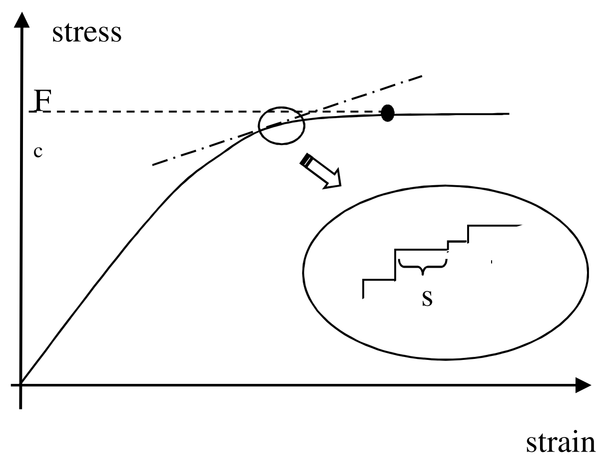
Plastic deformation and crackling noise:

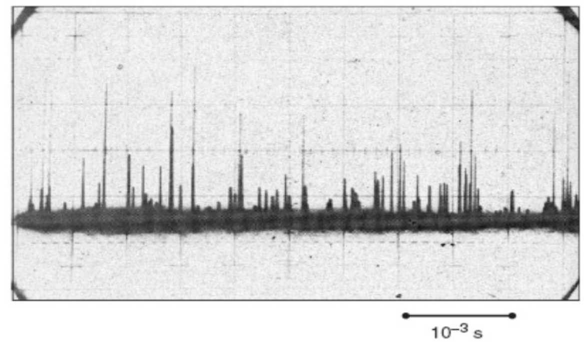
- Analytics: Braden Brinkman, Yehuda Ben-Zion, Jonathan Uhl
- Simulation: Georgios Tsekenis, Thomas Fehm, Patrick Chan, Jonathan Dantzig, Nigel Goldenfeld
- Experiment and Analysis: Nir Friedman, Andrew Jennings, Georgios Tsekenis, Ju-Yung Kim, Julia Greer, Tyler Earnest, Bob Behringer, Bob Hartley, Karen Daniels.



Funding/Equipment:
NSF, MCC, UIUC, SCEC, MGA

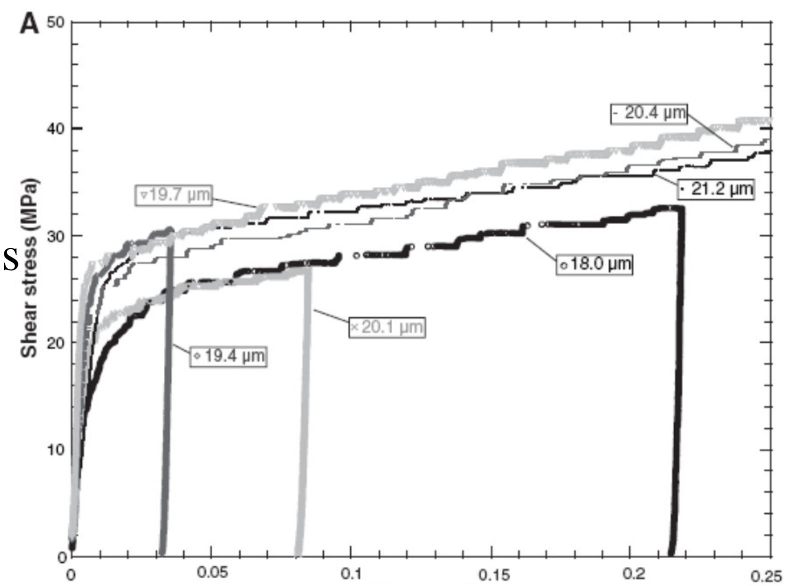
1. Motivation: Slowly sheared metals show discrete jumps !

Acoustic emission



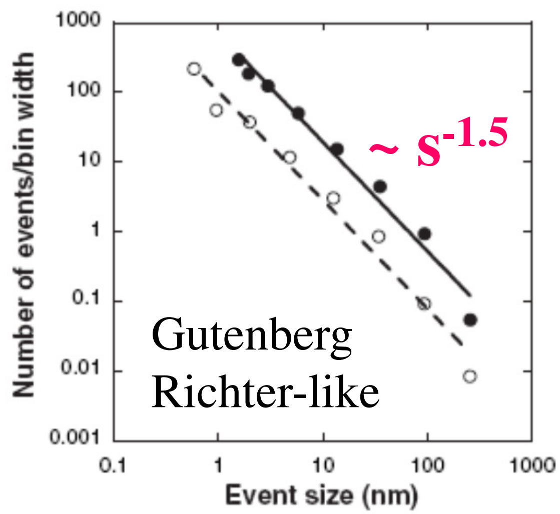
Imanaka et al.

stress



strain

jump size distribution



s

Universal!

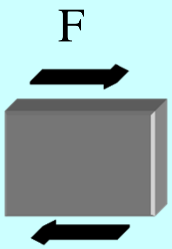
(Dimiduk, et al Science 2006)

2. THEORY (PRL 2009, 2010)

Simple Analytical MODEL, 1 tuning parameter (weakening ε)

➡ EXACT RESULTS for 2 different boundary conditions:

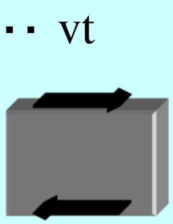
Main Results:



- For slowly increasing stress boundary condition:

brittle ($\varepsilon>0$), ductile ($\varepsilon=0$), & hardening materials ($\varepsilon<0$),

Avalanche size- and duration-distributions, power spectra, …



- For slow tangential velocity boundary condition:

Models earthquake fault zones & fixed strainrate experiments

- **Agrees with experiments and with simulations at zero Temperature and at finite Temperature.**

Dynamic equation for slip evolution in heterogeneous medium:

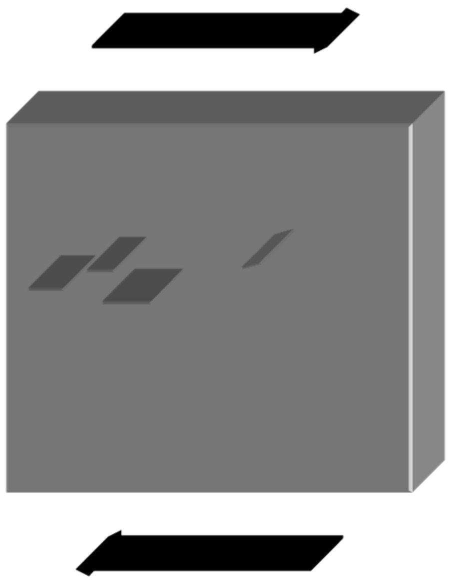
$$\eta\, \partial u(\mathbf{r},\mathbf{t})/\partial t = F + \sigma_{int}(\mathbf{r},t) - f_R[u, r, \text{history}, \varepsilon]$$

interaction:

Pinning due to heterogeneities

$$\sigma_{int}(\mathbf{r},t) = \int_{-\infty}^{t} dt' \int d^d r' \; J(\mathbf{r}-\mathbf{r}', t-t') \times [u(\mathbf{r}',t') - u(\mathbf{r},t)]$$

F



Renormalization Group:

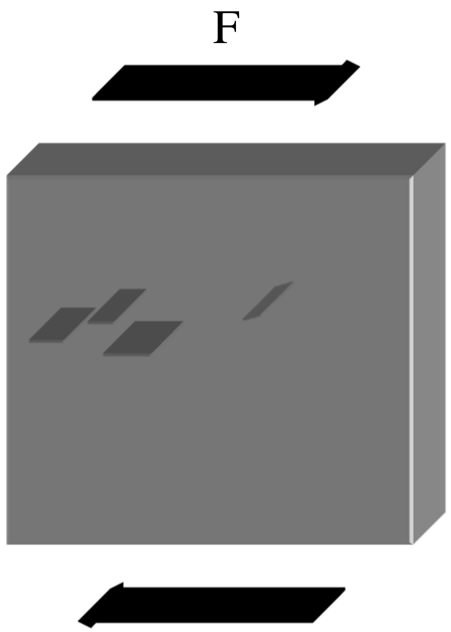
Interaction sufficiently long range $\int dt\, J(\mathbf{r},t) \sim r^{-2}$

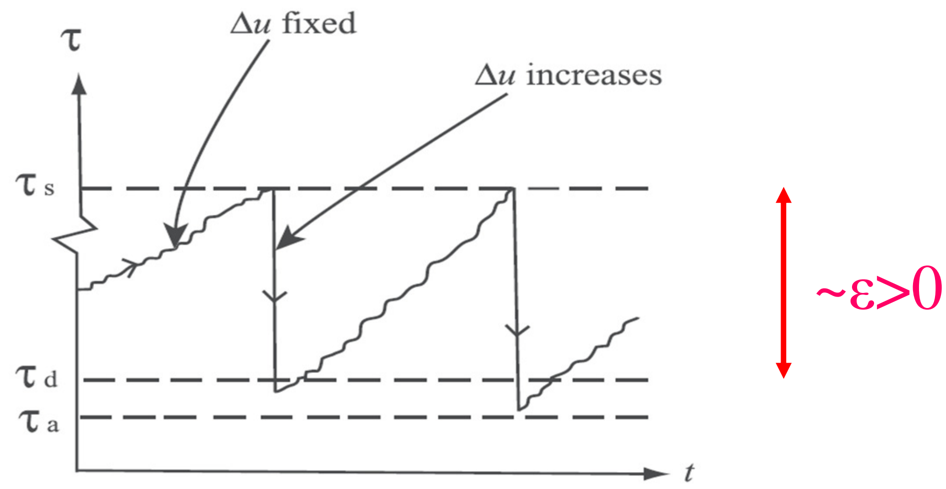
MEAN FIELD THEORY

EXACT RESULTS !

Related models: Chen, Bak, Obukhov (PRA 1991), Zaiser, Adv. of Phys, **55**, 185-245 (2006).

Threshold pinning (f_R[u, r, history,ε])





$$\varepsilon = (\tau_s - \tau_d)/\tau_s = \text{dynamic weakening}$$

weakening (ε>0)

during failure avalanche:

failed regions get weakened by O(ε)

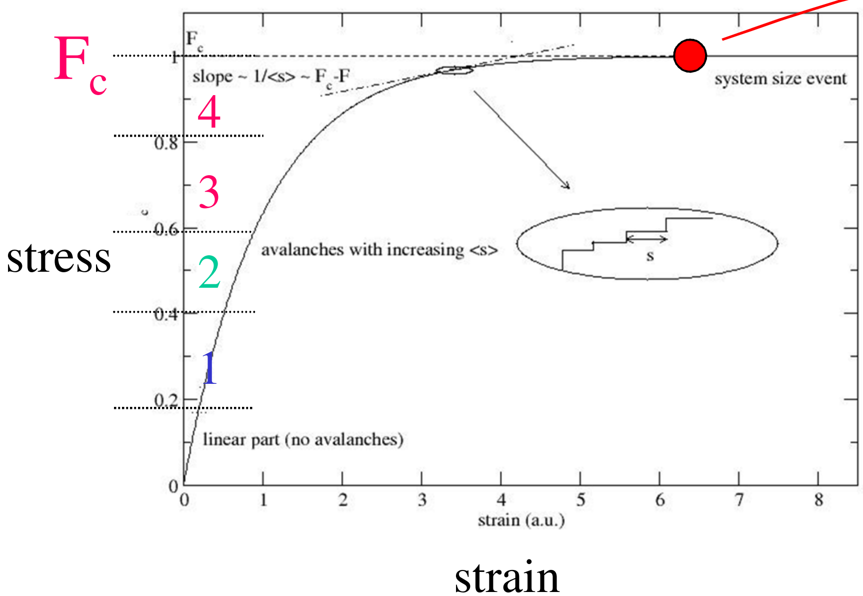
reheal to old strength after avalanche

hardening (ε<0)

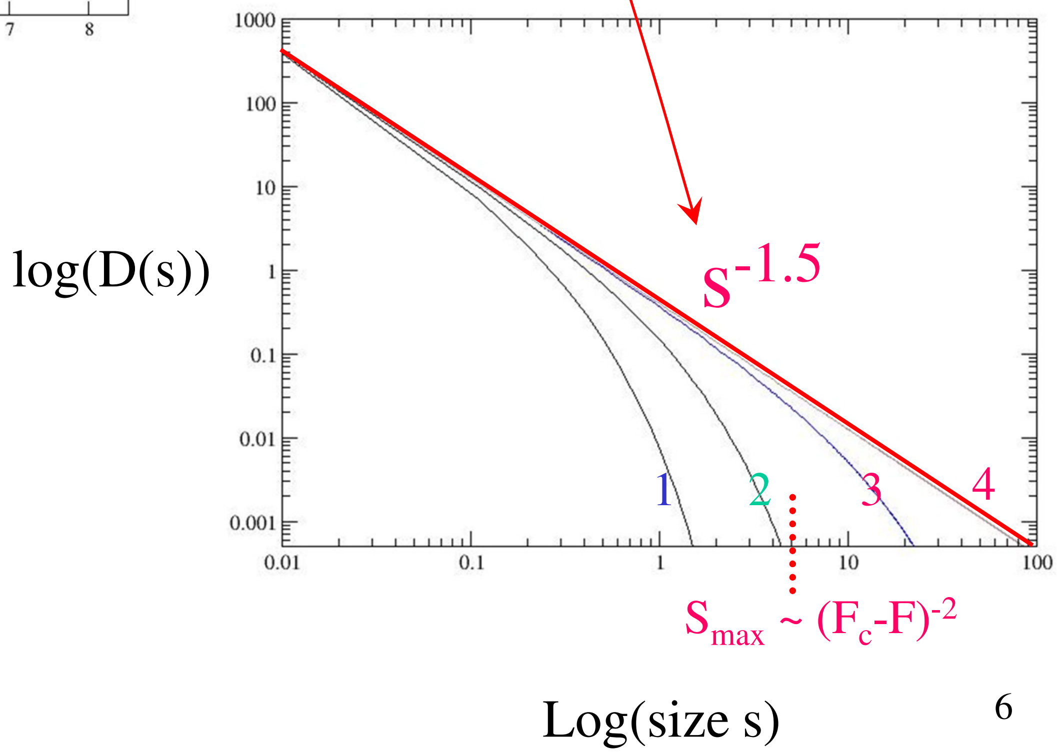
each failure event raises failure threshold everywhere by ~|ε/N|.
used to model aftershocks
(Mehta, KD, Ben-Zion, PRE 2006)

Fixed stress boundary condition: Results for ε=0:

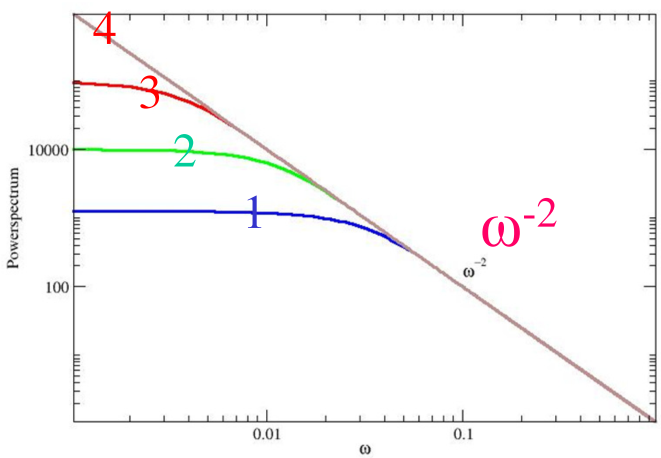
Stress strain curve



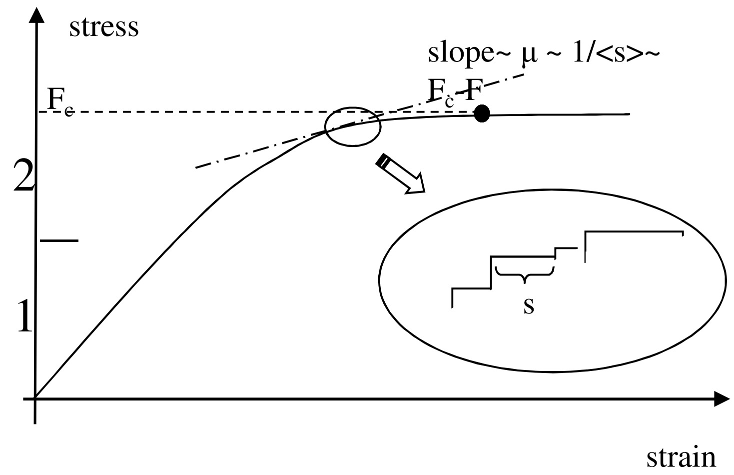
Avalanche size distribution:



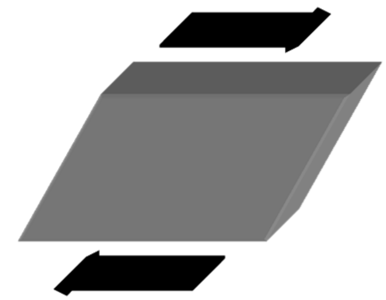
Power spectrum



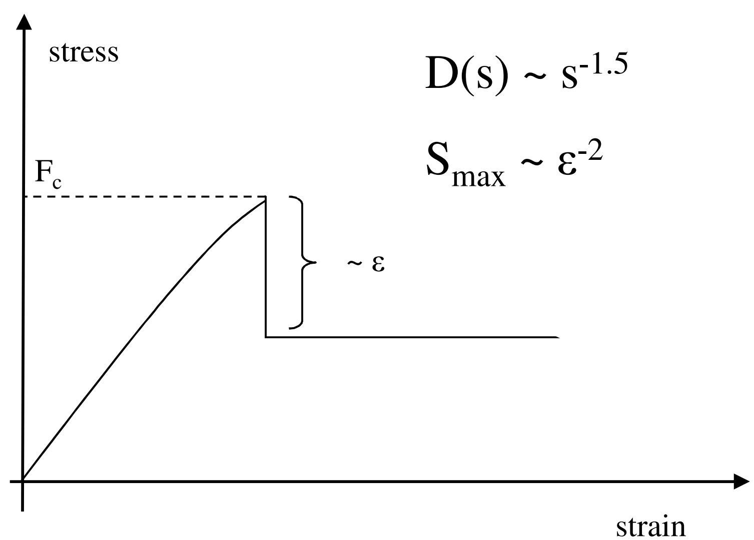
Without weakening ("ductile"): (ε=0):



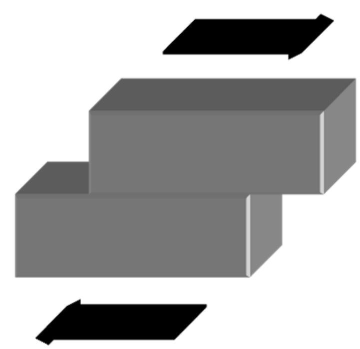
continuous depinning transition and distributed slip



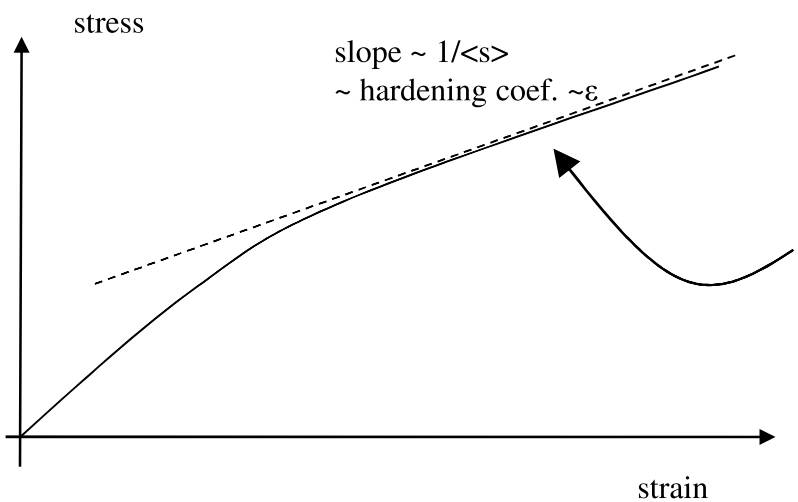
With weakening ("brittle"): (ε>0):



first order depinning transition and slip localization



Stress Strain Curve in the presence of hardening (“ductile”): $(\varepsilon < 0)$



$D(s) \sim s^{-1.5}$

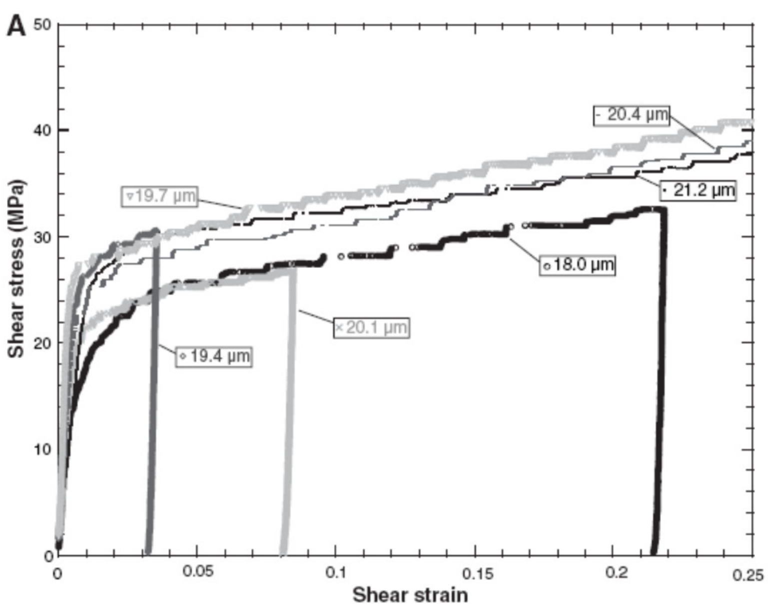
$P(\omega) \sim \omega^{-2}$

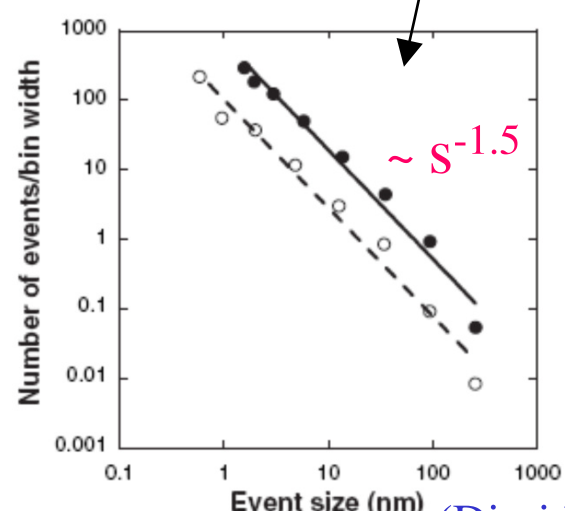
$D(T) \sim T^{-2}$

etc.

distributed deformation

Agrees with experiment

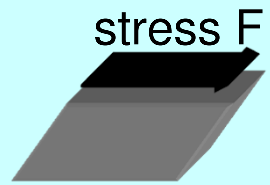




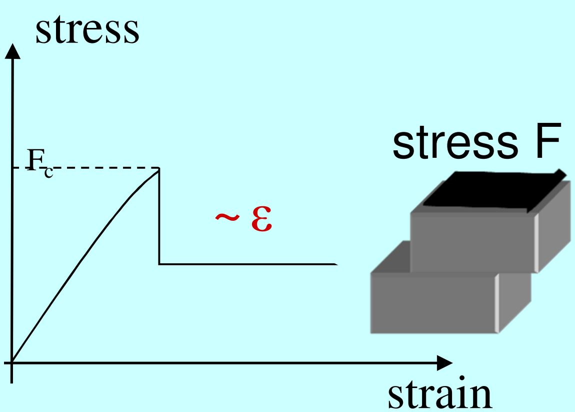
[(Dimiduk, et al Science 2006)](#)

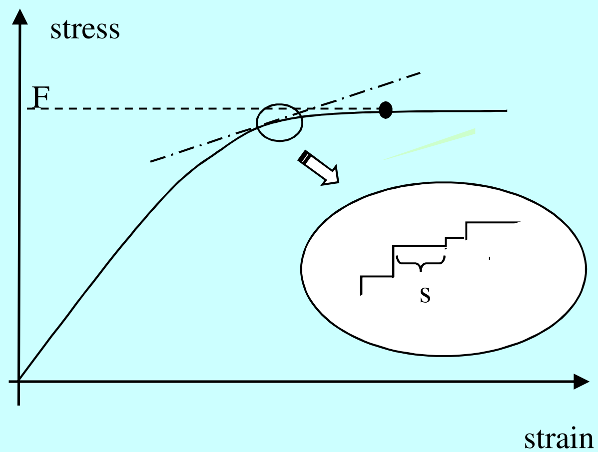
Simple *Analytic* Model for deformation under shear:

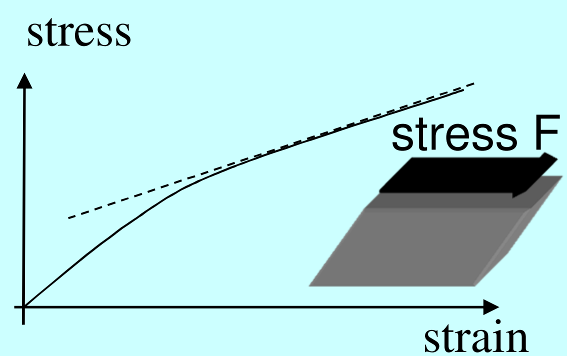
Main Results:



- For slowly increasing stress boundary condition:



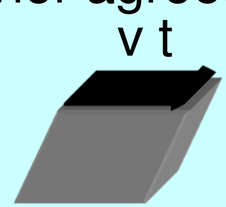




Brittle (ε>0) Plastic (ε=0) Hardening (ε<0)

Avalanche-size distributions (power laws with stress dependent cutoff)

Power spectra, Scaling Functions !! Scaling behavior agrees with experiments!

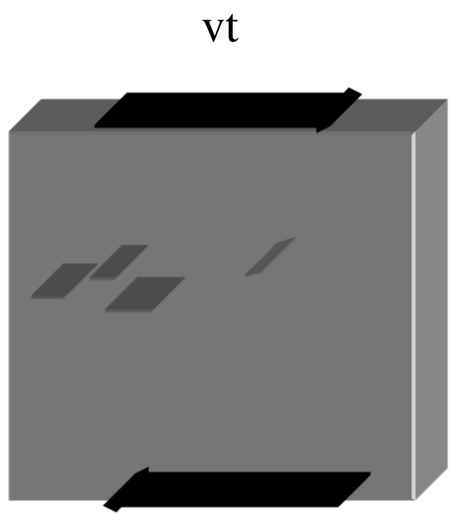


- For slow tangential velocity boundary condition:

same scaling behavior and phase diagram as Ben-Zion and Rice single earthquake fault zone model

For slowly moving boundary cond.: Same phase diagram and scaling as Ben-Zion and Rice single fault zone eq. model:

Ben-Zion and Rice, 1993, 1995; Ben-Zion, 1996; Fisher et al., 1997; KD,Ertas,Ben-Zion, 1998; Mehta et al., 06; Zöller et al., 05, 07; KD and Ben-Zion, 08; Bailey and Ben-Zion, 08)

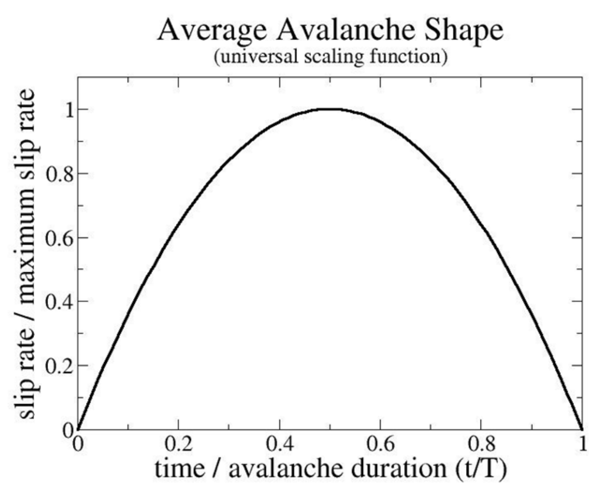


weakening ($\varepsilon>0$): "Stick slip" + mode switching

$\varepsilon=0$: Power law ("Gutenberg Richter")

hardening ($\varepsilon<0$): Power law (with Aftershocks)

There are more predictions for universal exponents, scaling functions, etc…



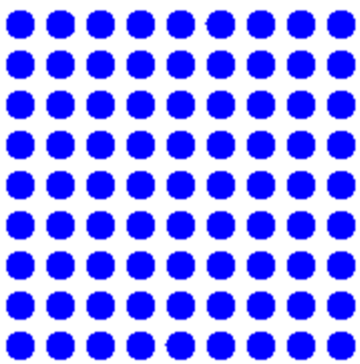
Simulations of slip avalanches in sheared crystals:

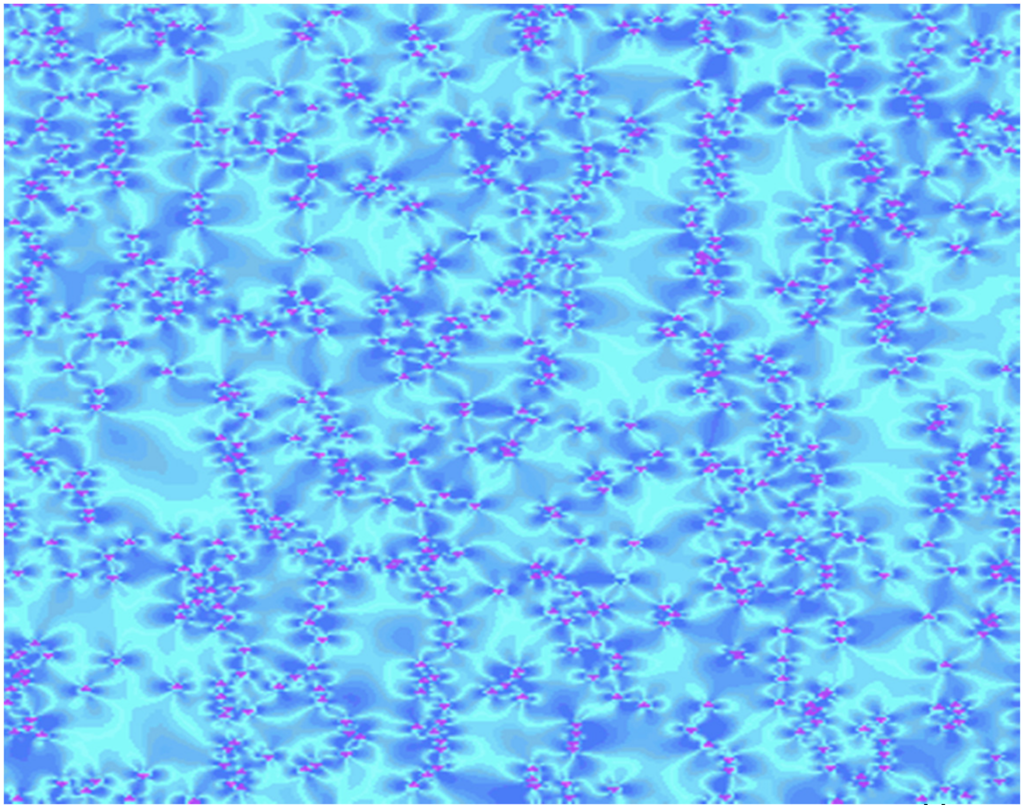
Dislocation Dynamics: Miguel et al Nature 410 (667-671) 2001, Csikor et al. Science (2007)
Georgios Tsekenis 2011;

Phase Field Crystal Models: Pak Yuen Chan, Georgios Tsekenis, J. Dantzig, KD, N. Goldenfeld (PRL 2010);

Phase Field Models: M. Koslowski (2006)…

Crystal with many edge dislocations under shear (Miguel et al. Nature 2001)



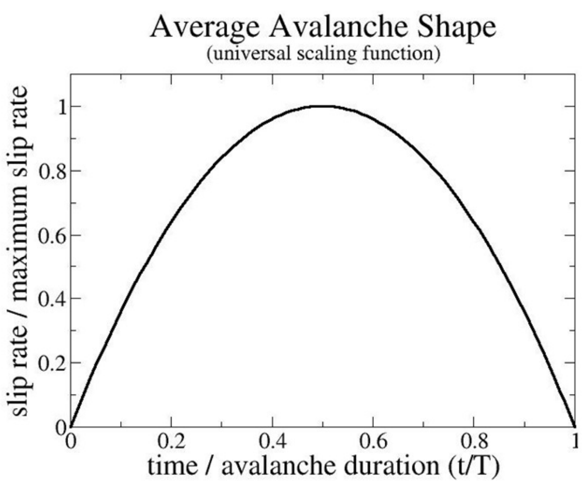


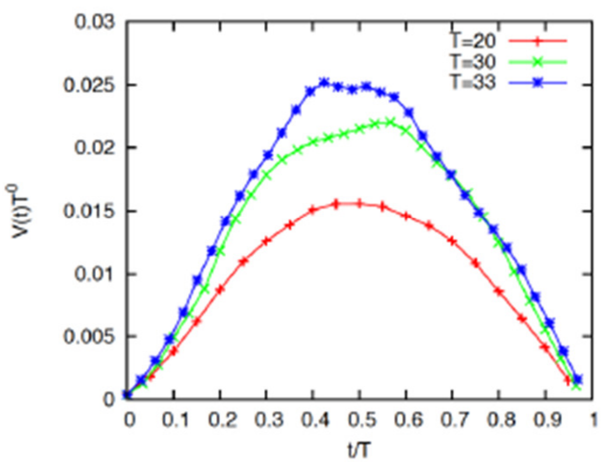
hhttp://www.tf.uni-kiel.de/matwis/amat/def_en/kap_5/illustr/a5_1_1.html

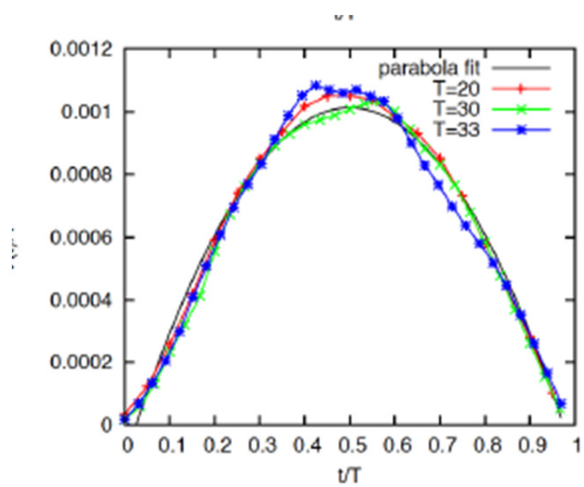
3. Comparison with simulations

There are more predictions for exponents, scaling functions, etc…

Mean Field Theory for ε=0 | Simulations: raw data | Collapse







Georgios Tsekenis: Collapse $V(t) \sim T^{1-1/\sigma\nu z} f(t/T)$

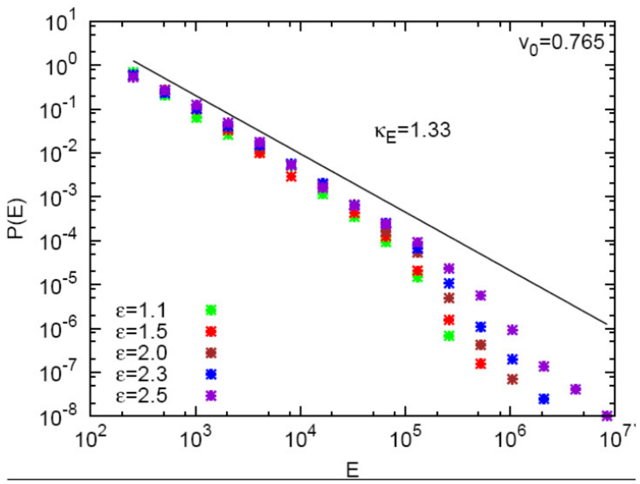
Discrete Dislocation Dynamics (DDD) Simulations

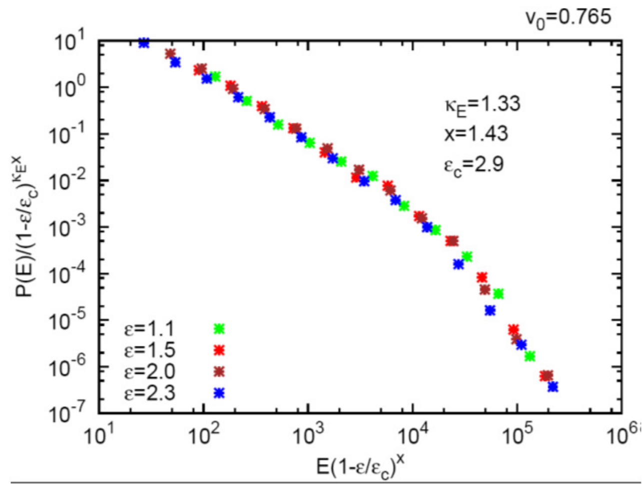
Georgios Tsekenis: Simulations, Mean Field Theory, Experiments

quantity	exponent	our simulations	MFT	simulations	experiments
$D_S(S,\tau)$	κ	~ 1.5	$\frac{3}{2}$	1.4[1, 2]	1.5[3]
$D(V_{\max})$	–	–	–	–	2.0 ± 0.1[4],1.5-2[5],1.2-2.2[6]
$D_S(S,\tau)$	$\frac{1}{\sigma}$	2	2	2[1, 2]	2[1]
$D_T(T,\tau)$	$1+\frac{\kappa-1}{\sigma\nu z}$	2.0	2		
$D_T(T,\tau)$	νz	1	1		
$D_E(E,\tau)$	$1+\frac{\kappa-1}{2-\sigma\nu z}$	1.3	$\frac{4}{3}$		
$D(V_{\max}^2)$	–	–	–	1.8 ± 0.2[7]	1.6[7],1.5 ± 0.1[4]
$D_E(E,\tau)$	$\frac{2-\sigma\nu z}{\sigma}$	3	3		
$\langle S\rangle \sim T^{1/\sigma\nu z}$	$1/\sigma\nu z$	~ 2.0	2		
$\langle T\rangle \sim S^{\sigma\nu z}$	$\sigma\nu z$	~ 0.5	$\frac{1}{2}$		
$\langle E\rangle \sim S^{2-\sigma\nu z}$	$2-\sigma\nu z$	~ 1.5	$\frac{3}{2}$		
$V(t)_{\text{shapes}} \sim T^{\frac{1}{\sigma\nu z}-1}$	$\frac{1}{\sigma\nu z}$	~ 2	2		
$PS_{\text{int}}(\omega)$	$\frac{1}{\sigma\nu z}$	~ 2	2		
$\langle v\rangle \sim (1-\frac{\tau}{\tau_c})^{\beta}$	β	~ 1.1	1	1.8[8]	

Agrees with Phase Field Crystal simulations at T>0

(Tsekenis, Fehm, Chan, Dantzig, Goldenfeld, KD, PRL 2010 and ongoing)

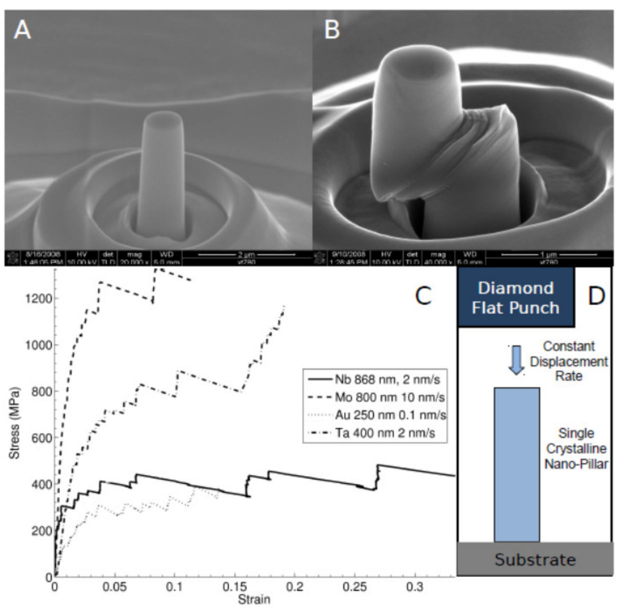


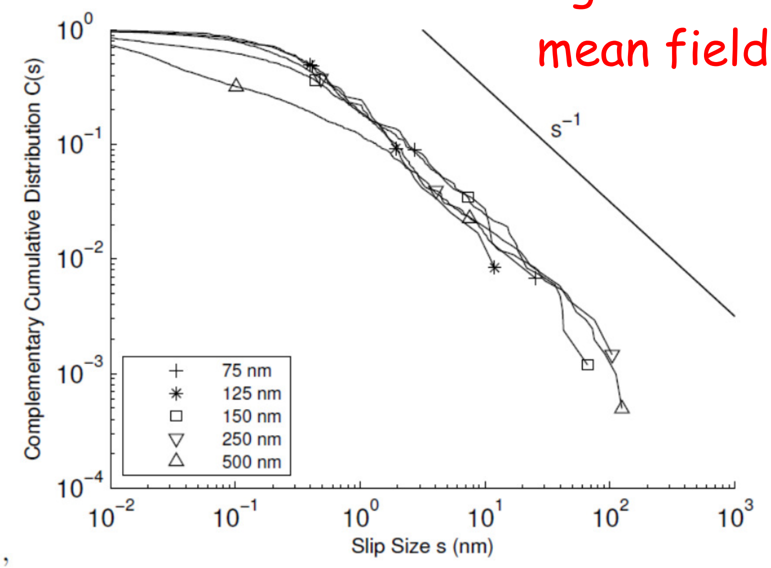


4. Agrees with Experiments on μm-size and nano-crystal

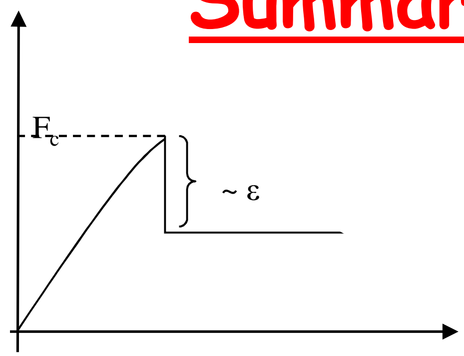
(Nir Friedman, Jennings, Tsekenis, Kim, Tao, Uhl, Greer, KD, submitted 2011)

Agrees with mean field theory





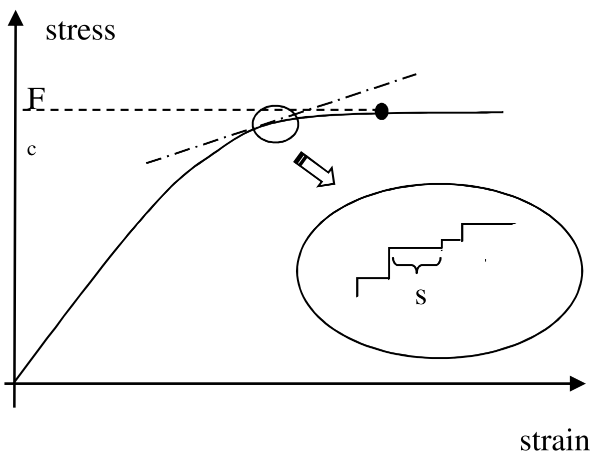
Summary on damage/dislocation model:



simple mean field theory exact

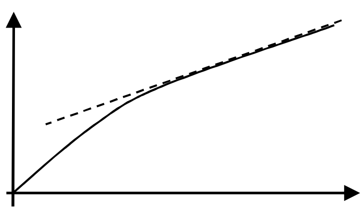
only 1 tuning parameter ε:

ε>0	ε=0	ε<0
brittle	ductile	hardening



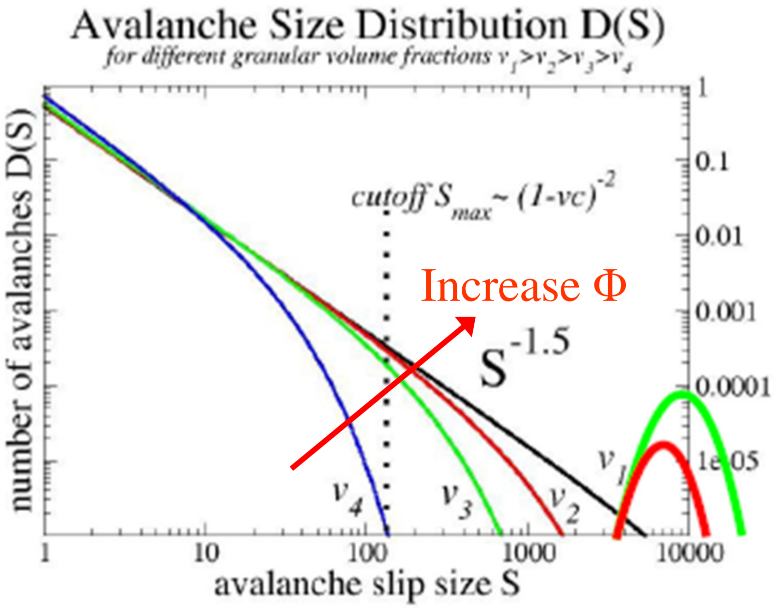
Agrees with:

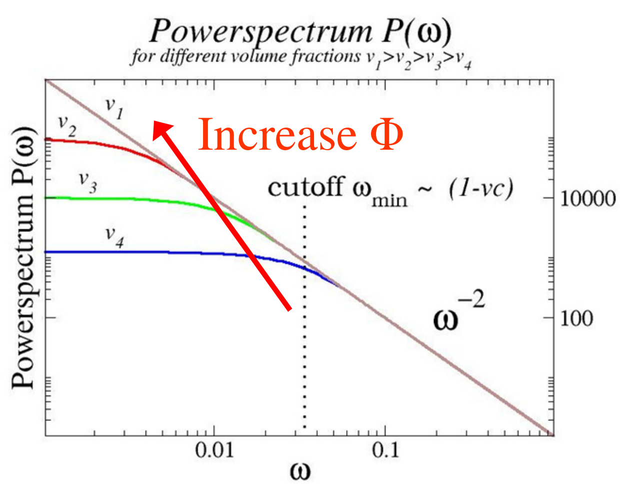
- ❑ Discrete Dislocation Simulations (T=0)
- ❑ Phase Field Crystal Simulations (T>0)
- ❑ Experiments on μm and nm samples
- ❑ Granular materials (Nature Physics 2011)
- ❑ Earthquake statistics (Brinkman et al.)

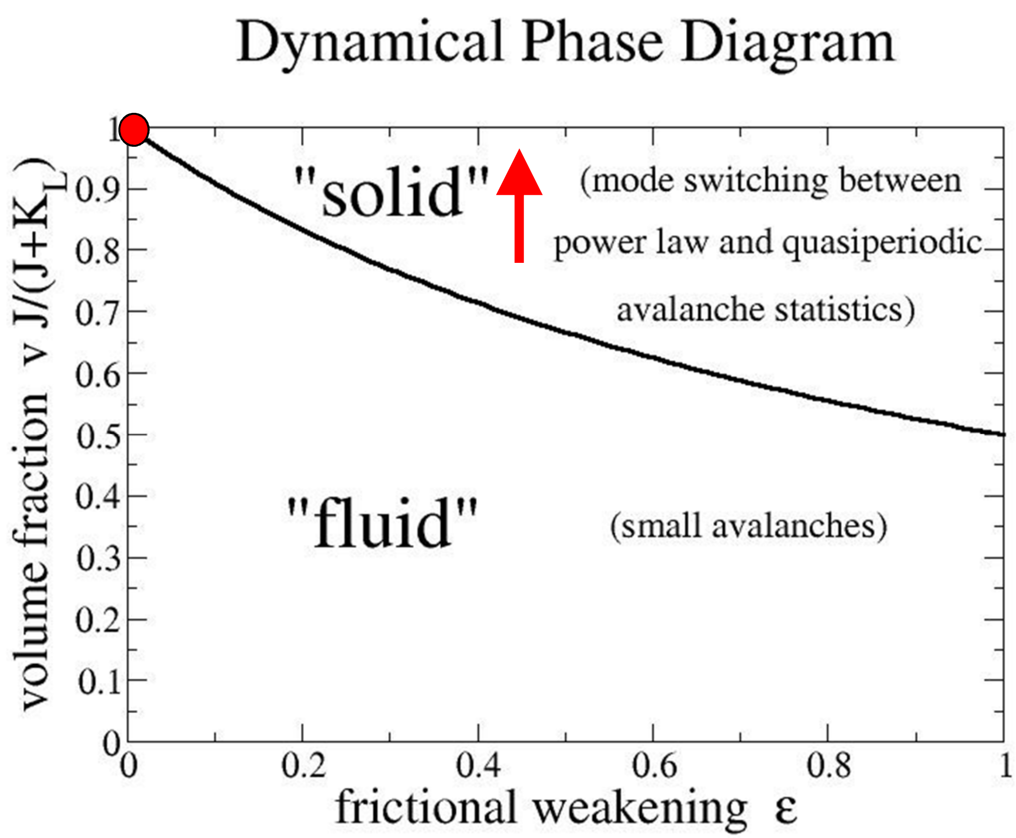


Universal Predictions for exponents, scaling functions: experiments ??

Similar SIMPLE model for avalanches in sheared granular materials:



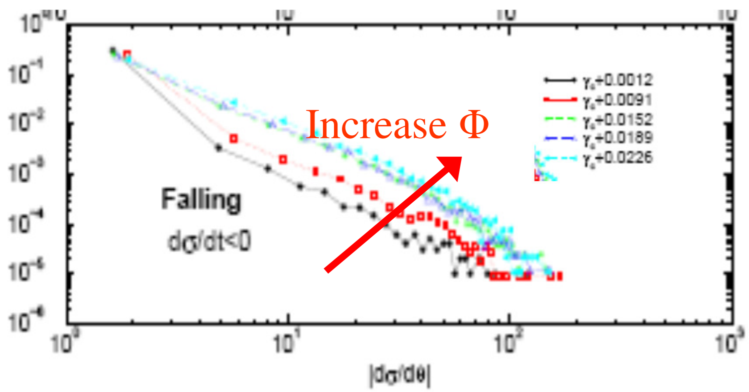


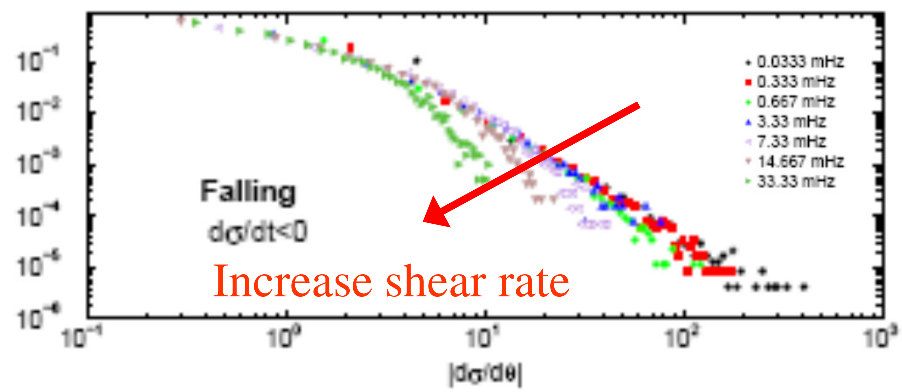


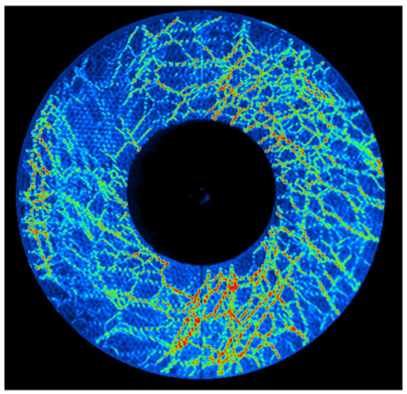
KD, D. Ertas, Y. Ben-Zion, PRE 1998

KD, Ben-Zion, Uhl, preprint 2010

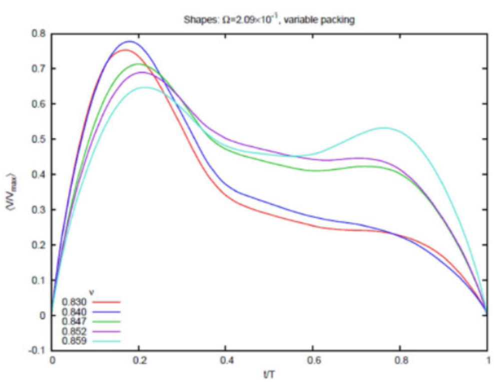
Matches Behringer and Hartley's experiments (Tyler Earnest):





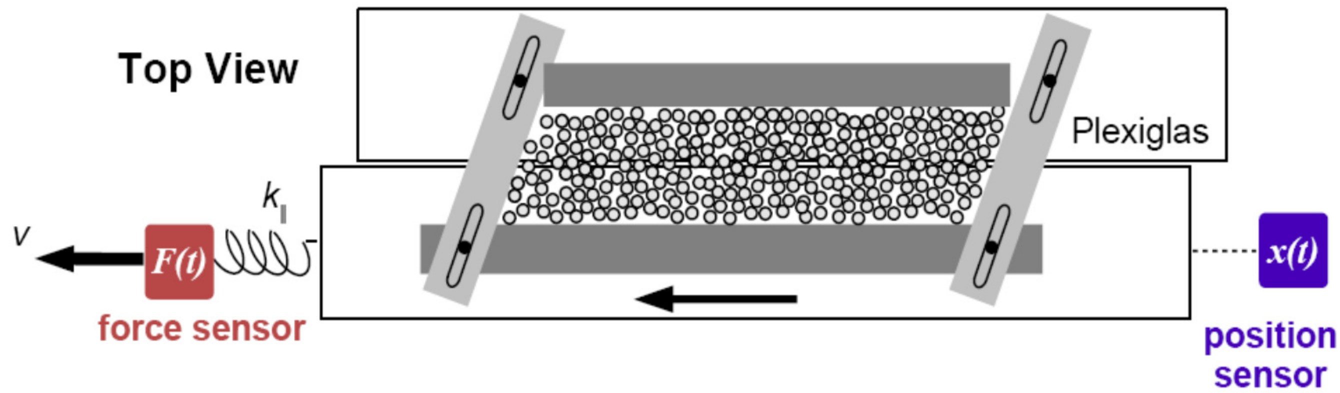


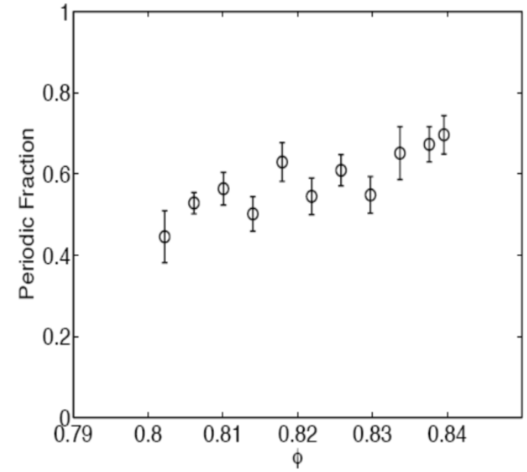
Avalanche Shapes (Tyler's poster)



Power law exponent or other universal quantity	Mean Field Theory (MFT)	granular experiment [6,8-10,20-21,29]	granular simula-tion [2-4]
avalanche size distribution $D(s) \sim s^{-\kappa}$	κ=1.5	κ=1.5	
avalanche duration distribution $D(T) \sim T^{-\alpha}$	α=2	α=2 or exponential?	
stress drop rate distribution ~ $D(V) \sim V^{-\psi}$	ψ=2	ψ=2 [29]	
power spectrum $P(\omega) \sim \omega^{\phi}$	ϕ=2 if $v \approx 1$; ϕ=0 if v <<1	ϕ=1.8-2.5, 2	ϕ=2 if solid ϕ=0 if fluid
Source time function averaged over avalanches of duration T.	Symmetric (parabola)	Symmetric (parabola ? Gaussian ?)	Symmetric sine fctn ?
Stick slip statistics	Yes, if ε>0 and $v>v^*$	Yes, sometimes	Yes (mode switching)
Mode switching (between powerlaw and stick slip)	Yes, if ε>0 and $v>v^*$	Yes, sometimes	Yes in solid regime

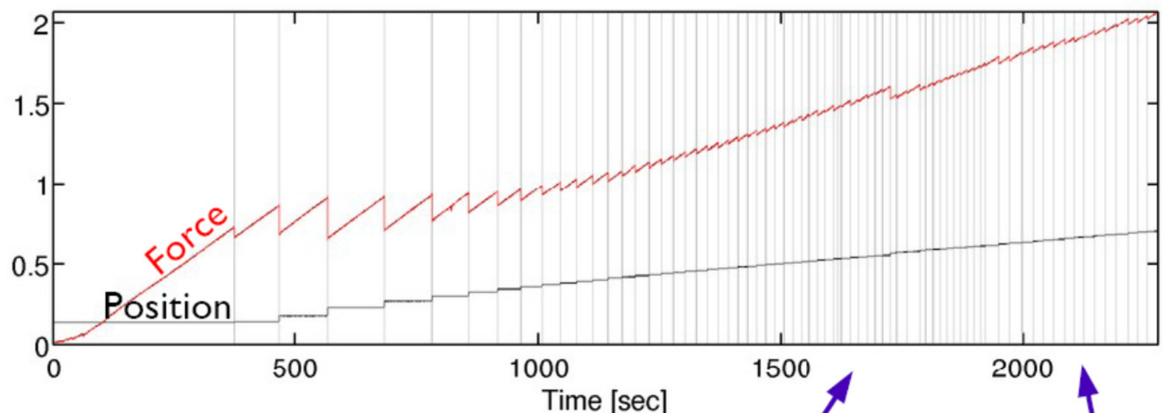
Matches Karen Daniels' and Kate Foco's experiments (in progress) :

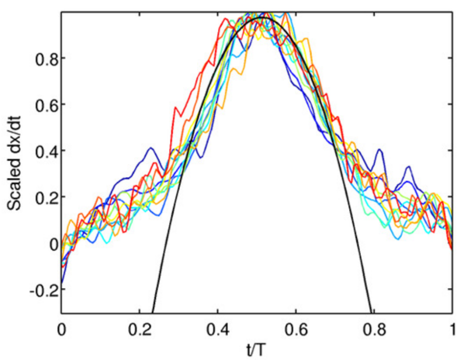


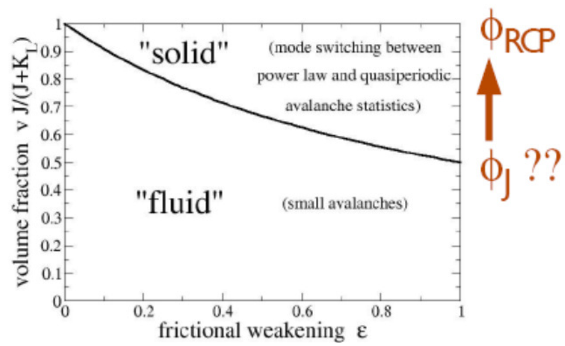


Mode Switching:

Daniels experiment:
For bigger Φ system spends longer times in the "quasi-periodic phase" Matches theory!







aperiodic quasi-periodic

mode-switching

KD, Ertas, Ben-Zion, PRE 1998

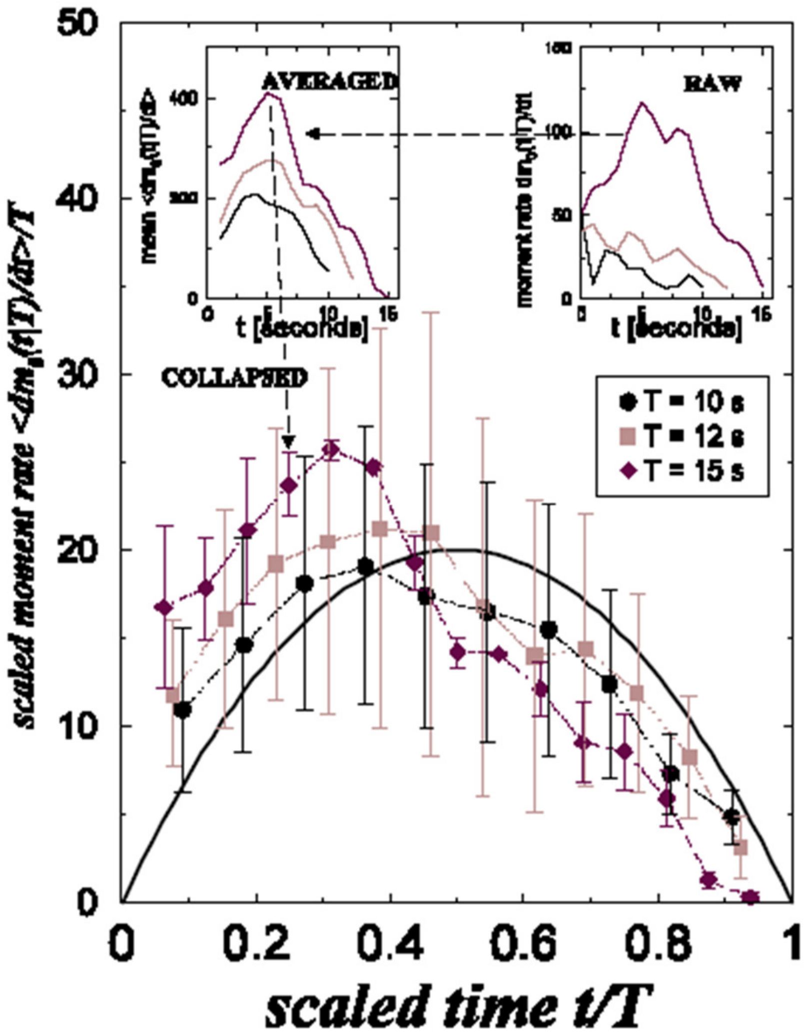
KD, Ben-Zion, Uhl, preprint 2010

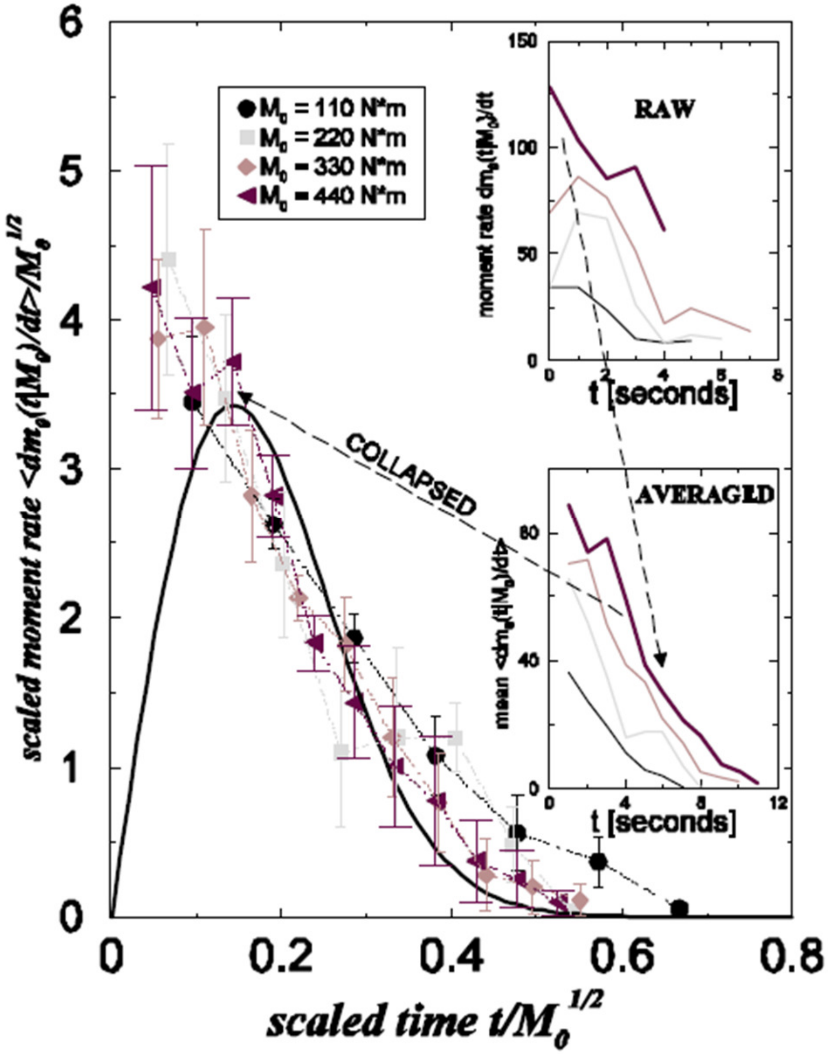
Earthquakes: Universal Scaling Functions:

(tidal triggering of earthquakes: Braden Brinkman)

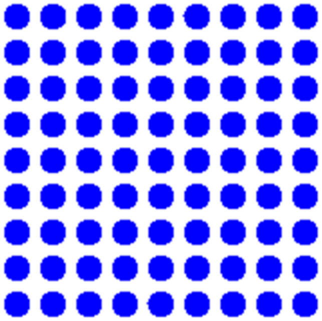
Data from Susan Bilek, see

Mehta, KD, Ben-Zion, 2005

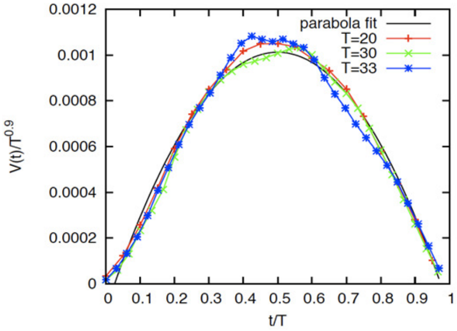




SUMMARY

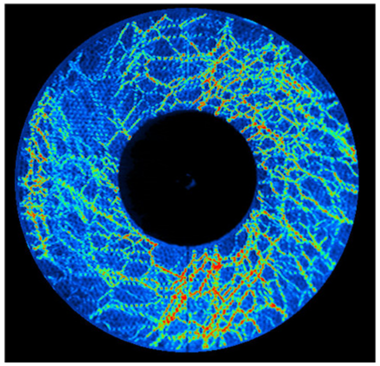


- **HUGE depinning universality class:**
- **Simple mean field theory gives exact & same scaling results for:**
  - ✓ magnetic domain wall motion
  - ✓ plasticity
  - ✓ granular materials
  - ✓ Earthquakes



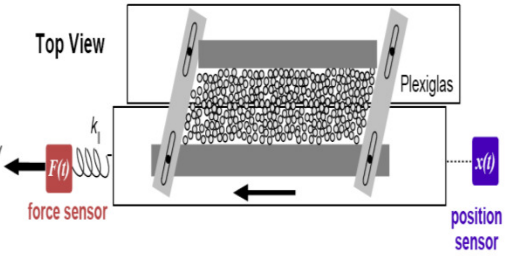
- ✓ Different universality class for systems with many interacting interfaces:

hard magnets
equilibrium and nonequilibrium
plastic charge density wave depinning
high Tc superconductors(?)…**ERICA CARLSON**



Many more predictions to be tested!

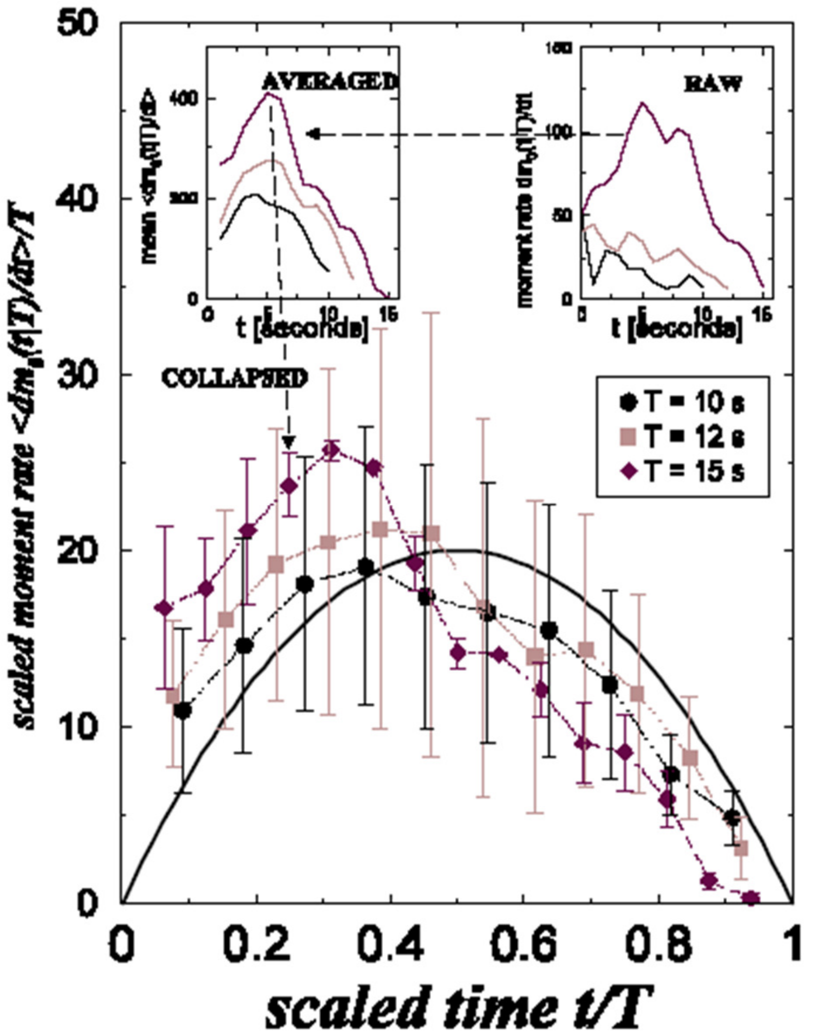
Experimental tests ?! Got DATA ?

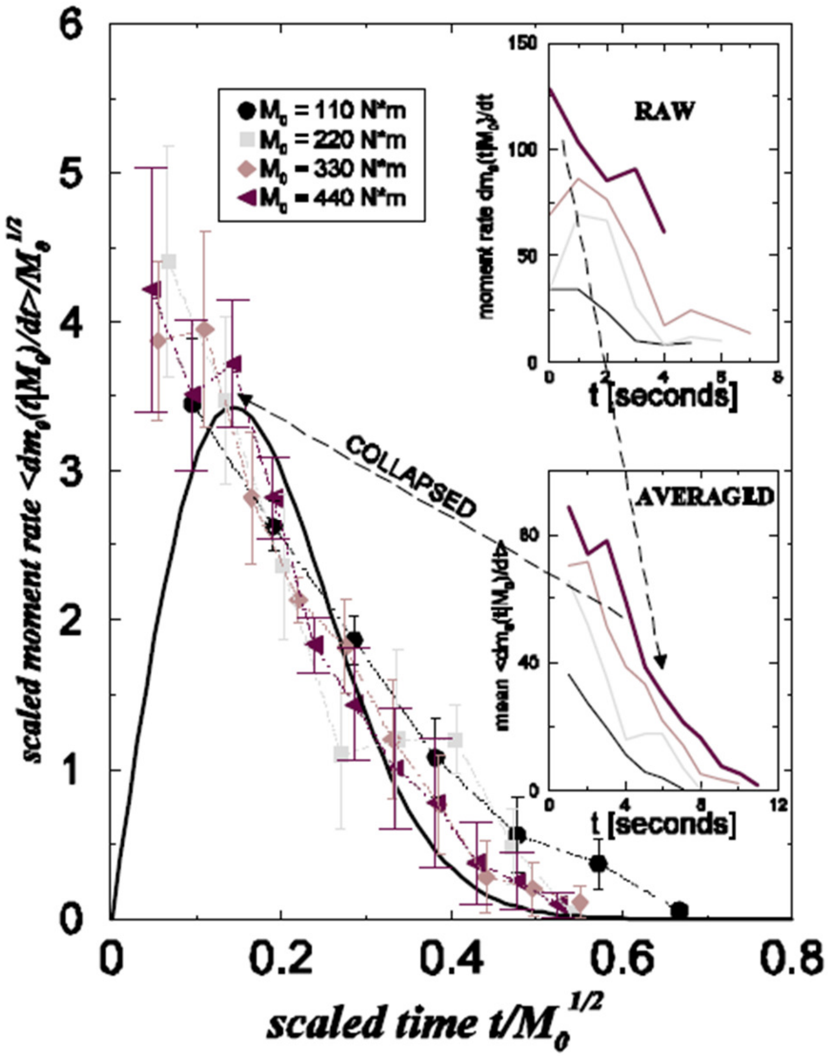


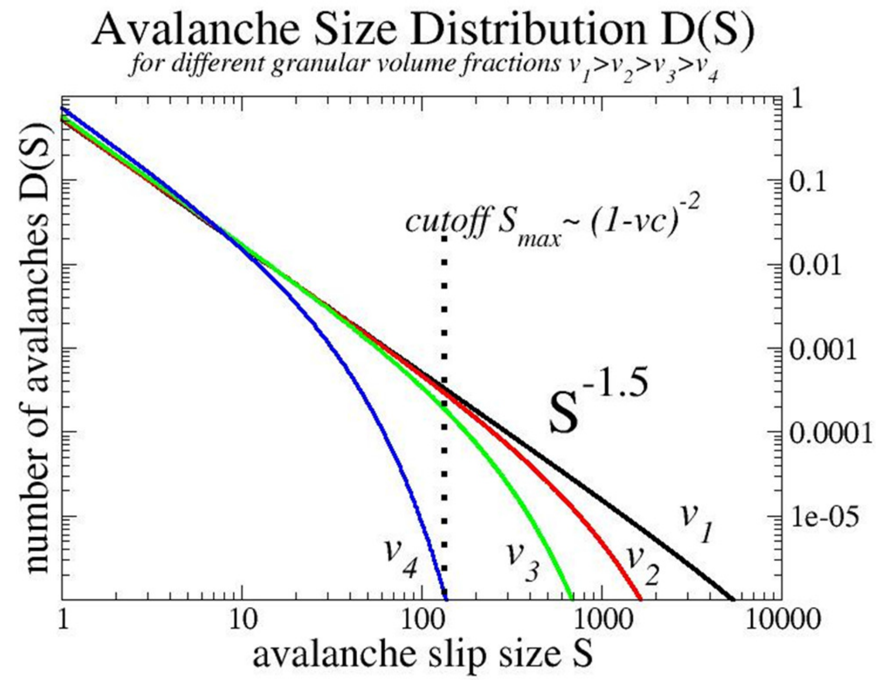
Earthquakes: Universal Scaling Functions:

Data from Susan Bilek, see

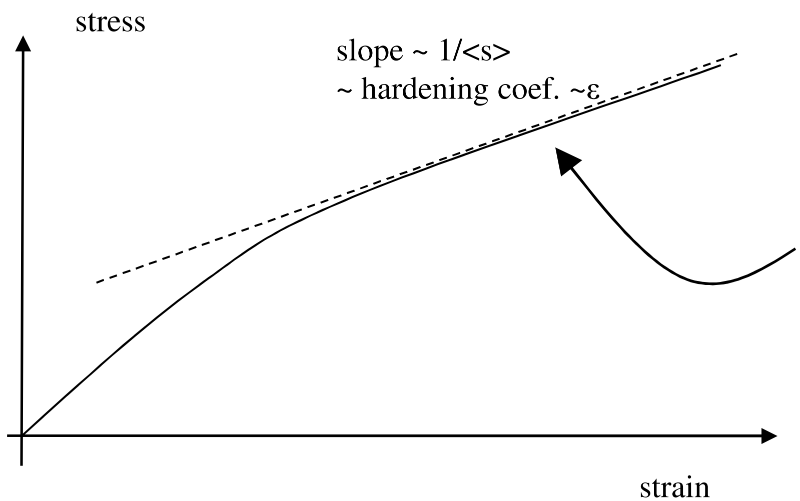
Mehta, KD, Ben-Zion, 2005






Avalanche Size Distribution D(S)
for different granular volume fractions v1>v2>v3>v4
number of avalanches D(S)
cutoff Smax~ (1-vc)-2
S-1.5
v1
v2
v3
v4
1
0.1
0.01
0.001
0.0001
1e-05
1
10
100
1000
10000
avalanche slip size S

Stress Strain Curve in the presence of hardening ("ductile"): ($\epsilon < 0$)



$D(s) \sim s^{-1.5}$

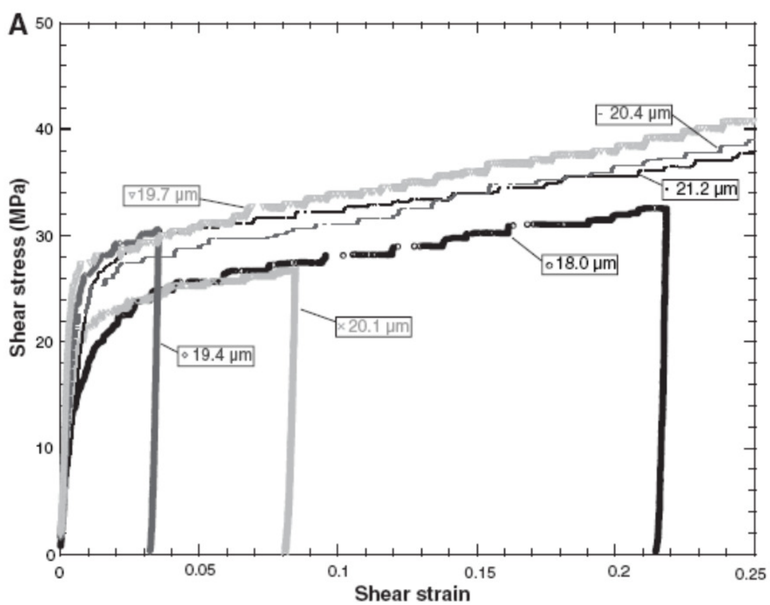
$P(\omega) \sim \omega^{-2}$

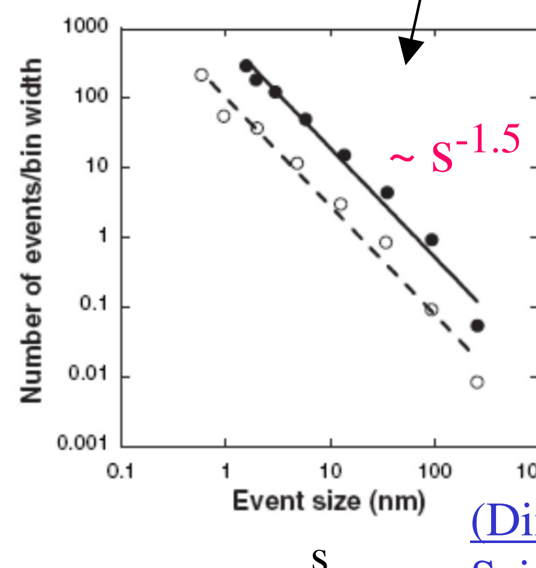
$D(T) \sim T^{-2}$

etc.

distributed deformation

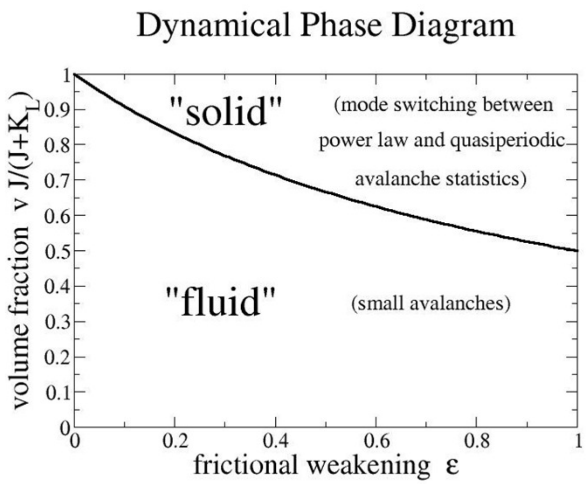
Agrees with experiment

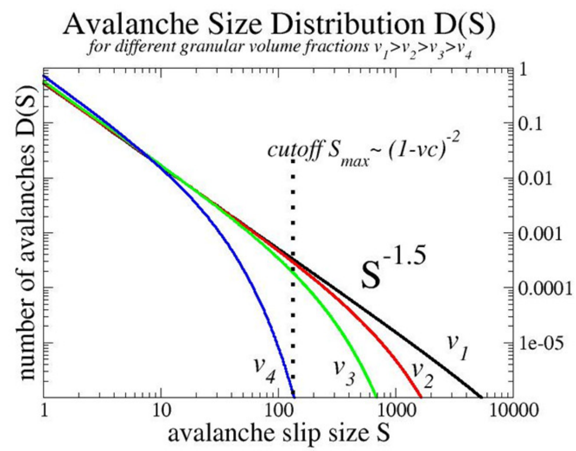


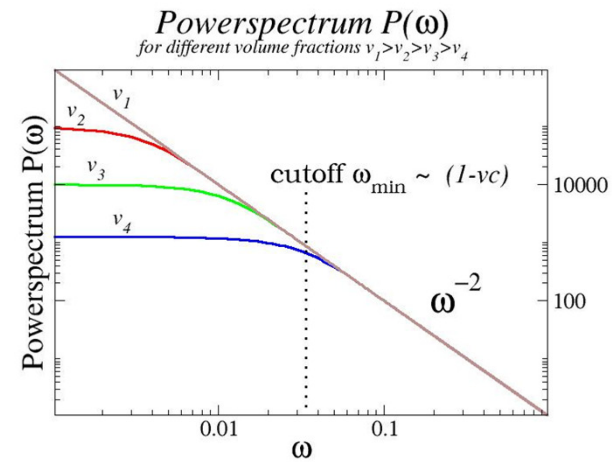


(Dimiduk, et al Science 2006)

Similar model for sheared granular materials tuning volume fraction:







Exponent or other universal quantity	**Mean Field Theory**	**granular experiment [6,8-10,20-21]**	**granular simulations [2-4]**
κ (size distribution)	1.5	1.5	?
1/ρυz (power spectrum)	2 if v ≈ 1; 0 if v <<1	1.8-2.5, 2	2 in solid regime 0 in fluid regime
α (duration distribution) (^)	2	2 or exponential ?	?
Source time function averaged over all avalanches of same duration T.	Symmetric (parabola)	?	Symmetric: fit by sine function (?)
Quasiperiodic event statistics	Yes, if ε>0 and v>v*	some-times	during mode switching
Mode switching (between powerlaw and quasiperiodic)	Yes, if ε>0 and v>v*	?	Yes, in solid regime