al, 2010

Stripe Order in HTSC





"Fluctuating Stripes" in LSCO
Tranquada et al, 1995

Static Charge Stripes in LBCO
Abbamonte et al 2004

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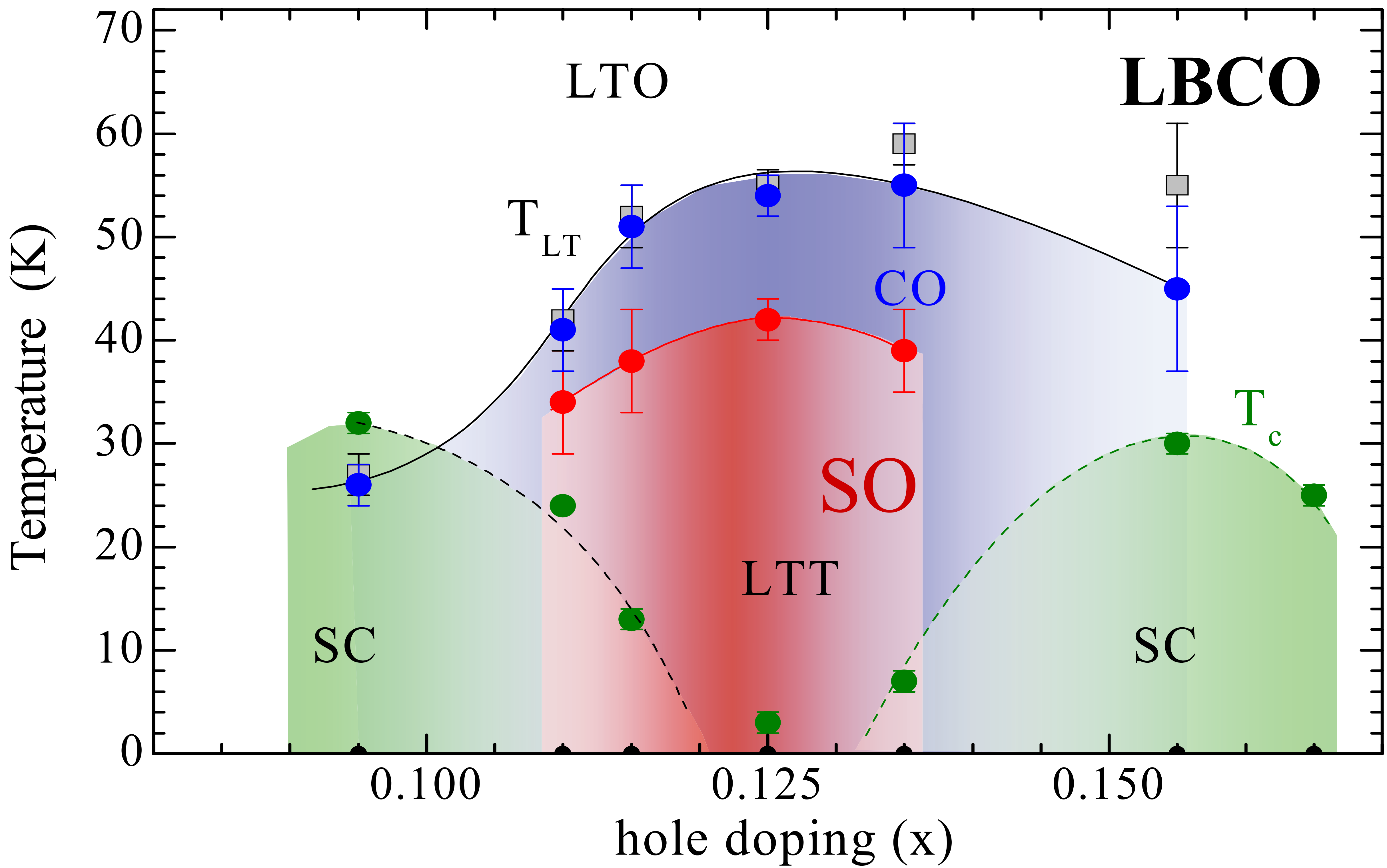
- It has a very low T_c near x=1/8 where it shows static stripe order in the LTT crystal structure
- Experimental evidence for superconducting layer decoupling in LBCO at x=1/8
- Layer decoupling, long range charge and spin stripe order and superconductivity: a novel striped superconducting state, a ***Pair Density Wave***, in which charge, spin, and superconducting orders are intertwined!

Phase Diagram of LBCO



M. Hücker et al (2009)

Li *et al* (2007): Dynamical Layer Decoupling in LBCO

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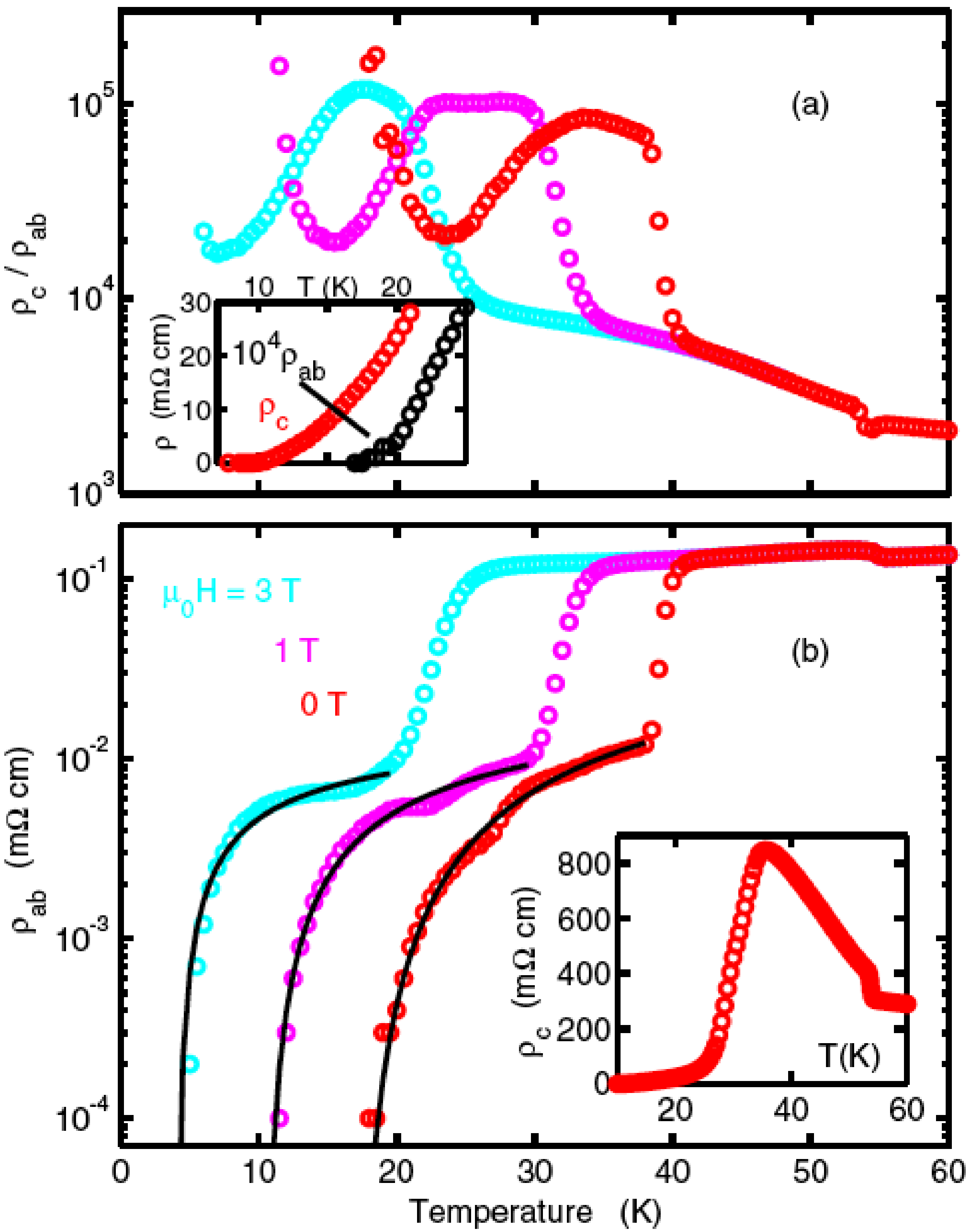
- Meissner state only below $T_c= 4$K

Anisotropic Transport Below the Charge Ordering transition



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- This can only happen if there is a special symmetry of the superconductor in the striped state that leads to an almost complete cancellation of the c-axis Josephson coupling.

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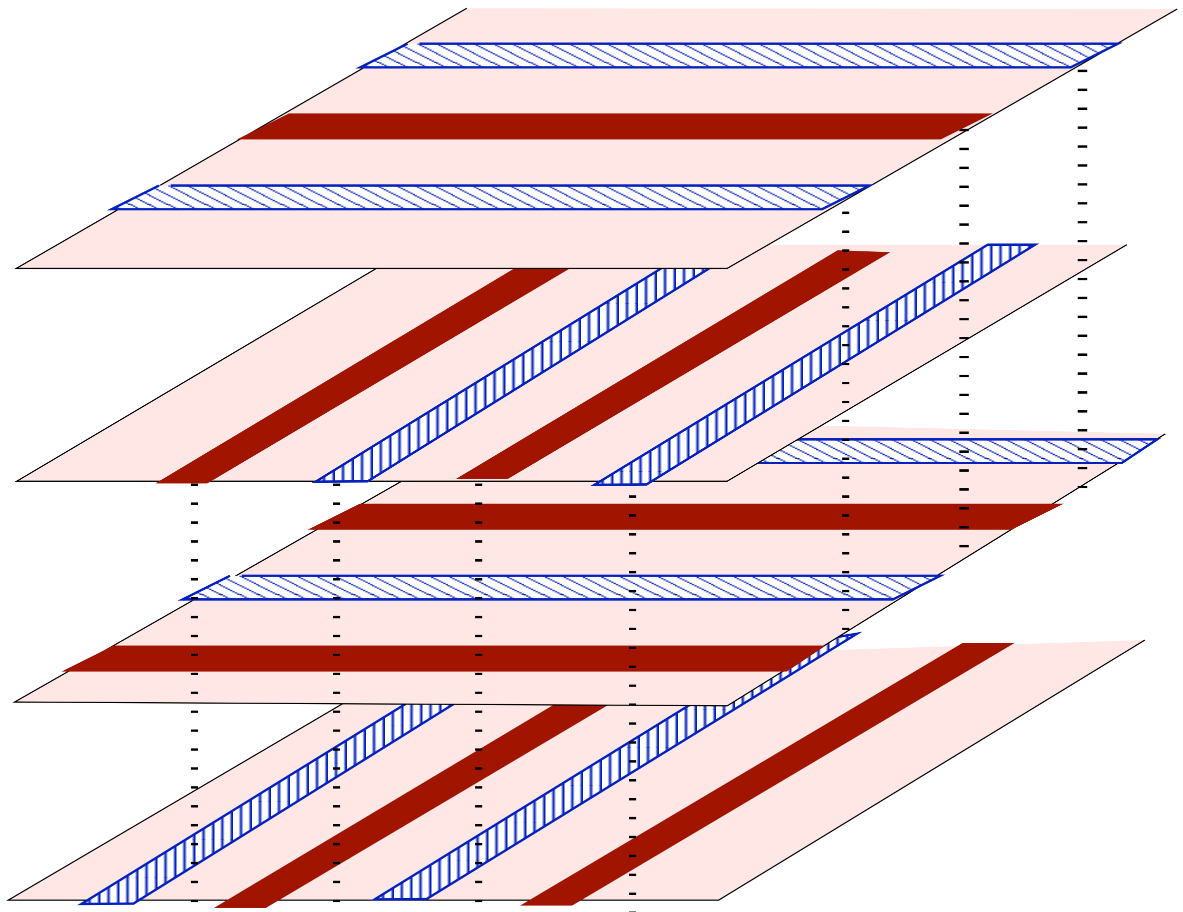
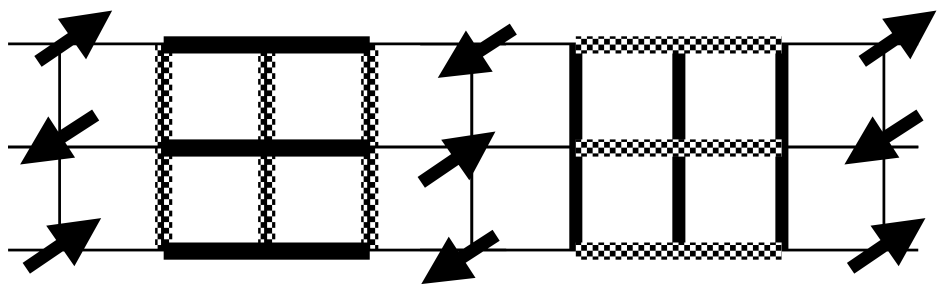
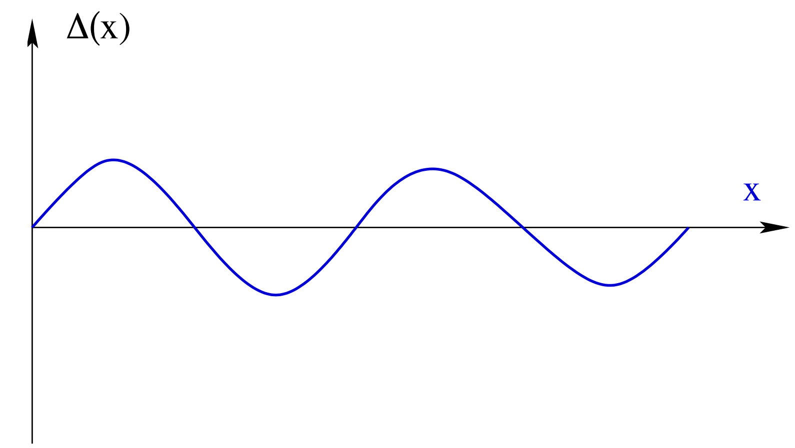
- We propose that the superconducting order is also striped and also suffers a π phase shift.
- The superconductivity resides in the spin gap regions and there is a π phase shift in the SC order across the AFM regions

Period 4 Striped Superconducting State

E. Berg et al, 2007







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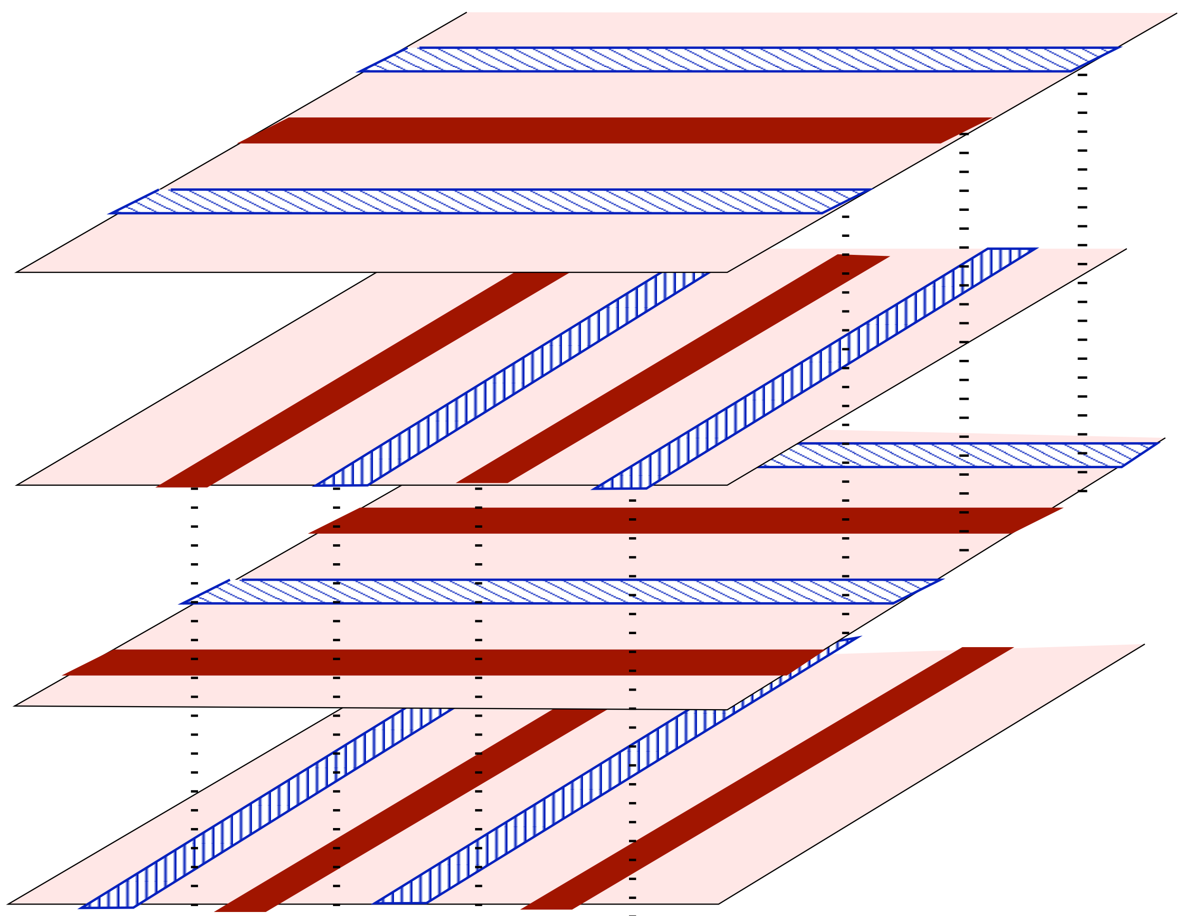

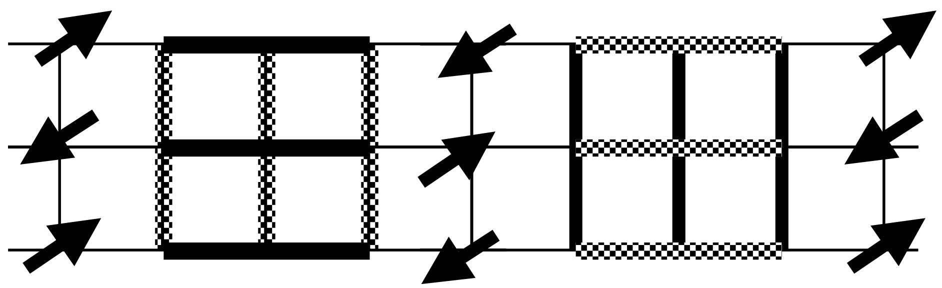

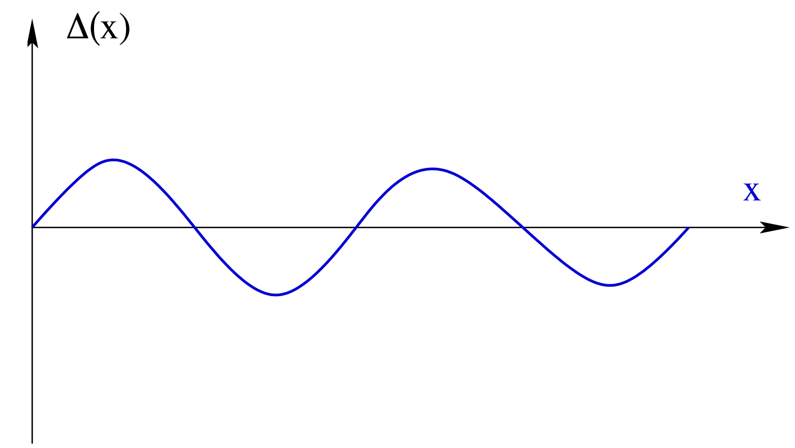

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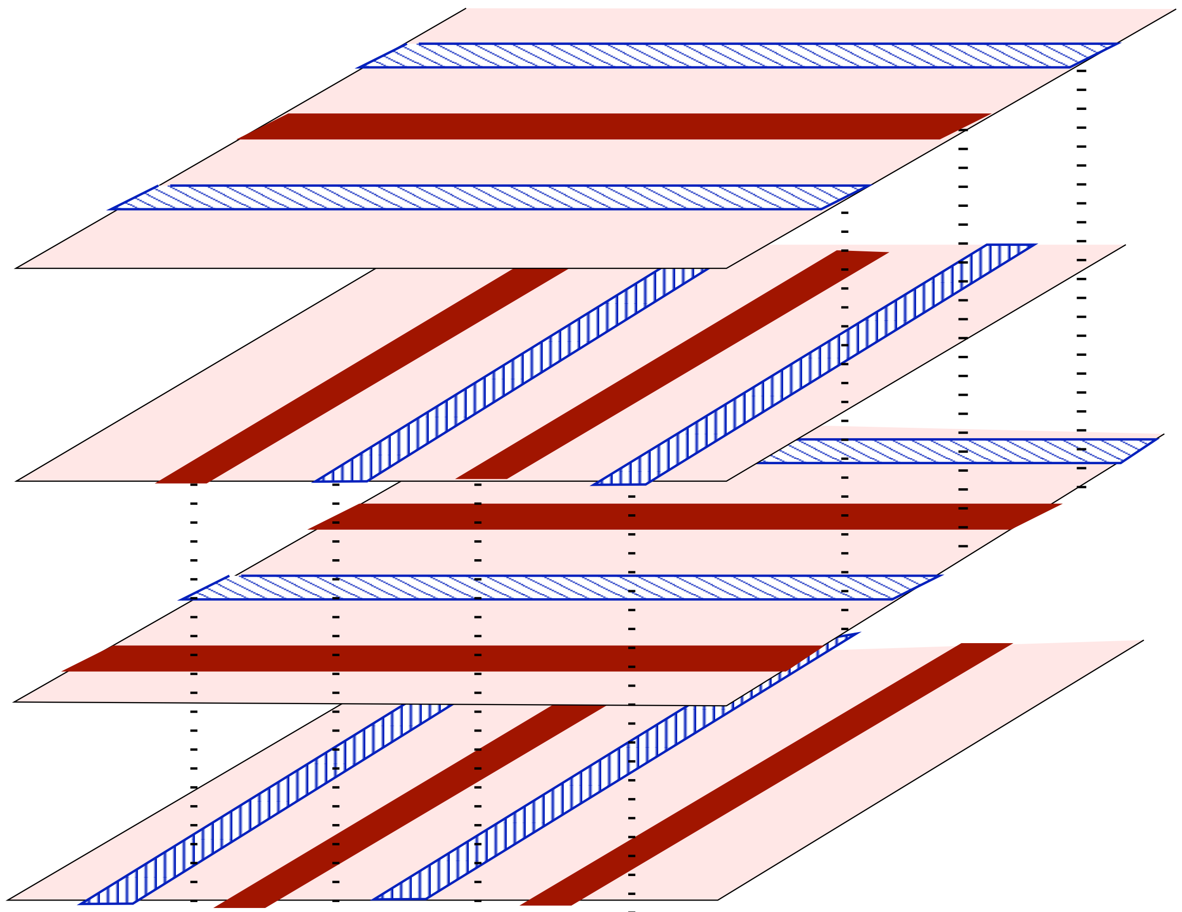

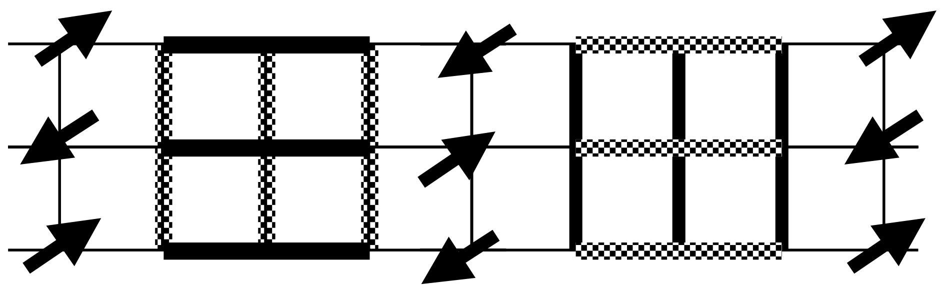

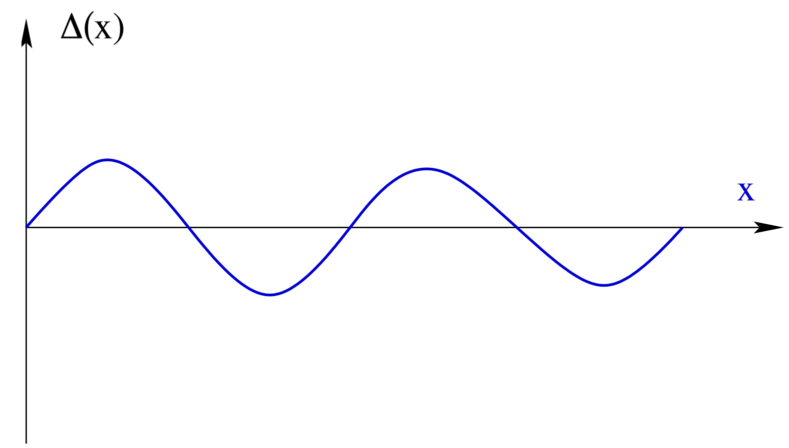

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- A state of this type was found in variational Monte Carlo (Ogata *et al* 2004) and MFT (Poilblanc *et al* 2007)

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- Defects and/or discommensurations gives rise to small Josephson coupling J_0 neighboring planes

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- $F'_3 = g_4 \, [\Delta_4^* \, (\Delta_{\mathbf{Q}} \Delta_{-\mathbf{Q}} + \text{rotation}) + \text{c.c.}]$
- Striped SC order (PDW) $\Rightarrow$ uniform charge 4e SC order

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- If $\Delta_0 \neq 0$ and $\Delta_{\mathbf{Q}} \neq 0 \Rightarrow$ there is a $\rho_{\mathbf{Q}}$ component of the charge order!
- The small uniform component Δ_0 removes the sensitivity to quenched disorder of the PDW state

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- ϕ and θ_{-} are locked $\Rightarrow$ topological defects of ϕ and θ_{+}

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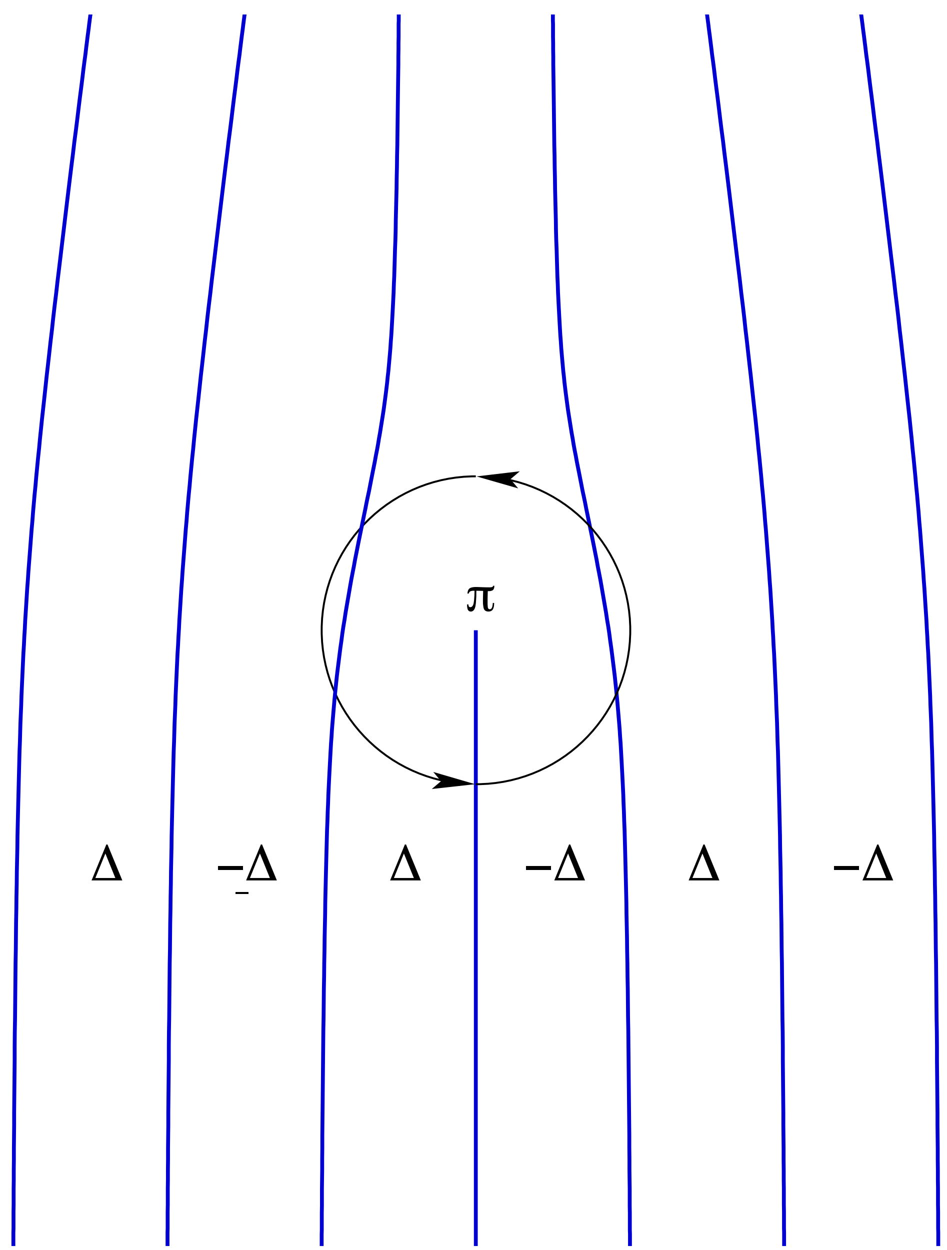
- All three topological defects have logarithmic interactions

Half-vortex and a Dislocation



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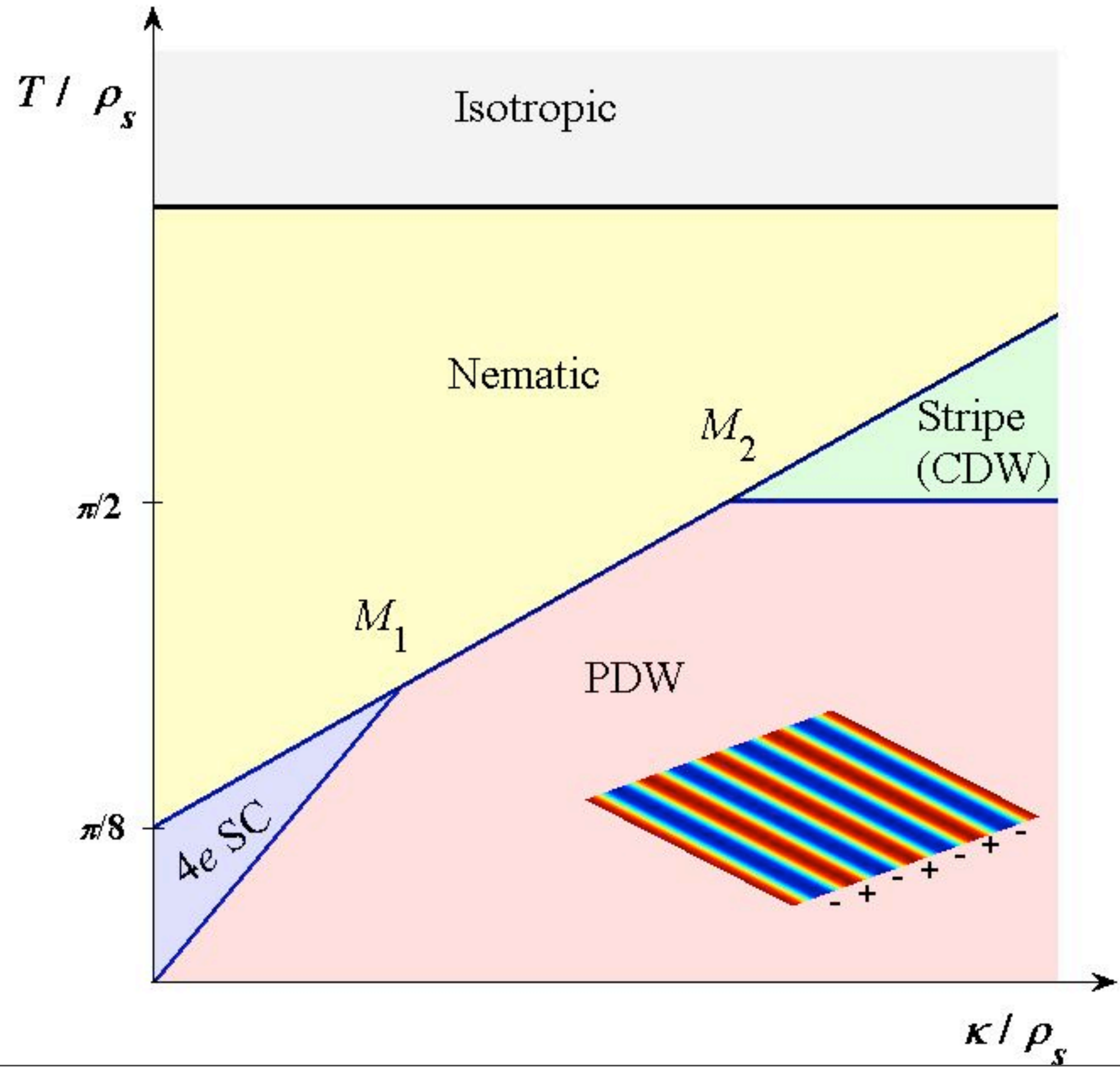
- Phases: PDW, Charge 4e SC, CDW, and normal (Ising nematic)

Schematic Phase Diagram



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- the charge 4e SC order is unaffected by the Bragg glass of the pinned CDW

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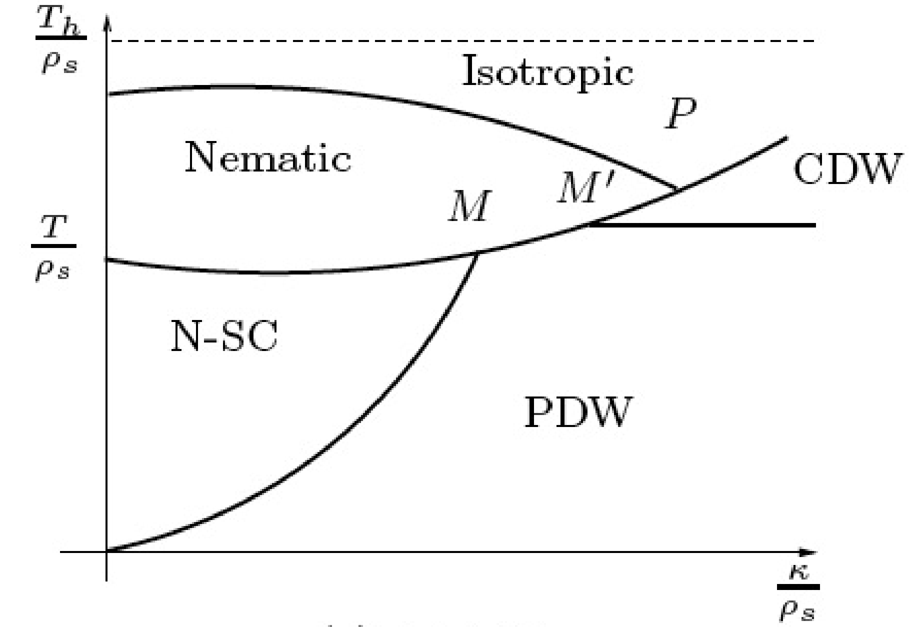
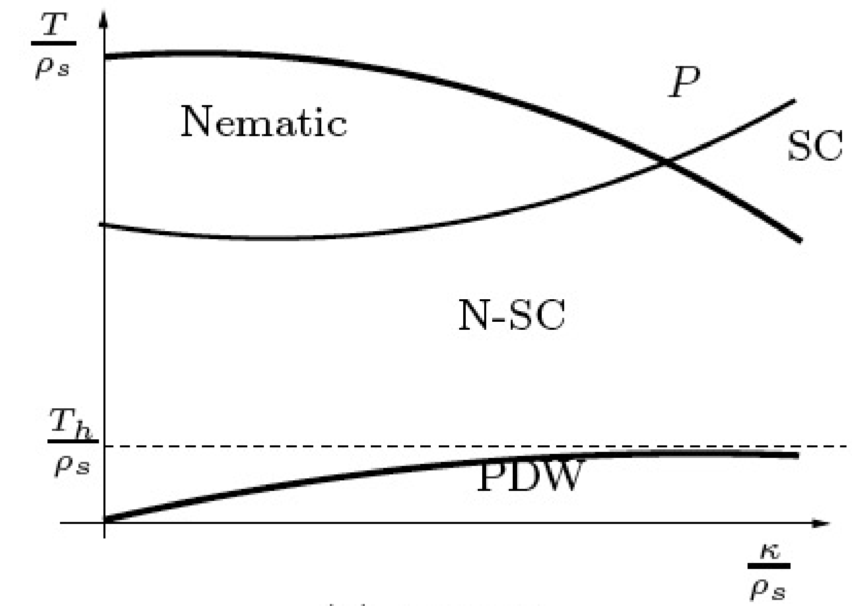
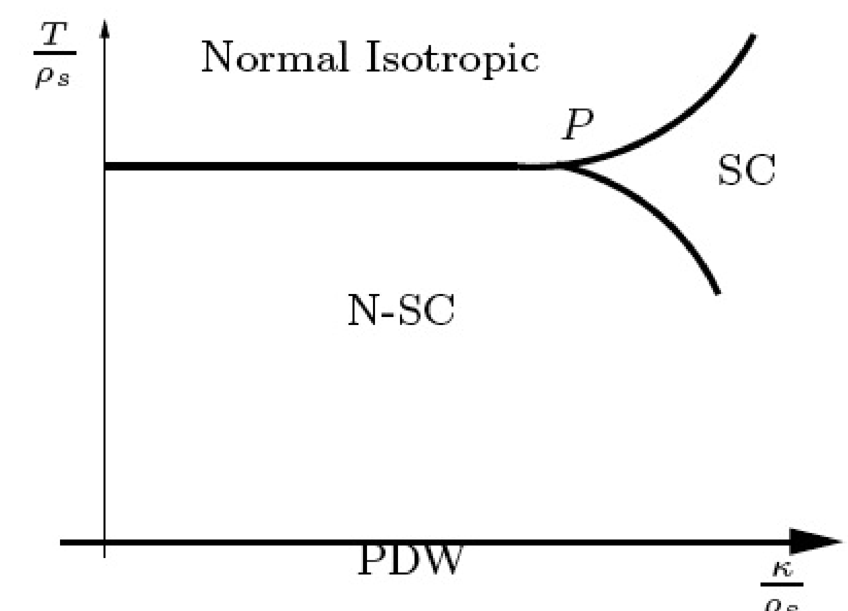
- For a square lattice the point group is C_4 and the nematic-isotropic transition is 2D Ising
- As the coupling to the lattice is weakened the structure of the phase diagram changes and the nematic transition is pushed to lower temperatures



(a) high T_h



(b) Low T_h



(c) $T_h = 0$

PDW melting as a McMillan-de Gennes theory

- The electronic nematic order parameter $N_{ij} = N(\hat{n}_i \hat{n}_j - \delta_{ij}/2)$ is an antisymmetric traceless tensor
- Its presence amounts to a local change of the geometry, the local metric, seen by the charge 4e SC order parameter Δ_{4e} the CDW order parameter ρ_K and the PDW order $\Delta_{\pm Q}$
- Metric tensor: $g_{ij} = \delta_{ij} + \lambda N_{ij}$
- Novel derivative couplings in the free energy

$$F_d = \int dx^2 \sqrt{\det g}\; g^{ij} \left\{ \left(D_i^{cdw} \rho_K\right)^* \left(D_j^{cdw} \rho_K\right) + \right.$$
$$\left. + \left(D_i^{sc} \Delta_{\pm Q}\right)^* \left(D_j^{sc} \Delta_{\pm Q}\right) + \left(D_i^{4e} \Delta_{4e}\right)^* \left(D_j^{4e} \Delta_{4e}\right) \right\}$$

$$D_i^{sc} = \nabla_i - i2eA_i \pm iQ\, \delta n_i$$
$$D_i^{cdw} = \nabla_i + i2Q\, \delta n_i$$
$$D_i^{4e} = \nabla_i - i4eA_i$$

- Deep in the PDW phase the amplitude of the SC, CDW and nematic order parameters are fixed but their phases are not

$$F = \int d^2x \, \left\{ \frac{\rho_s}{2} g^{ij} \, (\partial_i \theta - 2eA_i)(\partial_j \theta - 2eA_j) \right.$$
$$+ \frac{\kappa}{2} g^{ij} \, (\partial_i \varphi + Q\delta n_i)(\partial_j \varphi + Q\delta n_j)$$
$$\left. + K_1 \, (\nabla \cdot \delta \mathbf{n})^2 + K_3 \, (\nabla \times \delta \mathbf{n})^2 + h |\delta \mathbf{n}|^2 \right\}$$

$$\rho_s = |\Delta_{\pm Q}|^2 + |\Delta_{4e}|^2 \qquad \kappa = |\Delta_{\pm Q}|^2 + |\rho_K|^2$$

Effective CDW stiffness reduced by gapped nematic fluctuations

$$\kappa_{\text{eff}} = \frac{\kappa}{1 + \kappa Q^2 / h}$$

Deep in the charge 4e nematic superconducting phase

$$F = \int d^2x \left\{ K|\nabla\alpha|^2 + \rho_s|\nabla\theta|^2 + \frac{\rho_s N}{2} \left(\mathbf{n}\cdot\nabla\theta\right)^2 \right\}$$

- α and θ are the orientational nematic fluctuations and SC phase fluctuations
- in the absence of orientational pinning SC (half) vortices and nematic disclinations *attract each other* due to the fluctuations of the local geometry (nematic fluctuations): the only topological excitations are disclinations bound to half vortices
- If the nematic fluctuations are gapped by lattice effects there is a crossover to the pinned case discussed before
- Fermionic SC quasiparticles couple to the pair field which is uncondensed in the charge 4e SC. This leads to a "pseudogap" in the quasiparticle spectrum

Conclusions

- The static stripe order seen in LBCO and other LSCO related materials can be understood in terms of the PDW, a state in which charge, spin and superconducting orders are intertwined rather than competing
- This state competes with the uniform d-wave state
- Several predictions can be tested experimentally
- The observed nematic order is a state with "fluctuating stripe order", i.e. it is a state with melted stripe order. Is the nematic state a melted PDW?
- Is the PDW peculiar to LBCO? If so why? If it is generic, why?
- A microscopic theory of the PDW is needed. This state is not accessible by a weak coupling BCS approach. Strong evidence in 1d Kondo-Heisenberg chain and ladders.