Fluctuations in Granular Materials

Large Fluctuations *University of Illinois, UC*

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Roadmap

- What/Why granular materials?
- How do we think about granular systems

Use experiments to explore:

- Forces, force fluctuations ←
- Jamming ←
- Plasticity, diffusion
- Granular friction
- Force response—elasticity

What are Granular Materials?

- Collections of macroscopic 'hard' particles:
  - Classical h→ 0
  - Dissipative and athermal T→ 0
  - Draw energy for fluctuations from macroscopic flow
  - Physical particles are deformable/frictional
  - Show complex multi-scale properties
  - Large collective systems, but outside normal statistical physics
  - Exist in phases: granular gases, fluids and solids


Cheerios
Toasted Whole Grain Oat Cereal
Clinically PROVEN to Help Reduce Cholesterol!
Fuji Apples
1.0 mm
APR 20 2006

Questions

- Fascinating and deep statistical questions
  - What are the relevant macroscopic variables?
  - What is the nature of granular friction?
  - What is the nature of granular fluctuations—what is their range?
  - Is there a granular temperature?
  - Are threre granular phase transitions?
  - What are similarities/differences for jamming etc. in GM's vs. other systems: e.g. colloids, foams, glasses,…?
  - Is there a continuum limit i.e. 'hydrodynamics'—if so at what scales? (Problem of homogenization)
  - How to describe novel instabilities and pattern formation phenomena?

Assessment of theoretical understanding

- Basic models for dilute granular systems are reasonably successful—model as a gas—with dissipation
- For dense granular states, theory is far from settled, and under intensive debate and scrutiny

Statistical questions for dense systems:

How to understand order and disorder, fluctuations, entropy, and temperature, jamming?

What are the relevant length/time scales, and how does macroscopic (bulk) behavior emerge from the microscopic interactions? (Homogenization)

To what extent are dense granular materials like dense molecular systems (glasses), colloids, foams?

Collective behavior

When we push on granular systems, how do they respond?

- Granular Elasticity For small pushes, is a granular material elastic, like an ordinary solid, or does it behave differently?
- Jamming: Expand a granular solid enough, it is no longer a 'solid'—Compress particles that are far apart, reverse process, jamming, occurs—
- How should we characterize that process?

Collective behavior—continued

- Plasticity and response to shearing: For example, for compression in one direction, and expansion (dilation) in the perpendicular direction—i.e. pure shear.
- Under shear, solids deform irreversibly (plastically). Particles move ‘around’ each other
- What is the microscopic nature of this process for granular materials?

Jamming—and connection to other systems

How do disordered collections of particles lose/gain their solidity?

Common behavior may occur in glasses, foams, colloids, granular materials…

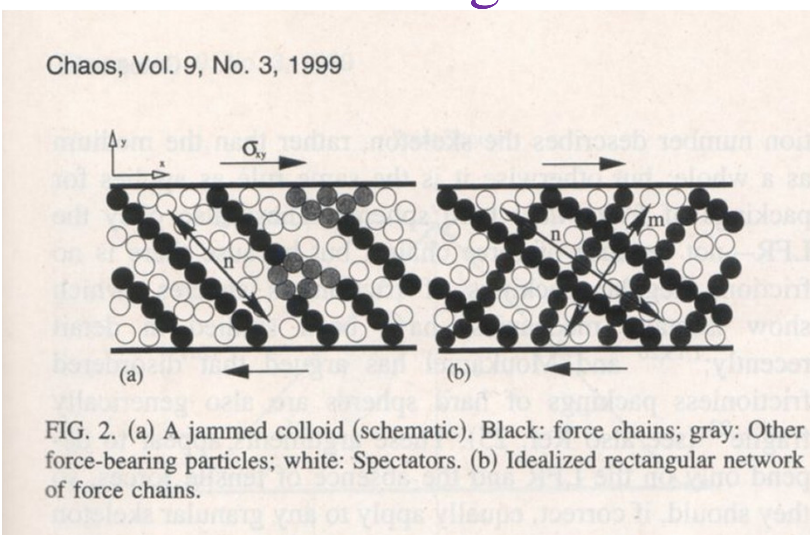
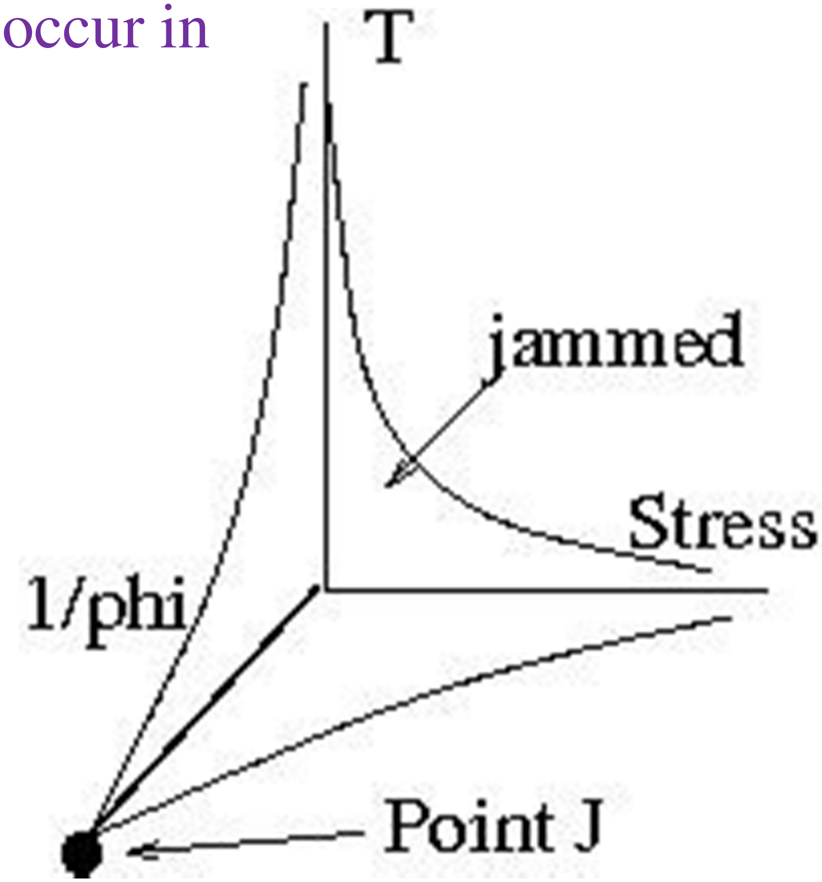

Chaos, Vol. 9, No. 3, 1999

FIG. 2. (a) A jammed colloid (schematic). Black: force chains; gray: Other force-bearing particles; white: Spectators. (b) Idealized rectangular network of force chains.

Bouchaud et al.



Liu and Nagel

Granular Properties-Dense Phases

Granular Solids and fluids much less well understood than granular gases

Forces are carried preferentially on force networks→multiscale phenomena

Friction and extra contacts → preparation history matters

Deformation leads to large spatio-temporal fluctuations

→ Need statistical approach

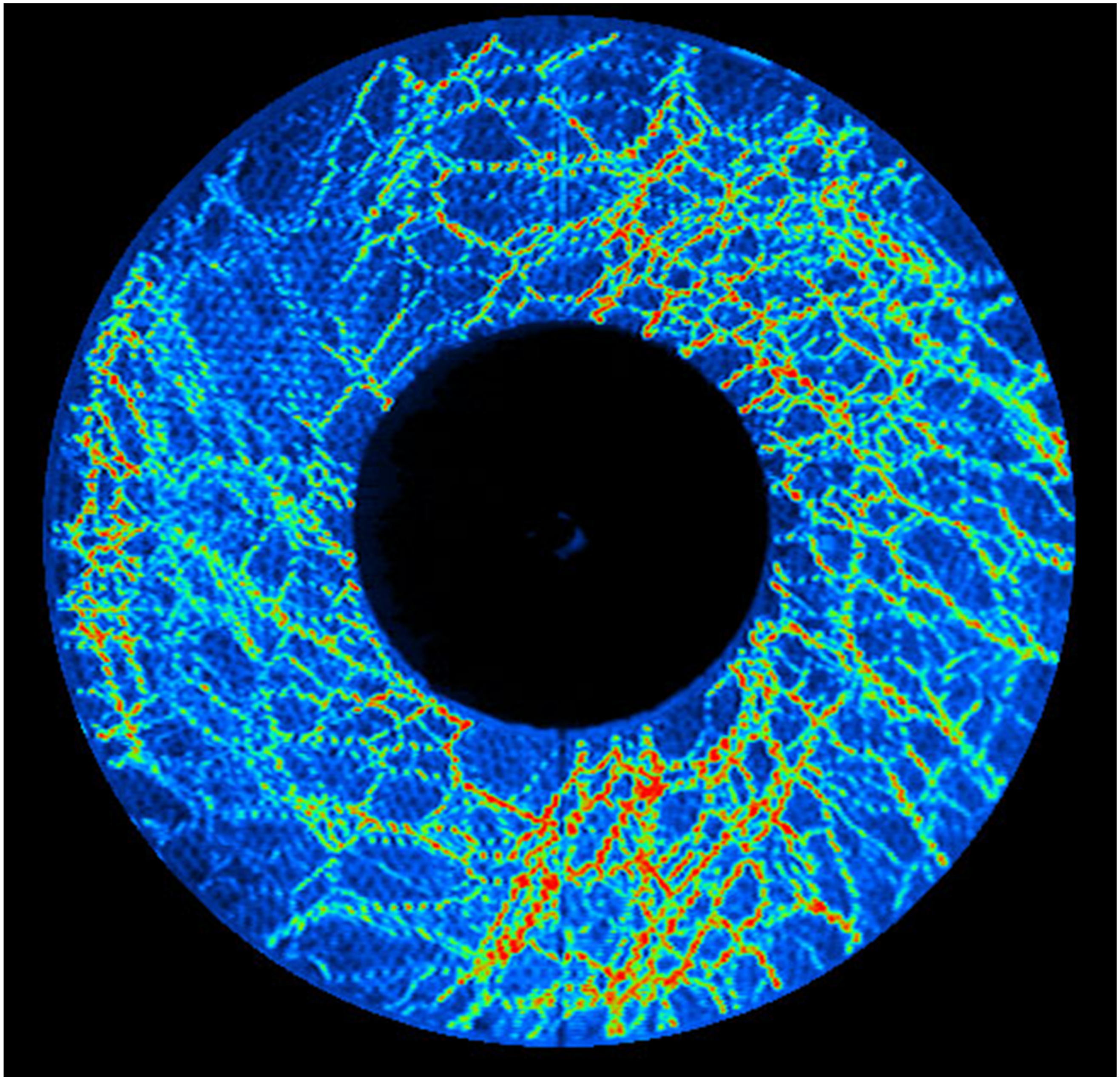
Illustrations follow…………

Experimental tools: what to measure, and how to look inside complex systems

- Confocal and laser sheet techniques in 3D—with fluid-suspended particles—for colloids, emulsions, fluidized granular systems
- Bulk measurements—2D and 3D
- Measurements at boundaries—3D
- 2D measurements: particle tracking, Photoelastic techniques (much of this talk)
- Promising new approach: MRI for forces and positions
- Numerical experiments—MD/DEM

GM's exhibit novel meso-scopic structures: Force Chains

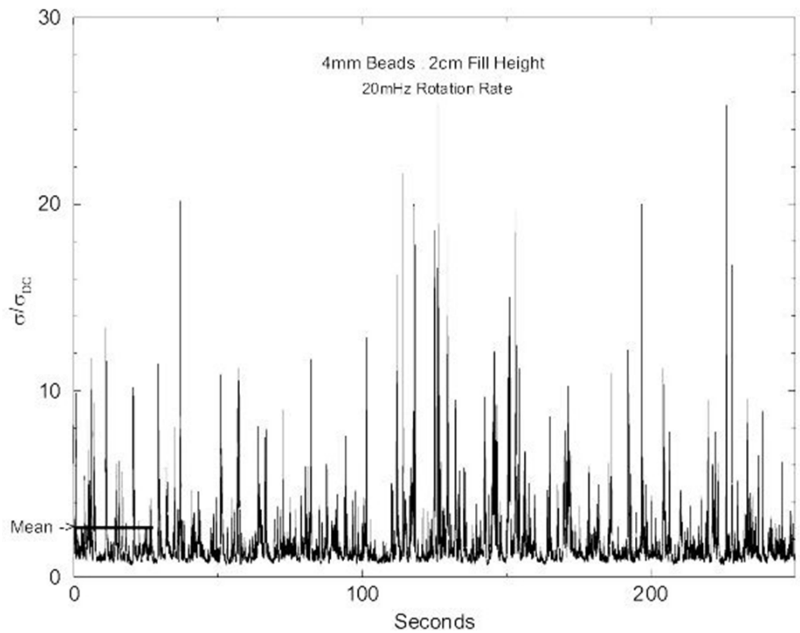
2d Shear →
Experiment



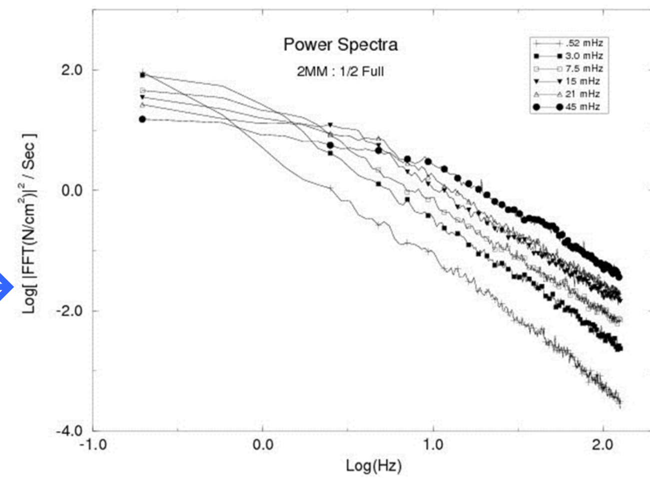
Howell et al.
PRL 82, 5241 (1999)

Experiments in 2D and 3D: Rearrangement of networks leads to strong force fluctuations

→Spectra-power-law falloff

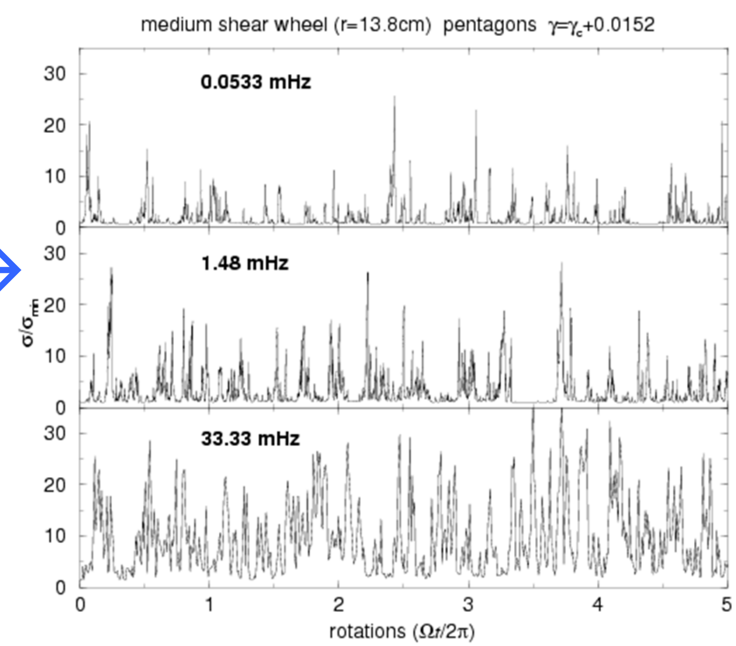


←3D→



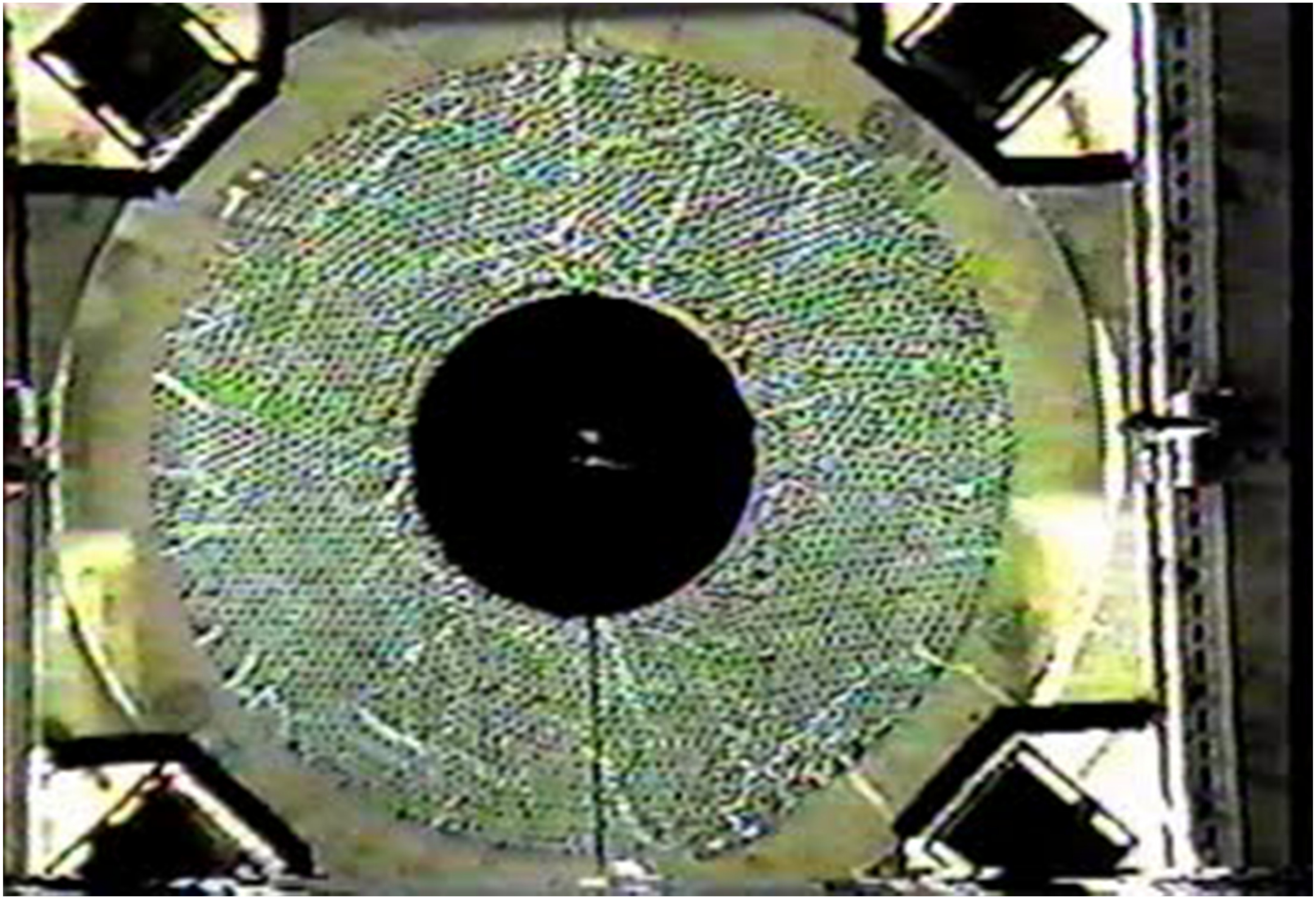
Time-varying Stress in 3D (above) and 2D (right) Shear Flow

2D→

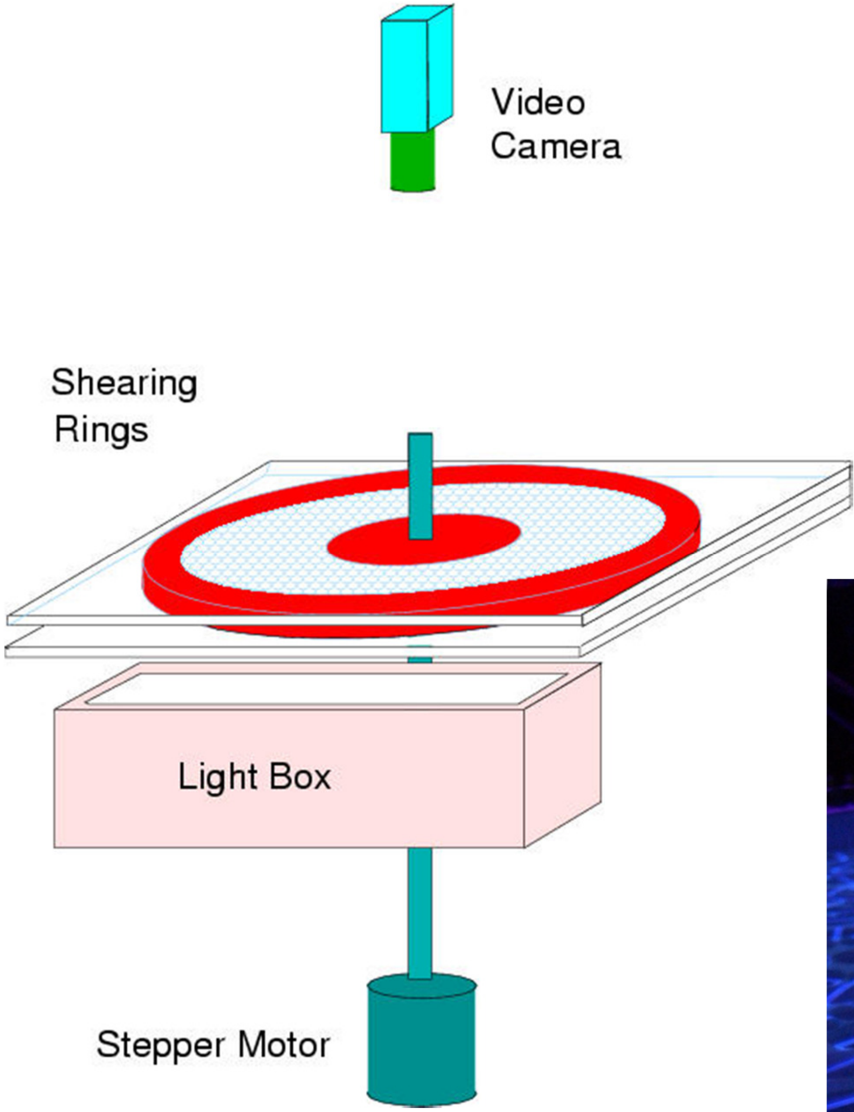


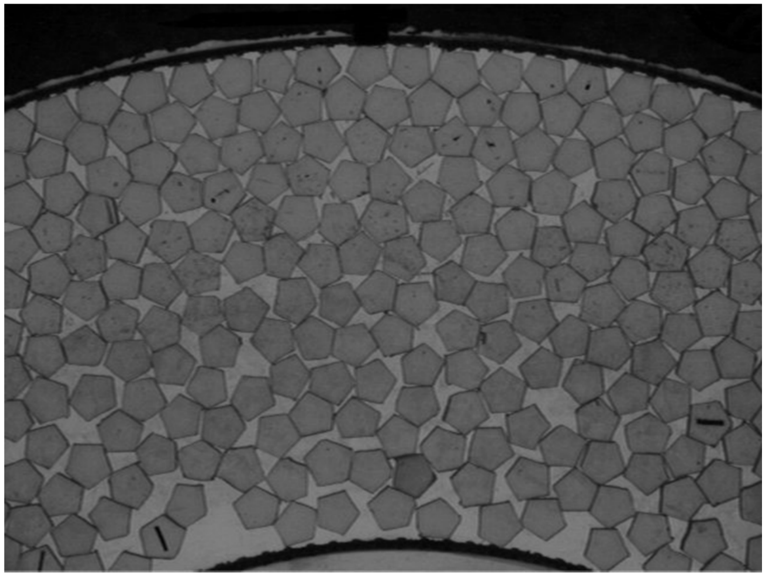
Miller et al. PRL 77, 3110 (1996)
Hartley & BB Nature, 421, 928 (2003)
Daniels & BB PRL 94, 168001 (2005)

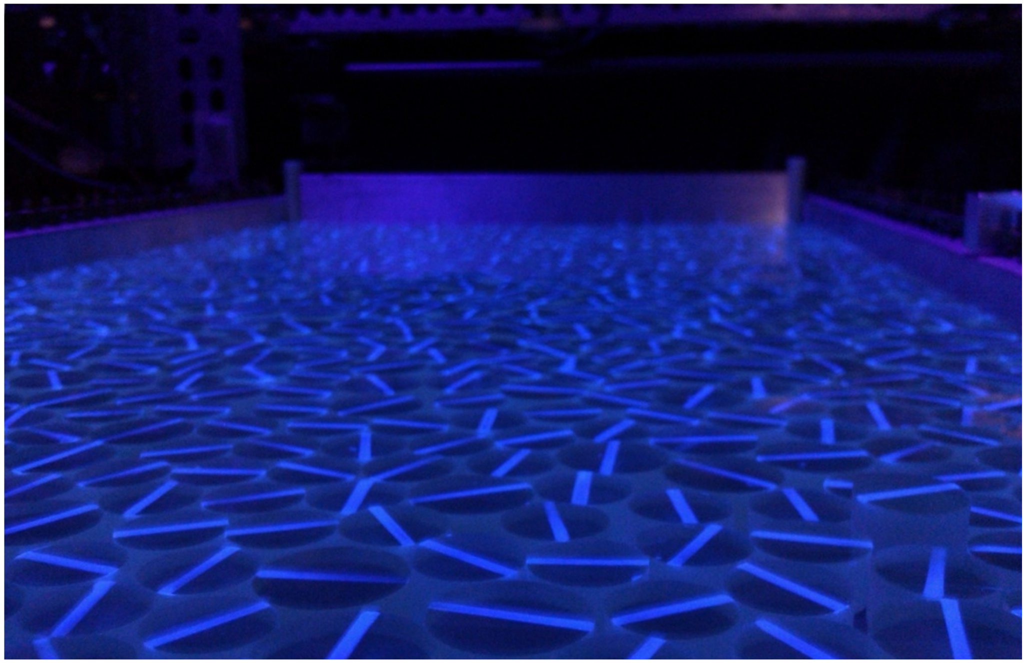
Shear experiment shows force chains—note sensitivity to shear



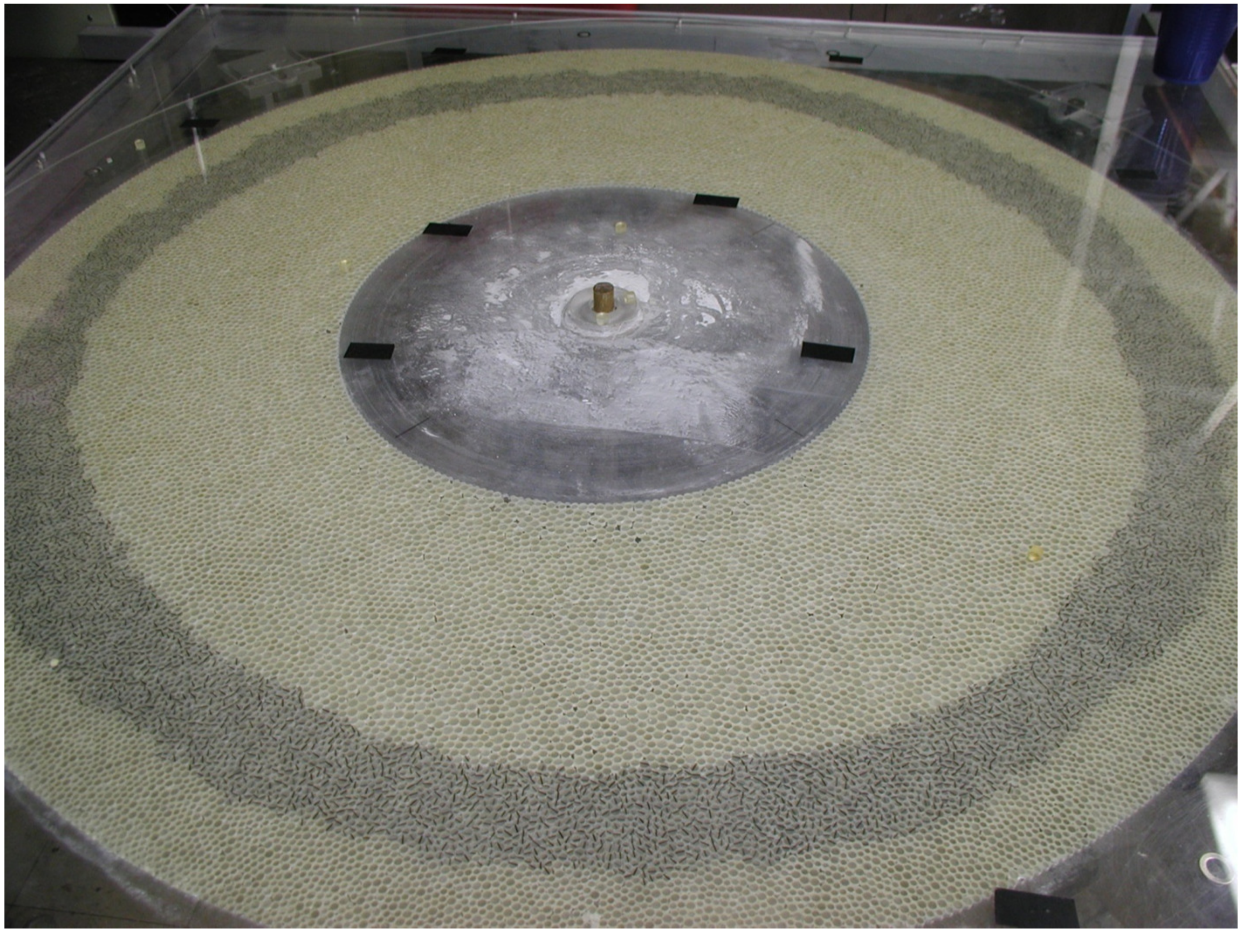
Schematic of apparatus—particles are pentagons or disks







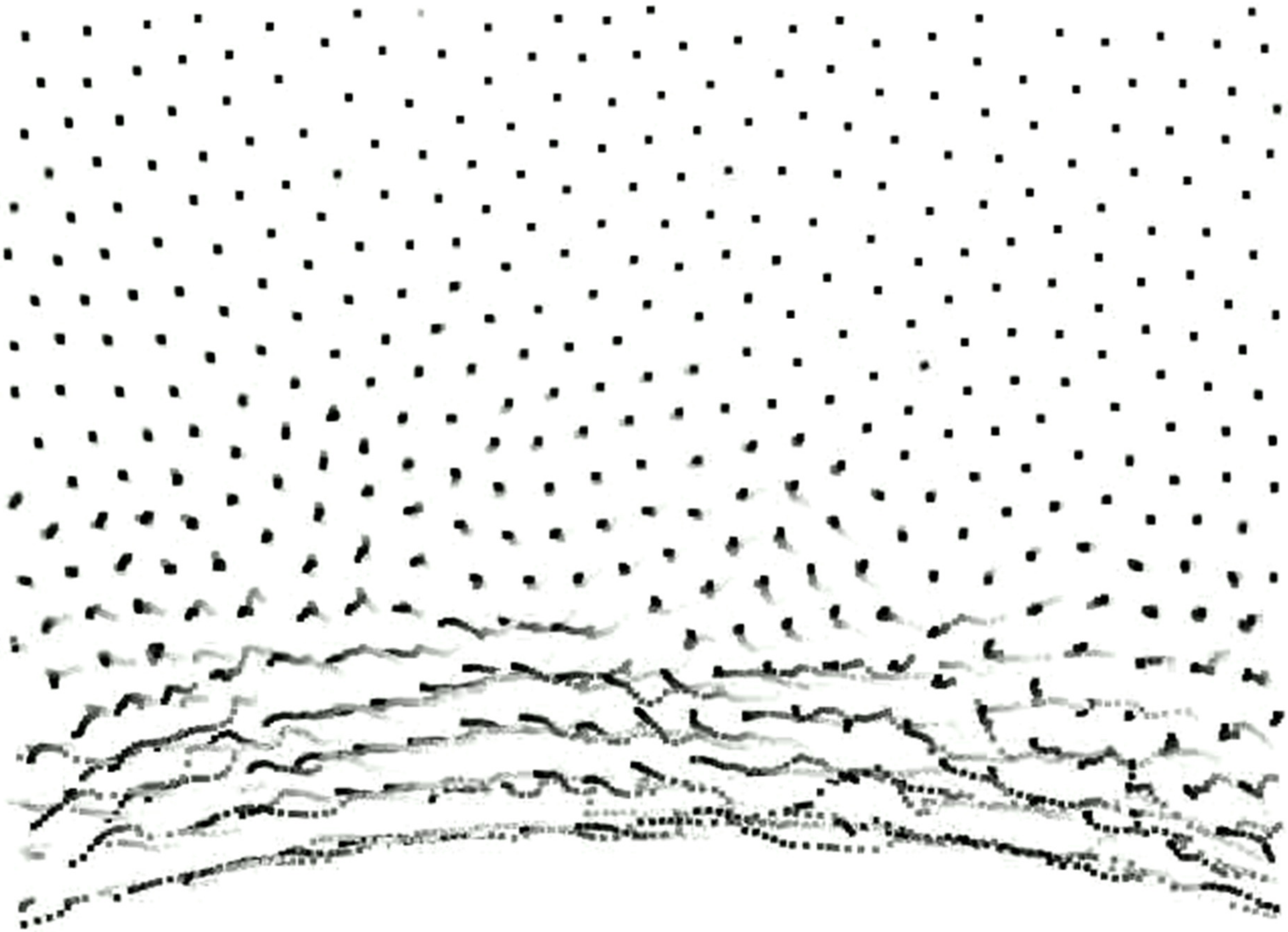
Couette apparatus—inner wheel rotates at rate Ω



~ 1 m

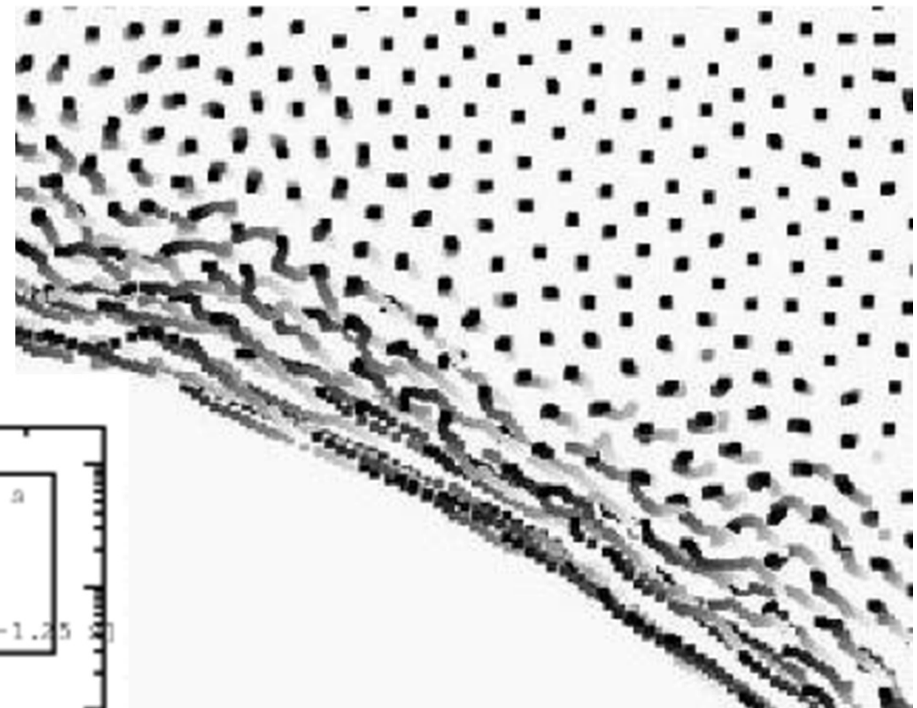
~50,000 particles, some have dark bars for tracking

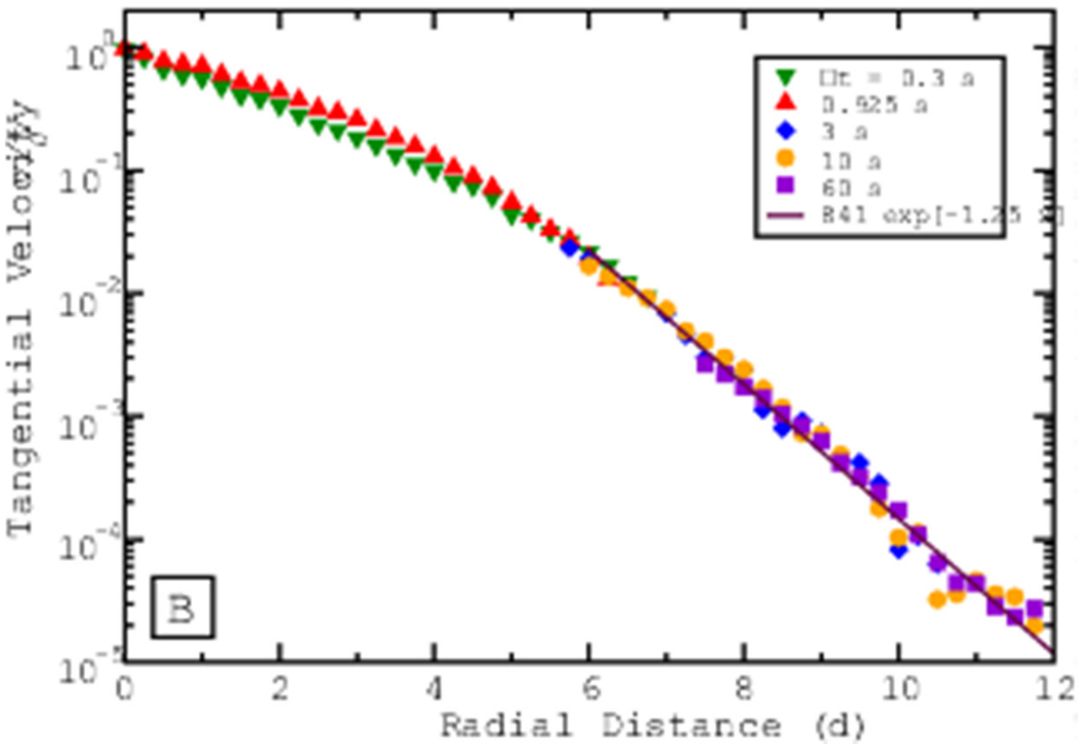
Trajectories



Motion in the shear band

Typical particle
Trajectories →

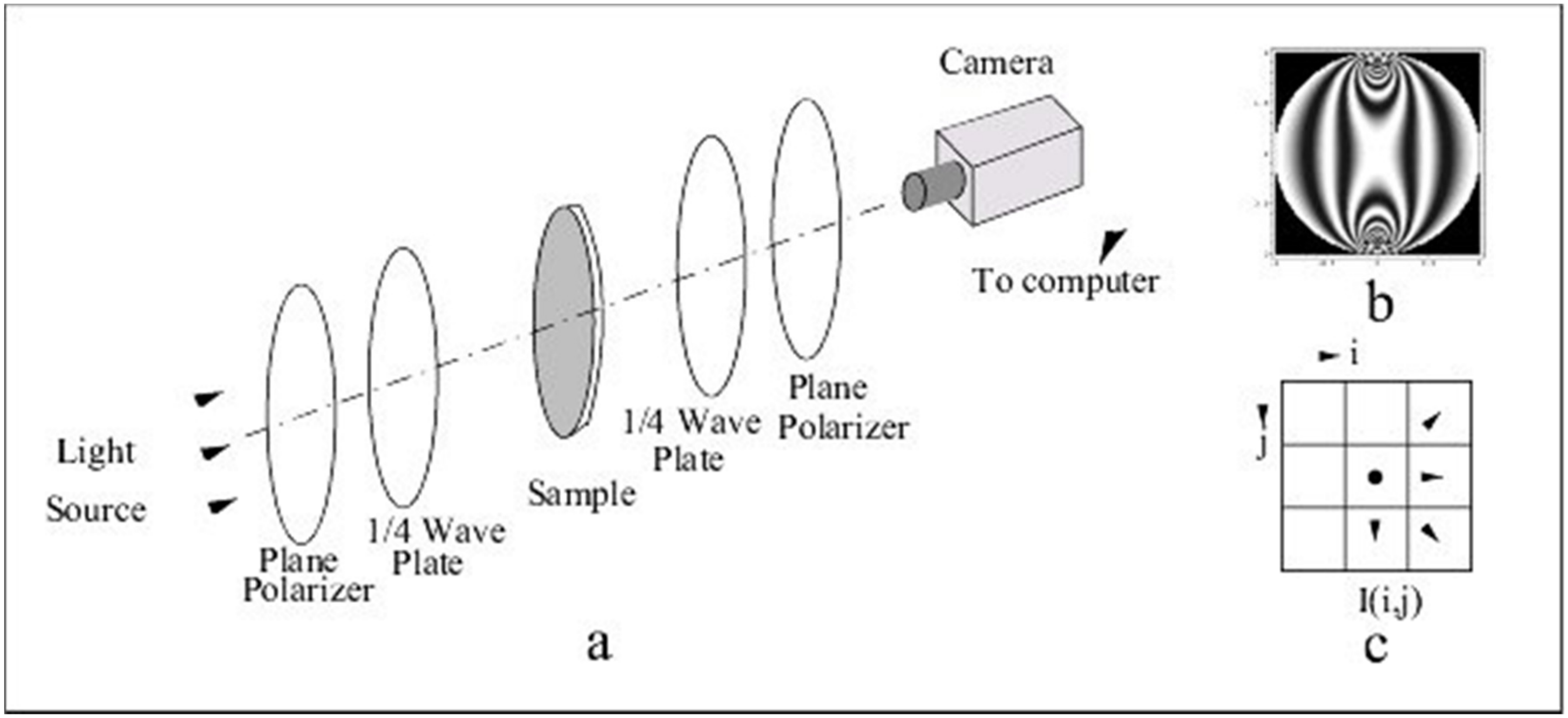




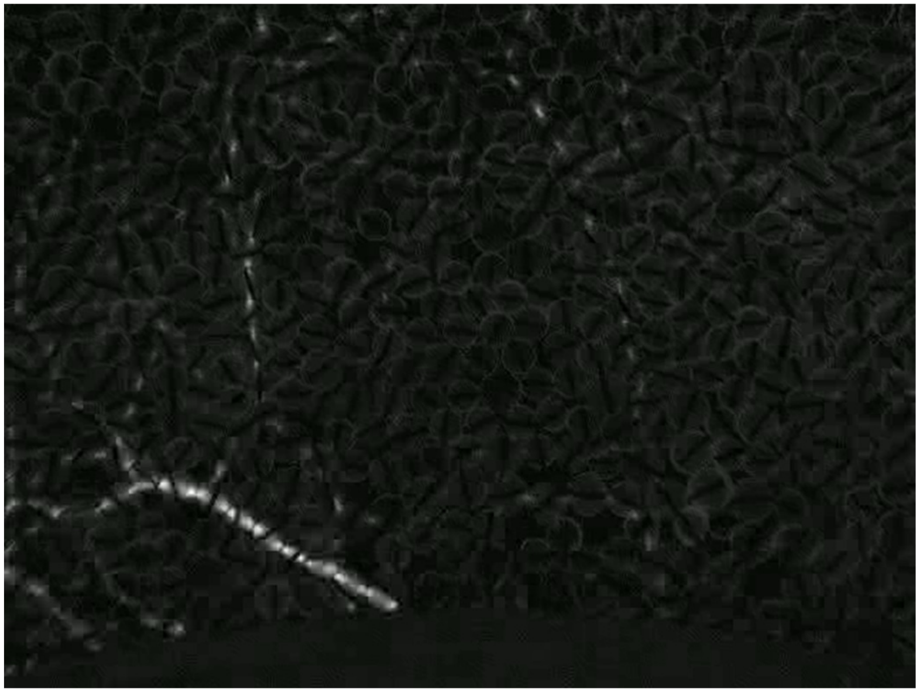
← Mean velocity profile

B.Utter and RPB PRE 69, 031308 (2004)
Eur. Phys. J. E 14, 373 (2004)
Phys. Rev. Lett 100, 208302 (2008)

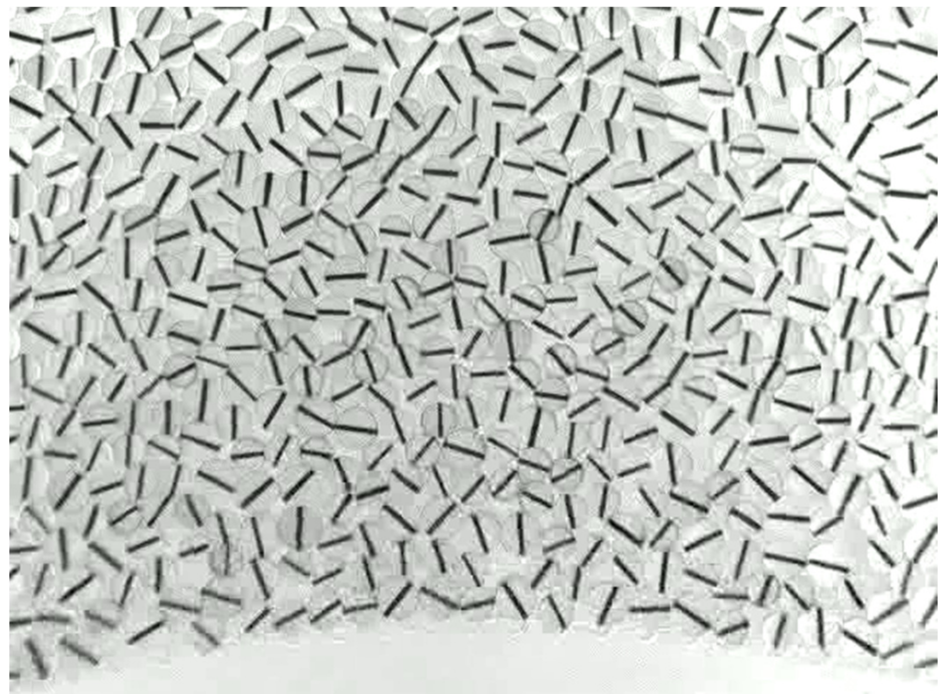
Measuring forces by photoelasticity



What does this system look like?



Photoelastic video

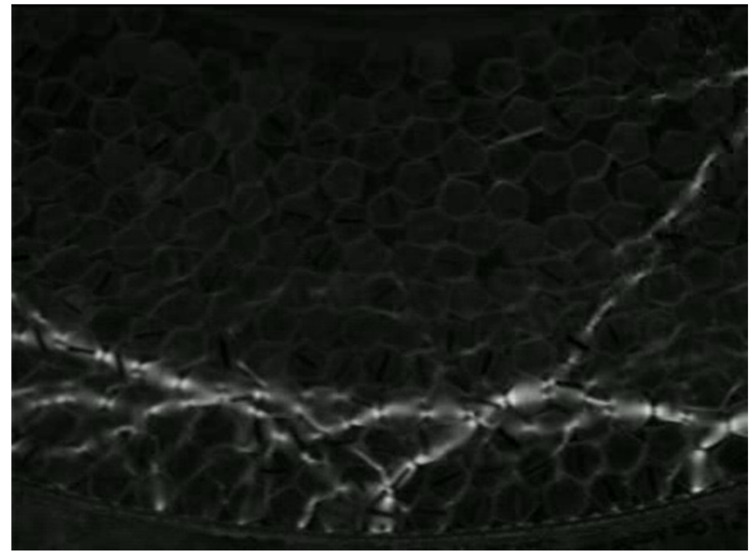


Particle and rotation tracking

Calibrate average local intensity gradient vs. pressure

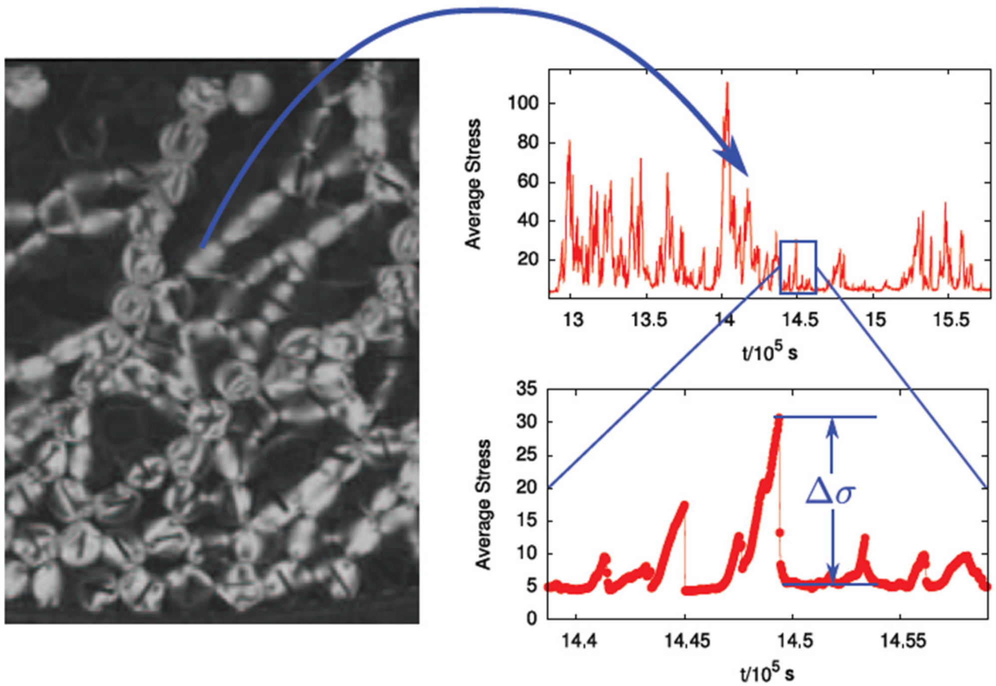
shear rate in mHz

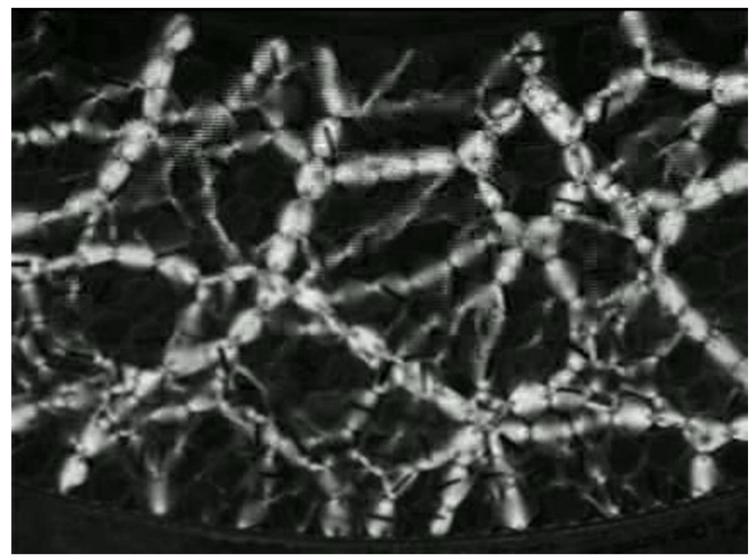
2.7 mHz



Experiment

$$\sum_{\text{frame}} |\vec{\nabla} I|^2$$

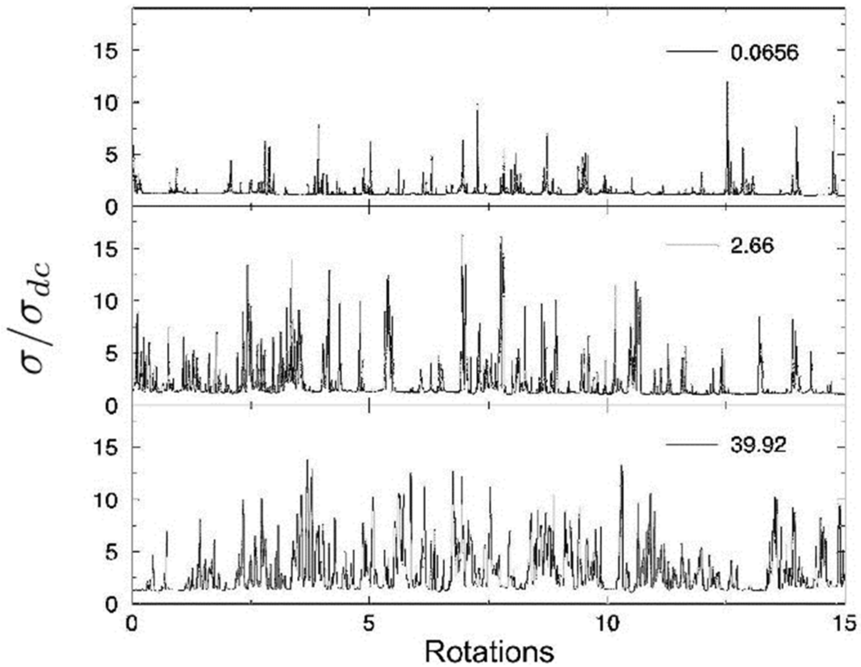


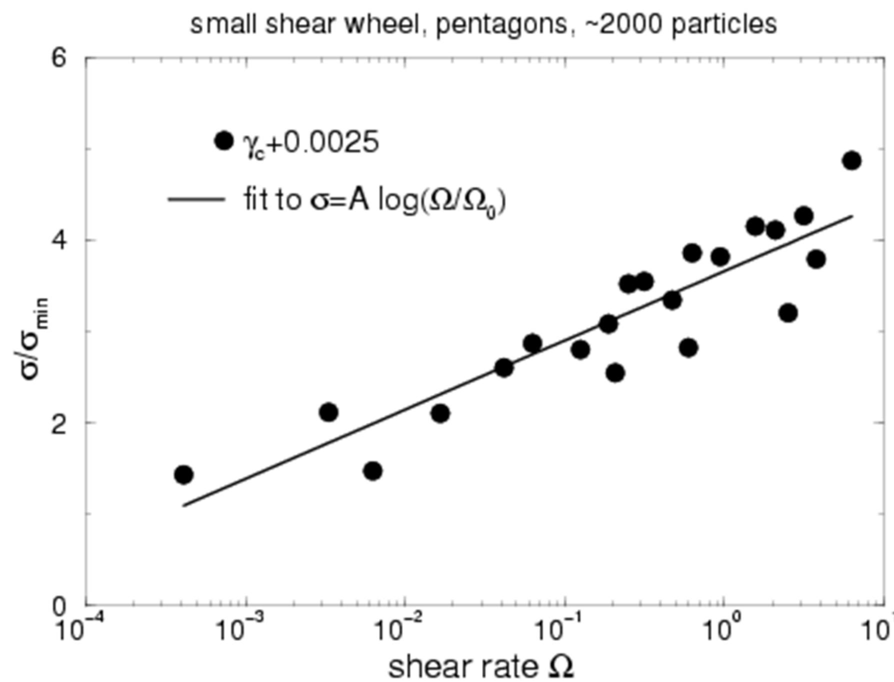


270mHz

Logarithmic rate dependence on shear rate

Hartley and RPB, Nature 2003





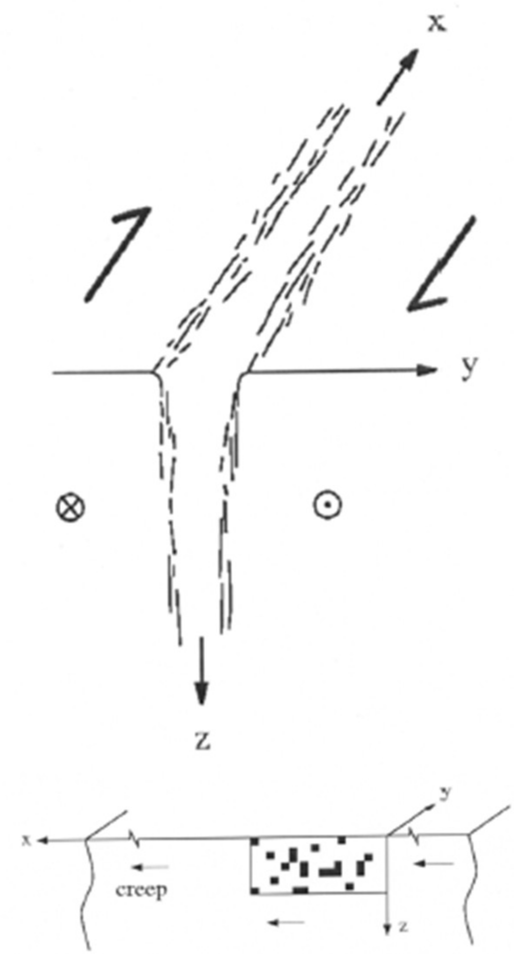
Mean Field model: Tyler Earnest and Karin Dahmen

Simple Model

Simple modification to
Ben-Zion/Rice earthquake model

$\sim \phi/\phi_{max}$

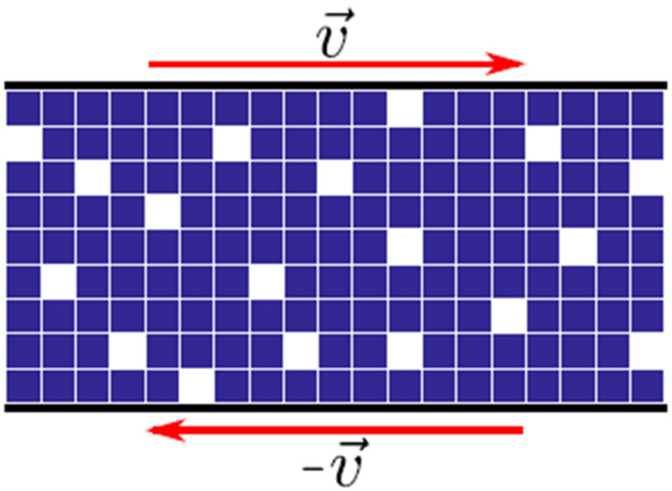
1. Partially occupied lattice, $\nu = N_{occ}/N$
2. Site slips if local $\tau > \tau_s$
3. Slips until its stress $\tau = \tau_a$.
4. Failed site weakens: $\tau_s \rightarrow \tau_d = \tau_s - \epsilon(\tau_s - \tau_a)$
5. Fraction of stress redistributed to other sites
   - Potentially triggering a slip avalanche



Simple Model

Stress on each site: $$\tau_i(t) = J o_i \left(\frac{1}{N} \sum_m o_m u_m(t) - u_i(t) \right) + K_L o_i [vt - u_i(t)]$$

Neglect dependence on distance between sites : MEAN FIELD THEORY



Consequences:

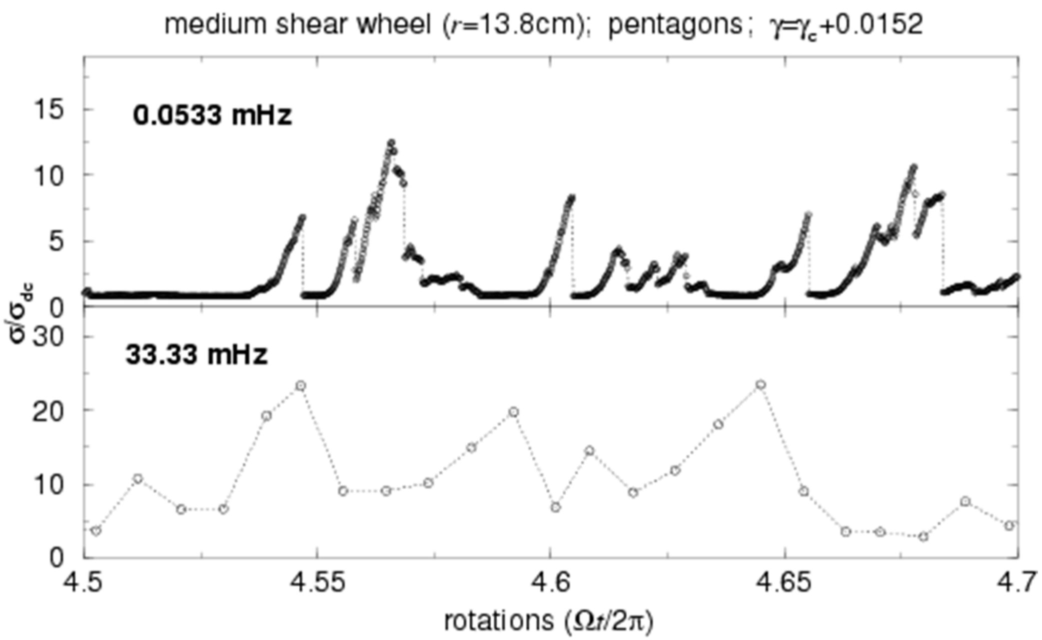
1. Mean field interface depinning universality class

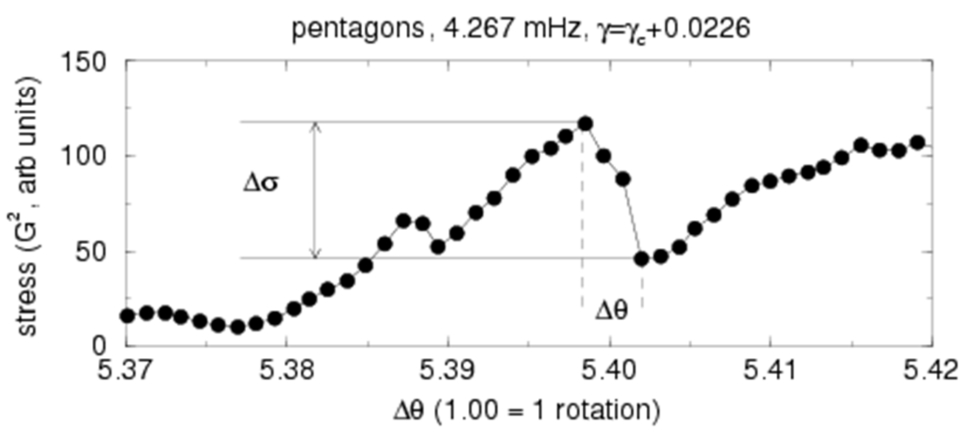
1. Voids dissipate fraction $(1-\phi/\phi_{max})$ of stress during slips

➡ mean avalanche size decreases with packing fraction ϕ

Results from Experiments:

Stress Drop Rate Distribution D(V)





Simple Mean Field Model Results:

(Ω= Shear rate)

Stress drop rate ($V=S/\Omega T$) distribution: $D(V) \sim V^{-\psi} D(V/V_{max})$

Avalanche duration (T) distribution: $D(T) \sim T^{-\alpha} D(T/T_{max})$

Power Spectra of stress time series: $P(\omega) \sim \omega^{-\phi} P(\omega/\omega_{min})$

Stress drop size (S) distribution: $D(S) \sim S^{-\kappa} D(S/S_{max})$

At high packing fraction $\phi \to \phi_{max}$	At slow shear rate $\Omega \to 0$
$V_{max} \sim \Omega^{-1-\rho}$	$V_{max} \sim (1-\phi/\phi_{max})^{-2\mu(1-1/\phi)}$
$T_{max} \sim \Omega^{-\lambda}$	$T_{max} \sim (1-\phi/\phi_{max})^{-\mu}$
$\omega_{min} \sim \Omega^{\lambda}$	$\omega_{min} \sim (1-\phi/\phi_{max})^{\mu}$
$S_{max} \sim \Omega^{-\lambda\phi}$	$S_{max} \sim (1-\phi/\phi_{max})^{-\mu\phi}$

$$\langle s^2 \rangle \sim \Omega^{-\eta_{s\Omega}} = \Omega^{-2\lambda(3-\kappa)}$$

$$\langle s^2 \rangle \sim (1-c\nu)^{-\eta_{s\nu}} = (1-c\nu)^{-2\mu(3-\kappa)}$$

$$\langle V^2 \rangle \sim \Omega^{-\eta_{V\Omega}} = \Omega^{-(\rho+1)(3-\psi)}$$

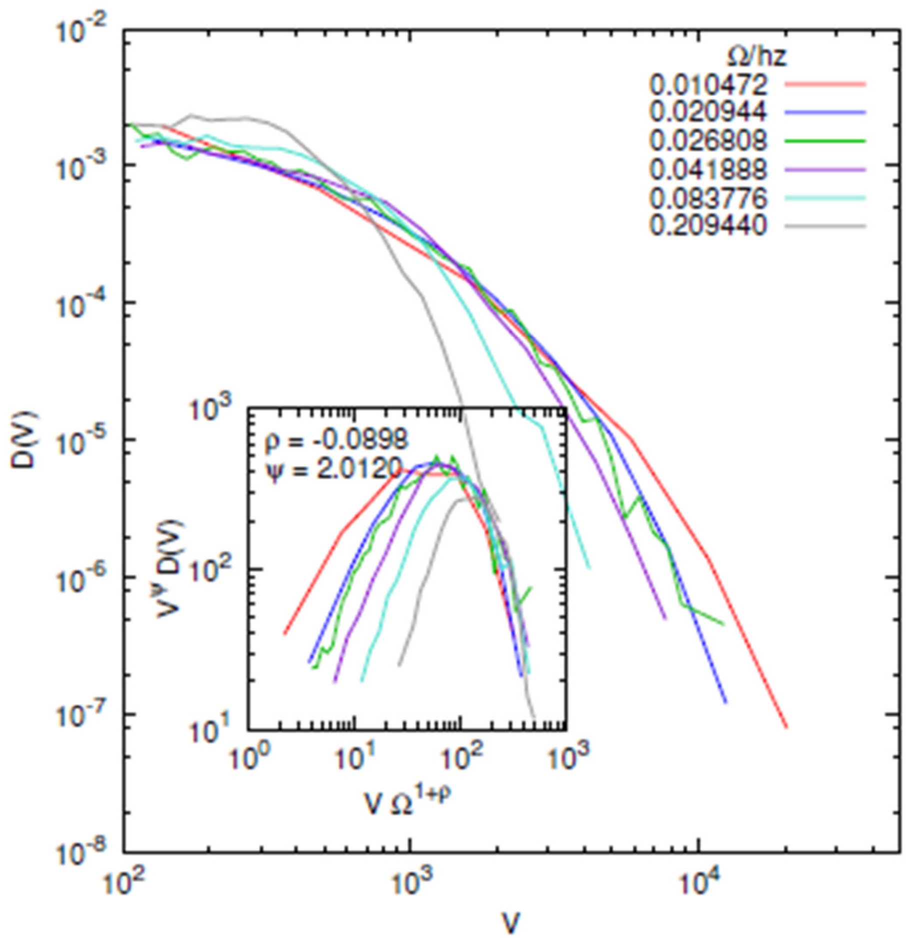
$$\langle T^2 \rangle \sim \Omega^{-\eta_{T\Omega}} = \Omega^{-\lambda(3-\alpha)}.$$

Exponent	MFT	Simulation	Experiment
ψ	2		2.01
α	2		1.75
ϕ	2		2.13
λ	1	0.3	0.3
ρ	1		0.09
$\eta_{V\Omega}$	2		1.00
$\eta_{V\nu}$	2		>0
$\eta_{s\Omega}$	3	-0.4	-0.1
$\eta_{T\Omega}$	1	0.7	0.67

" ="

Results from Experiments:

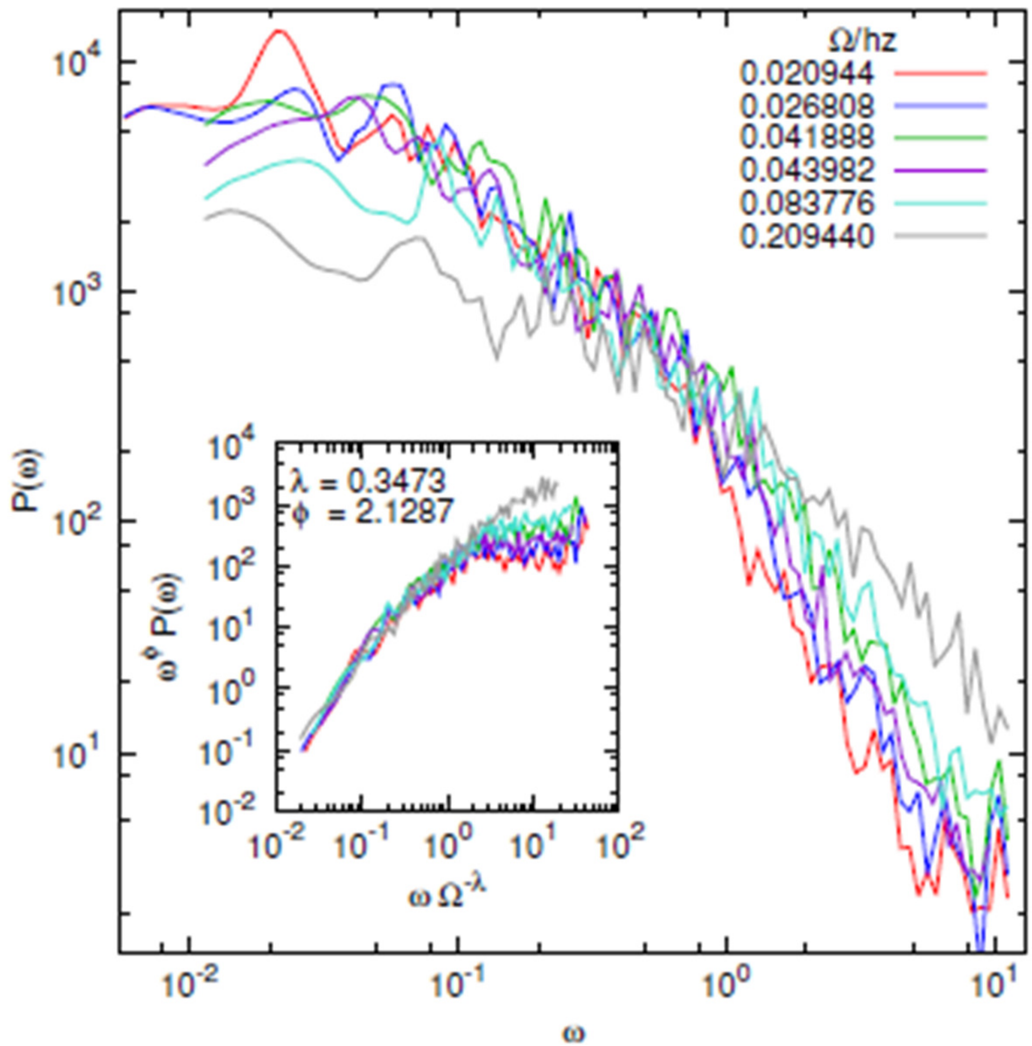
Stress Drop Rate Distribution D(V)



MFT: $\psi = 2$

Scaling collapse of stress drop rate $V := \frac{\Delta G^2}{\Omega \Delta T}$ with scaling form $P(V) \sim V^{-\psi} \mathcal{D}_{V\,r}(\Omega^{1+\rho} T)$.

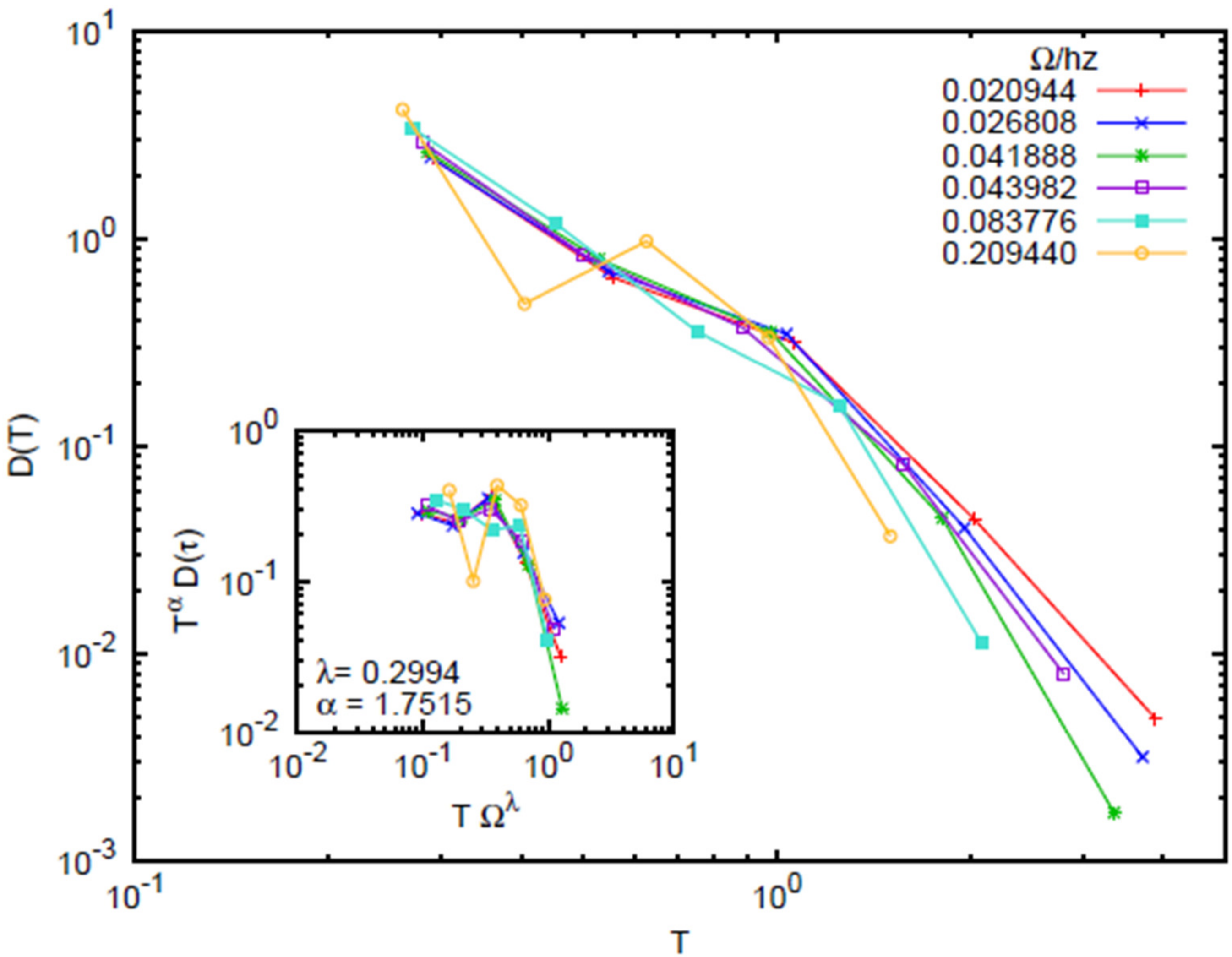
Power spectra P(ω) of stress time series:



MFT: φ = 2

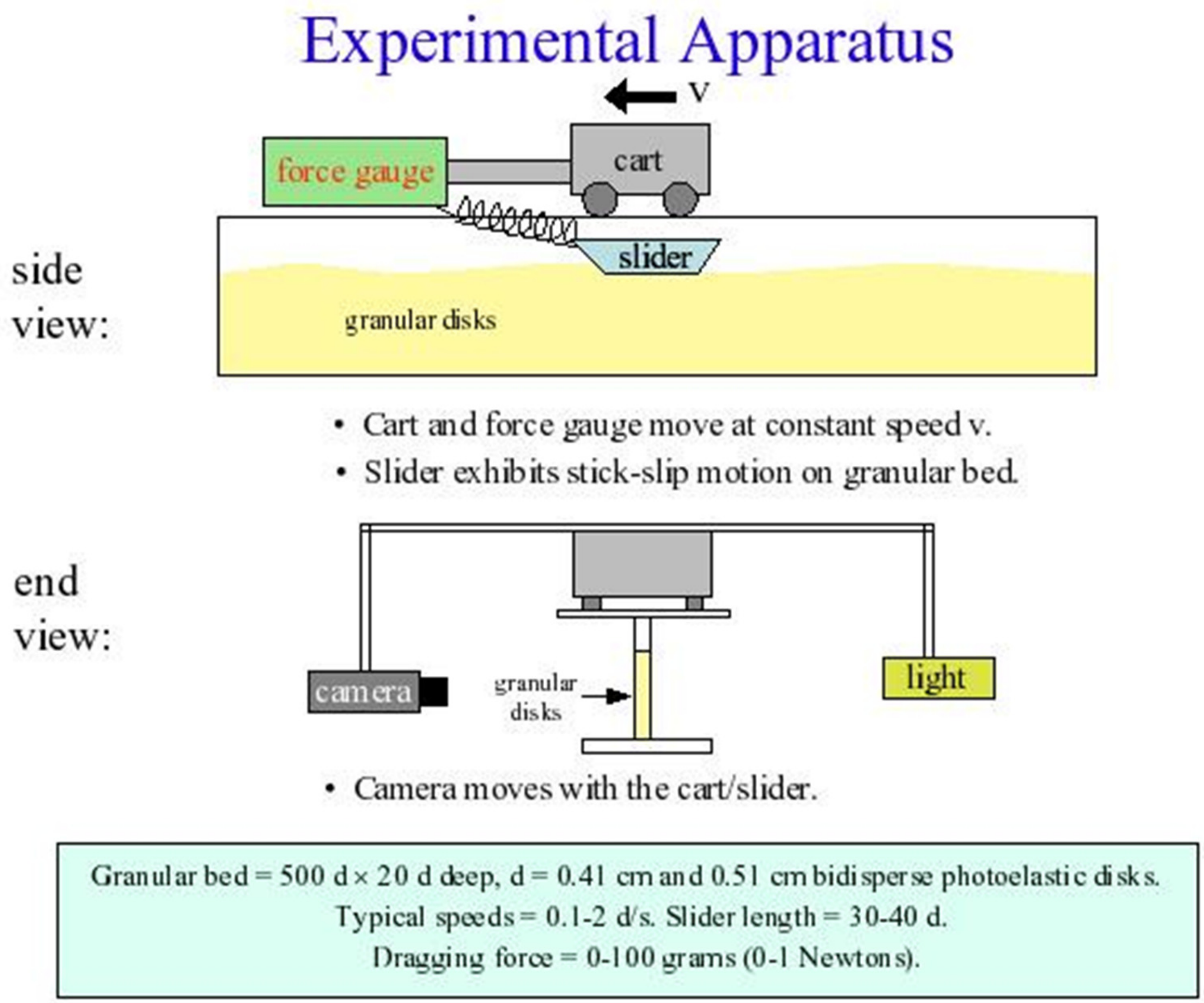
Figure 3: Scaling collapse of power spectra of G^2 at small system size at various shear rates calculated using an averaged Lomb periodogram. The scaling form is $S(\omega) \sim \omega^{-\phi}\mathcal{D}_{P_r}(\Omega^{-\lambda}\omega)$.

Stress Drop Duration Distribution D(T):



$$D(T) \sim T^{-\alpha} \mathcal{D}_{\mathrm{T}}(T/T_{\max}), \qquad T_{\max} \sim \Omega^{-\lambda}$$

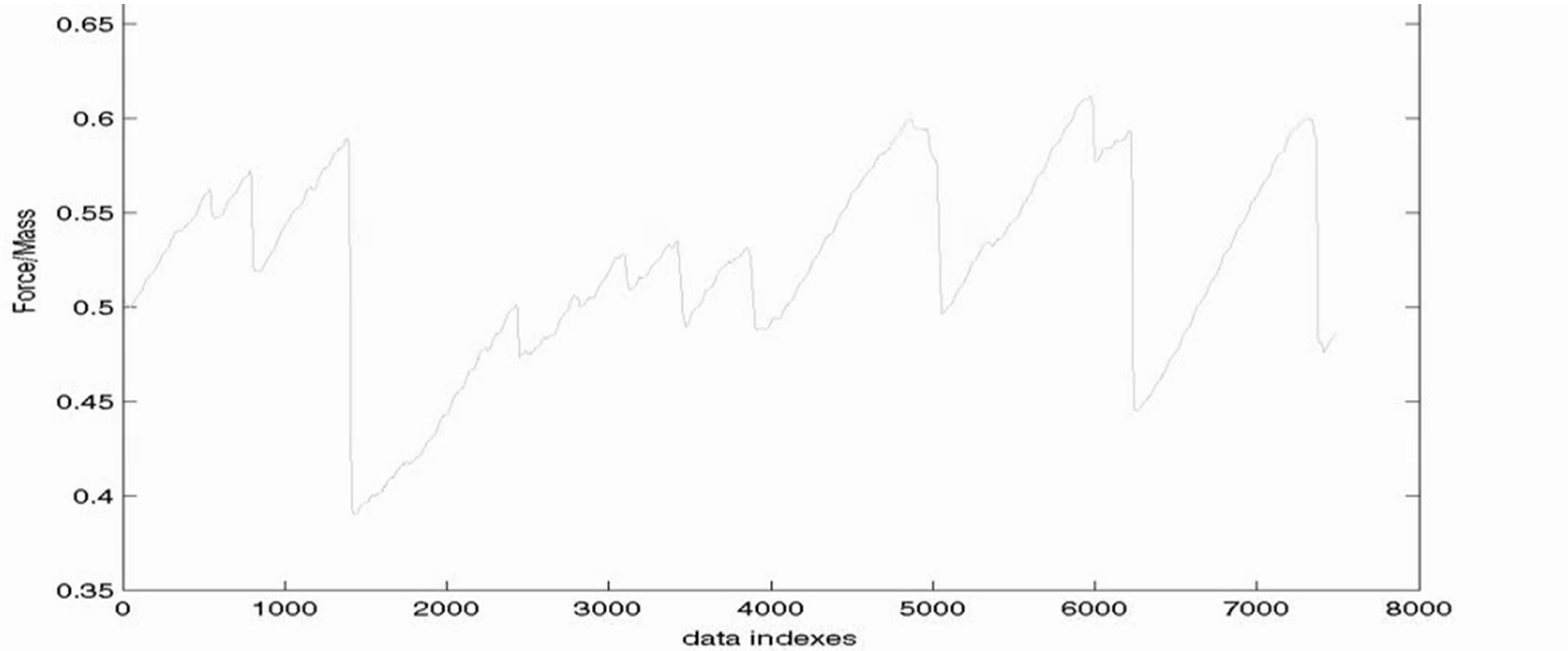
Granular Rheology—a slider experiment



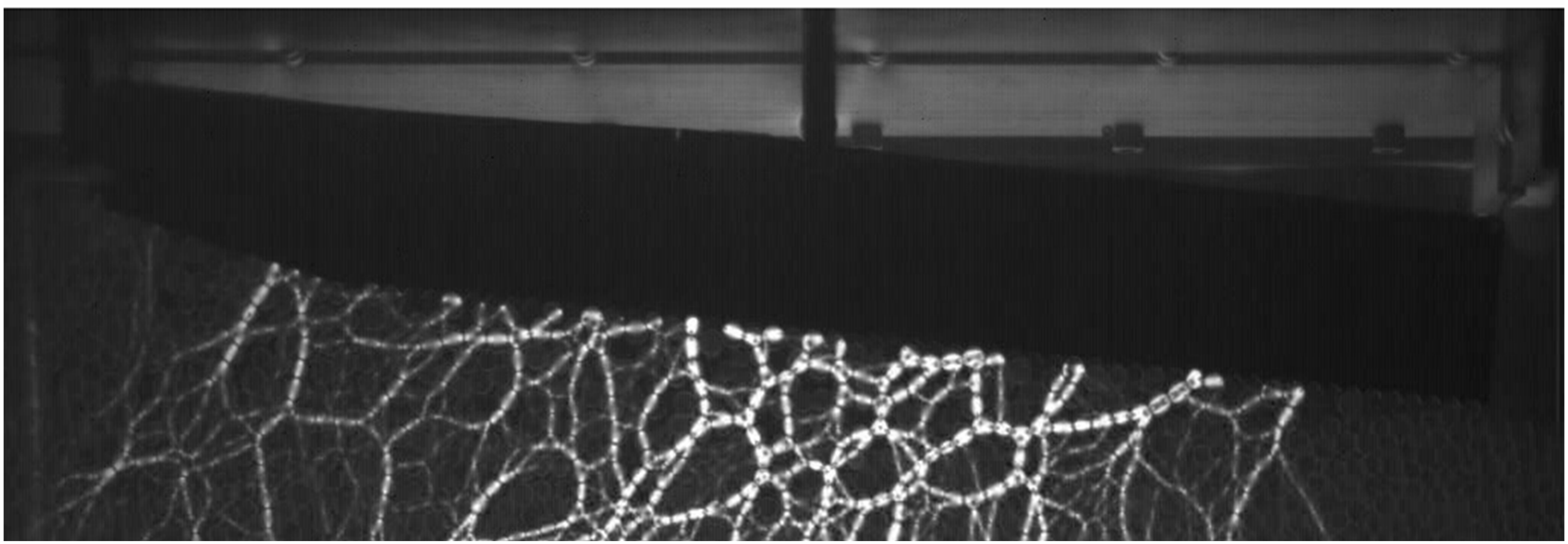
See e.g. Nasuno et al. Kudroli, Marone et al…

Experimental apparatus

What is the relation between stick slip and granular force structure?



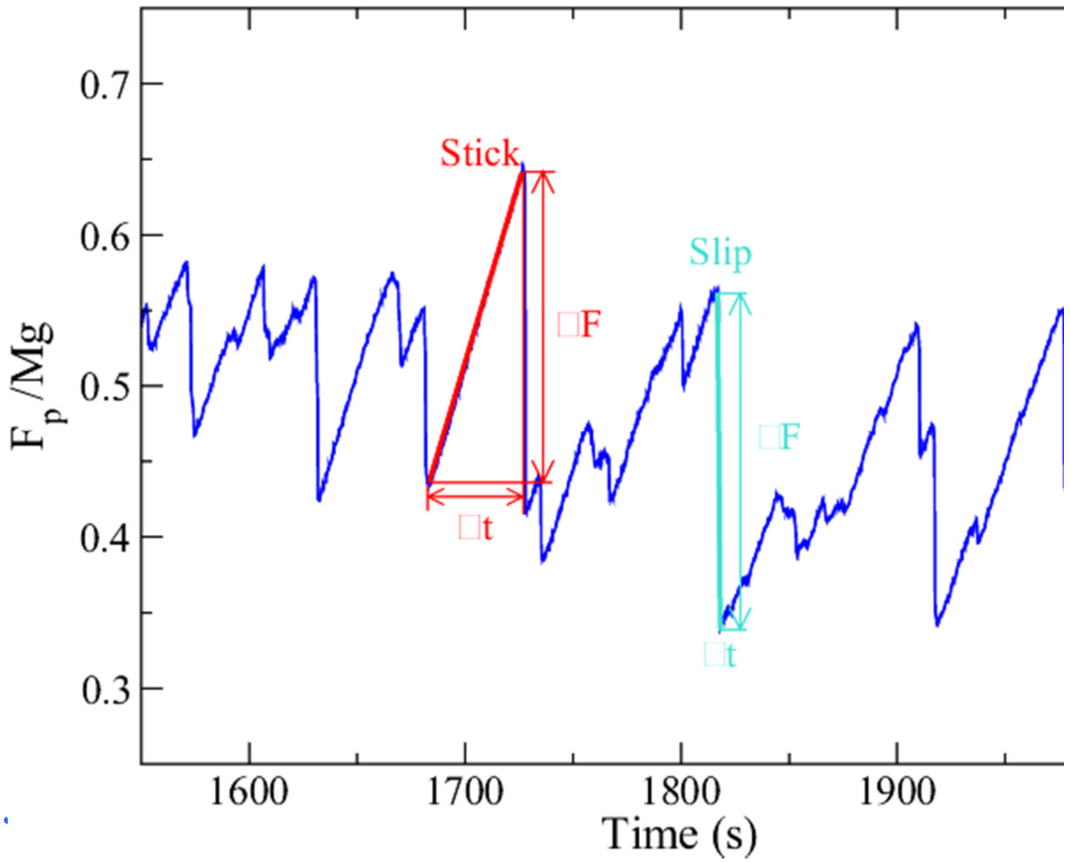
Force/Mass
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0
1000
2000
3000
4000
5000
6000
7000
8000
data indexes



Non-periodic Stick-slip motion

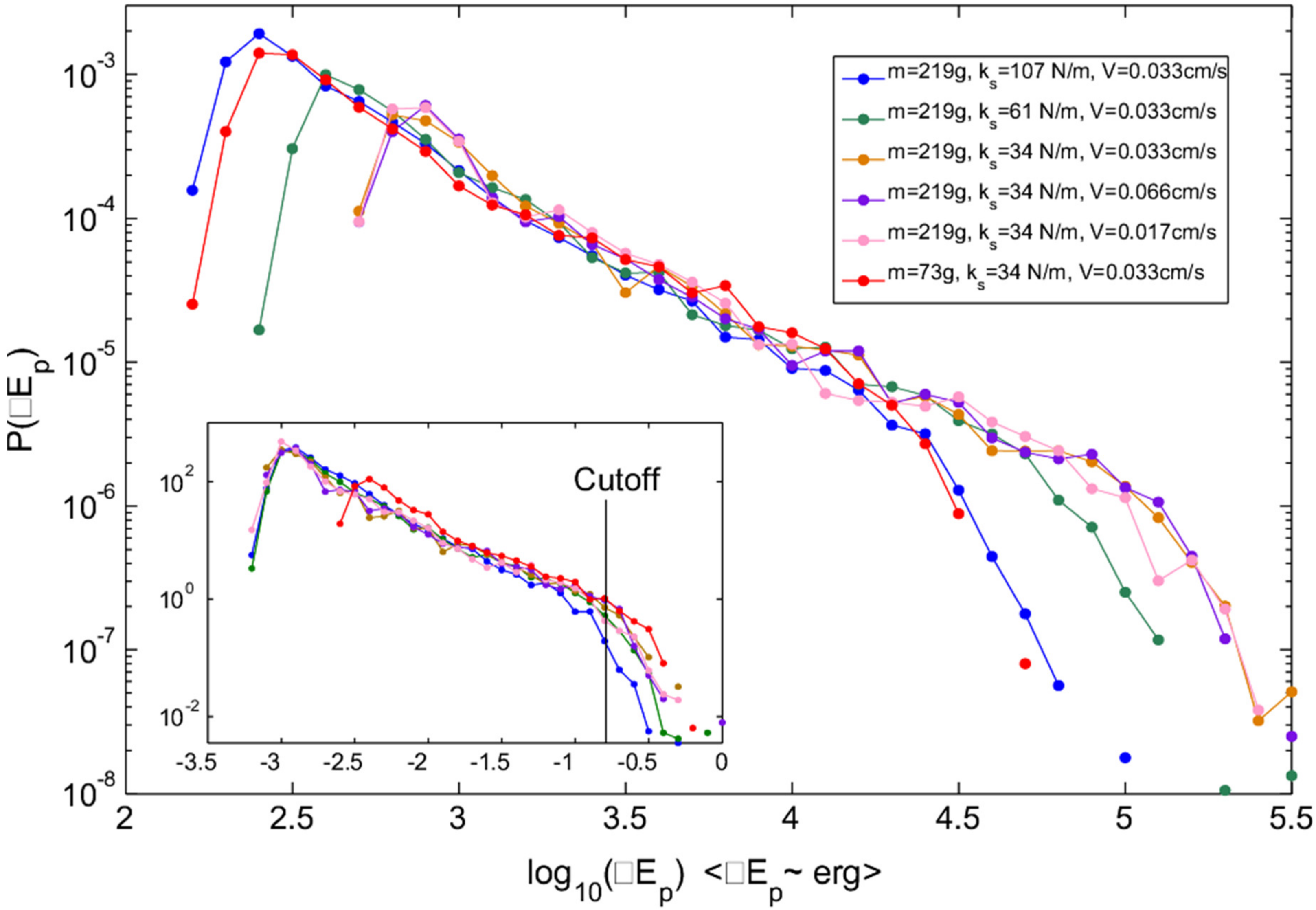
- Stick-slip motions in our 2D experiment are non-periodic and irregular
- Time duration, initial pulling force and ending pulling force all vary in a rather broad range
- Random effects associated with small number of contacts between the slider surface and the granular disks.

Size of the slider ~ 30-40 d

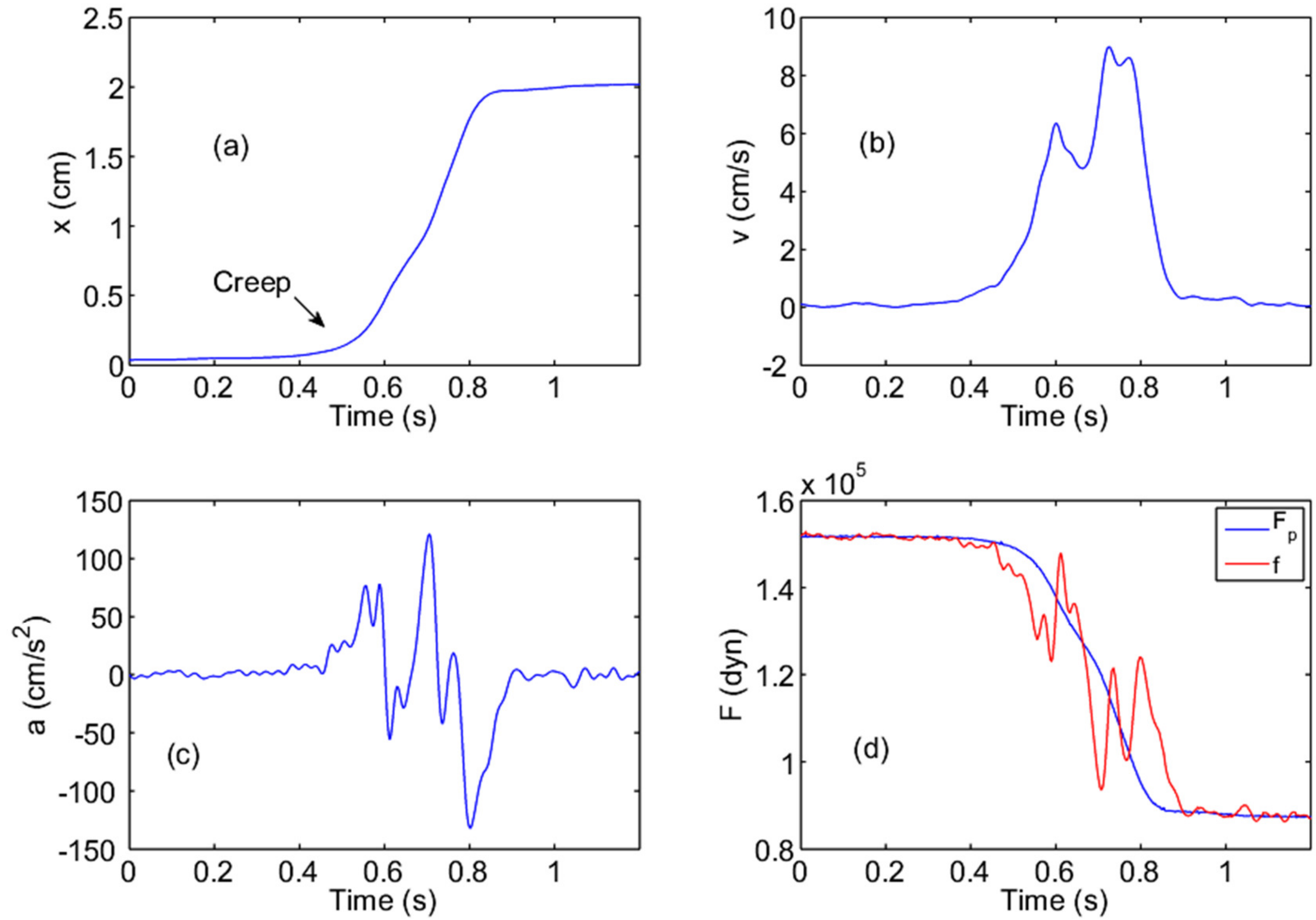


Definitions of stick and slip events

PDF of energy changes—exponent is ~1.2



Dynamics of actual slip events—note creep



Roadmap

- What/Why granular materials?
- Where granular materials and molecular matter part company—open questions of relevant scales

Use experiments to explore:

- Forces, force fluctuations ◄
- Jamming ◄
- Force response—elasticity
- Plasticity, diffusion
- Granular friction

Use experiments to explore:

- Forces, force fluctuations
- Jamming –distinguish isotropic and anisotropic cases◀

Isotropic (Standard) case

- Jamming—how disordered N-body systems becomes solid-like as particles are brought into contact, or fluid-like when grains are separated—thought to apply to many systems, including GM's foams, colloids, glasses…
- Density is implicated as a key parameter, expressed as packing (solid fraction) φ
- Marginal stability (isostaticity) for spherical particles (disks in 2D) contact number, Z, attains a critical value, Z_{iso} at φ_{iso}

Jamming

How do disordered collections of particles lose/gain their solidity?

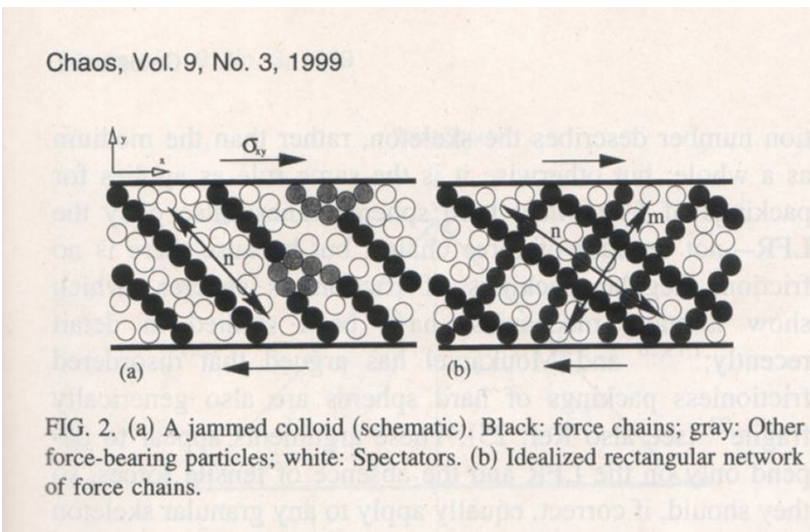
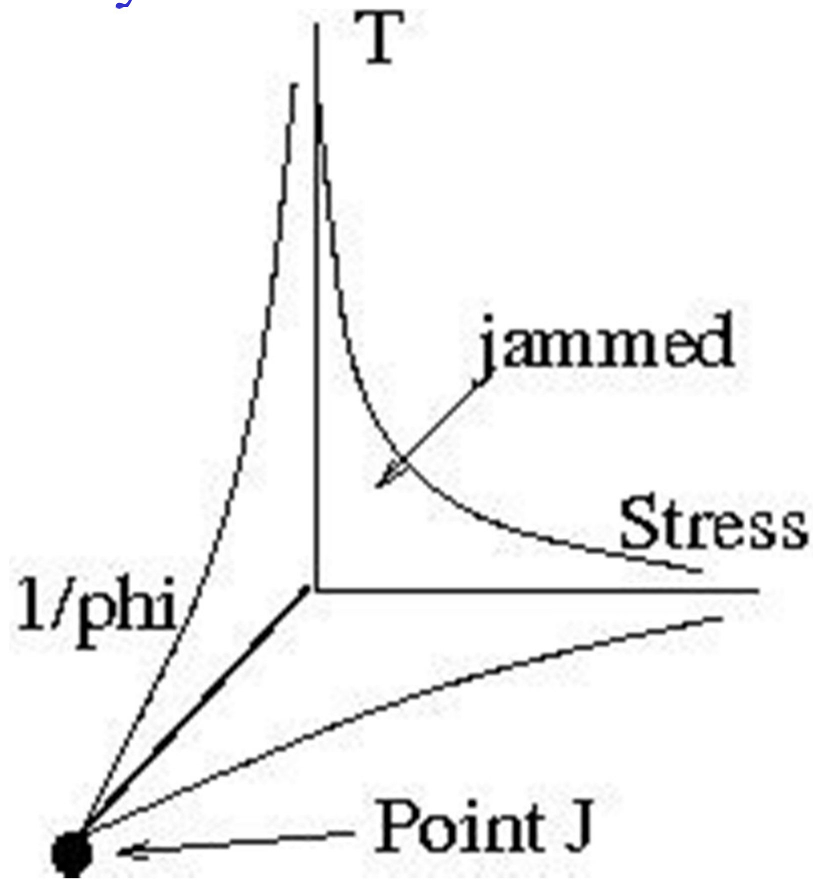

Chaos, Vol. 9, No. 3, 1999

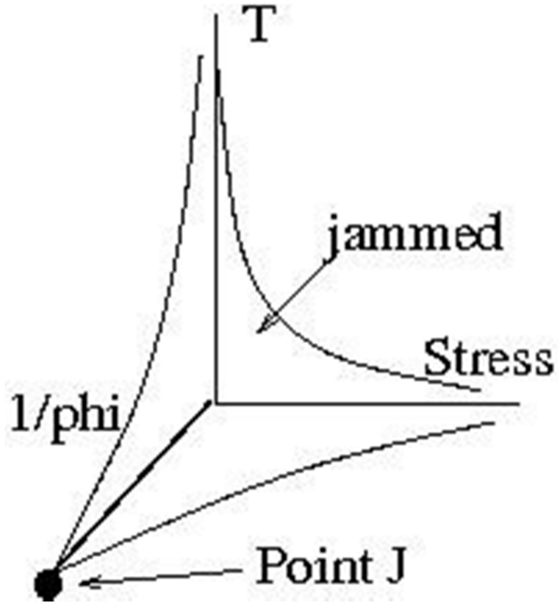
FIG. 2. (a) A jammed colloid (schematic). Black: force chains; gray: Other force-bearing particles; white: Spectators. (b) Idealized rectangular network of force chains.

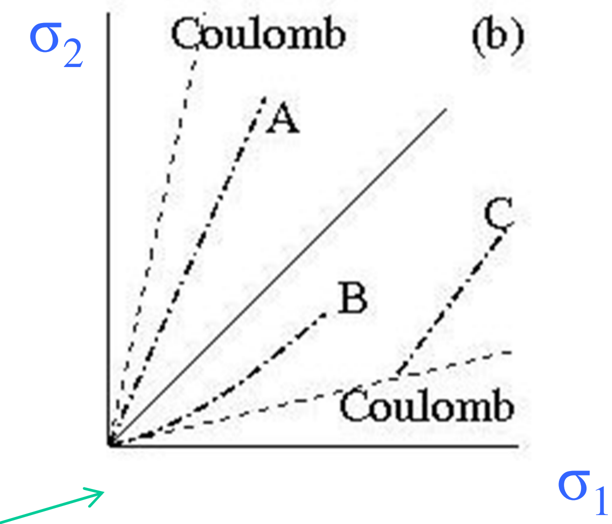
Bouchaud et al.



Liu and Nagel

Return to Jamming—now with Shear





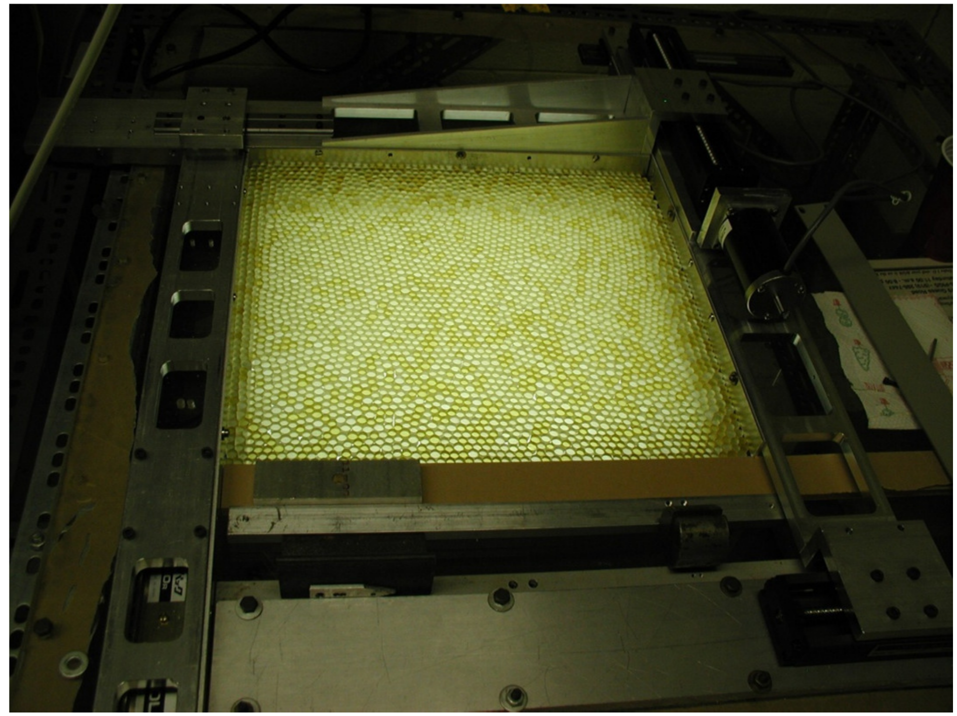
What happens here or here, when shear strain is applied to a GM?

Note: $P = (\sigma_2 + \sigma_1)/2$ $\quad :\tau = (\sigma_2 - \sigma_1)/2$

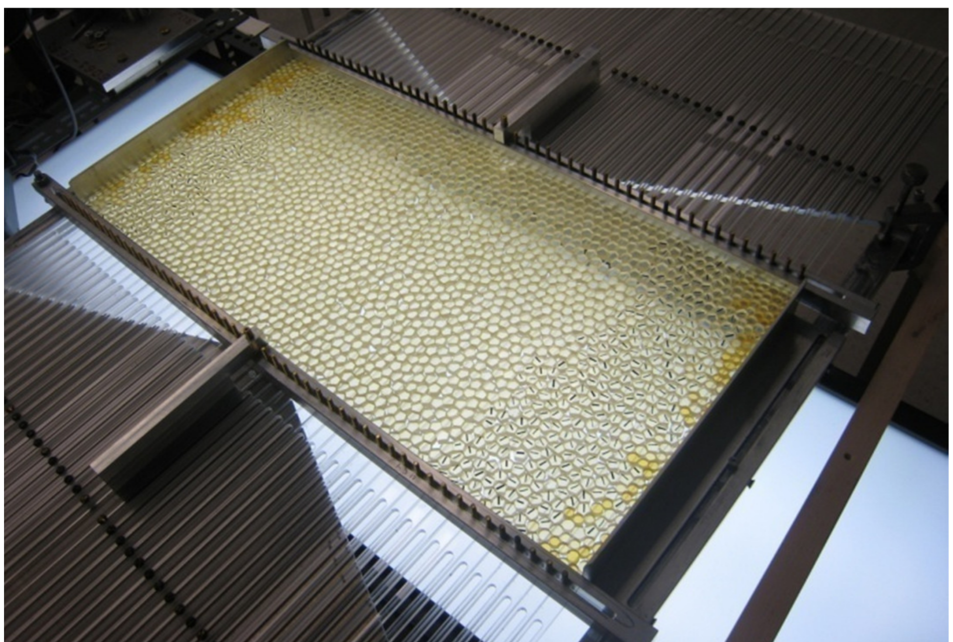
Coulomb failure: $\quad |\tau|/P = \mu$

Pure and simple shear experiments for photoelastic particles

Experiments use biaxial tester → and photoelastic particles



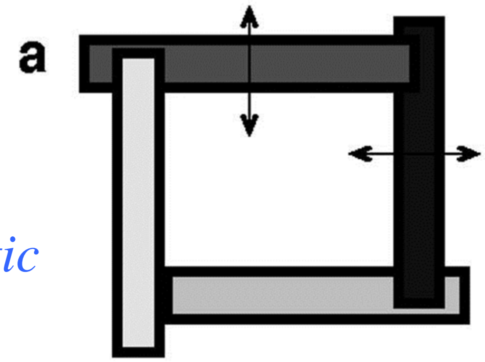
Majmudar and RPB, Nature, June 23, 2005)

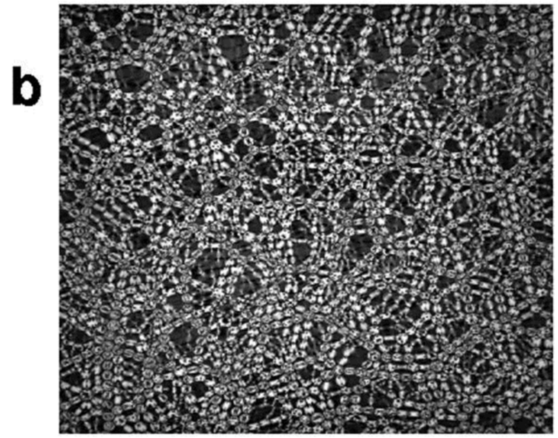


←...and simple shear apparatus with articulated base

Overview of Experiments

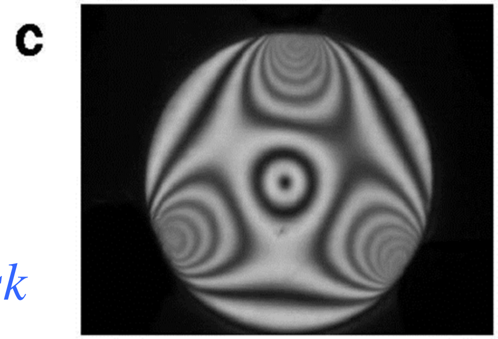
Biax schematic

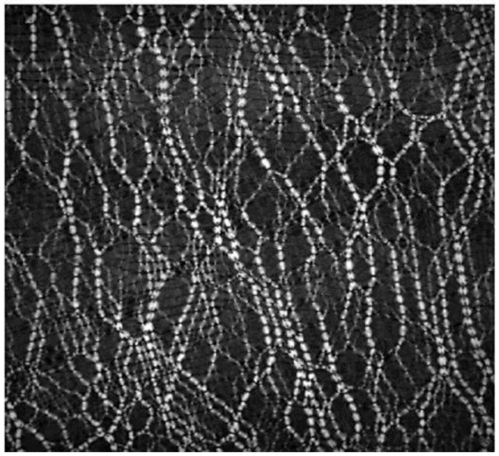




Compression

Image of Single disk

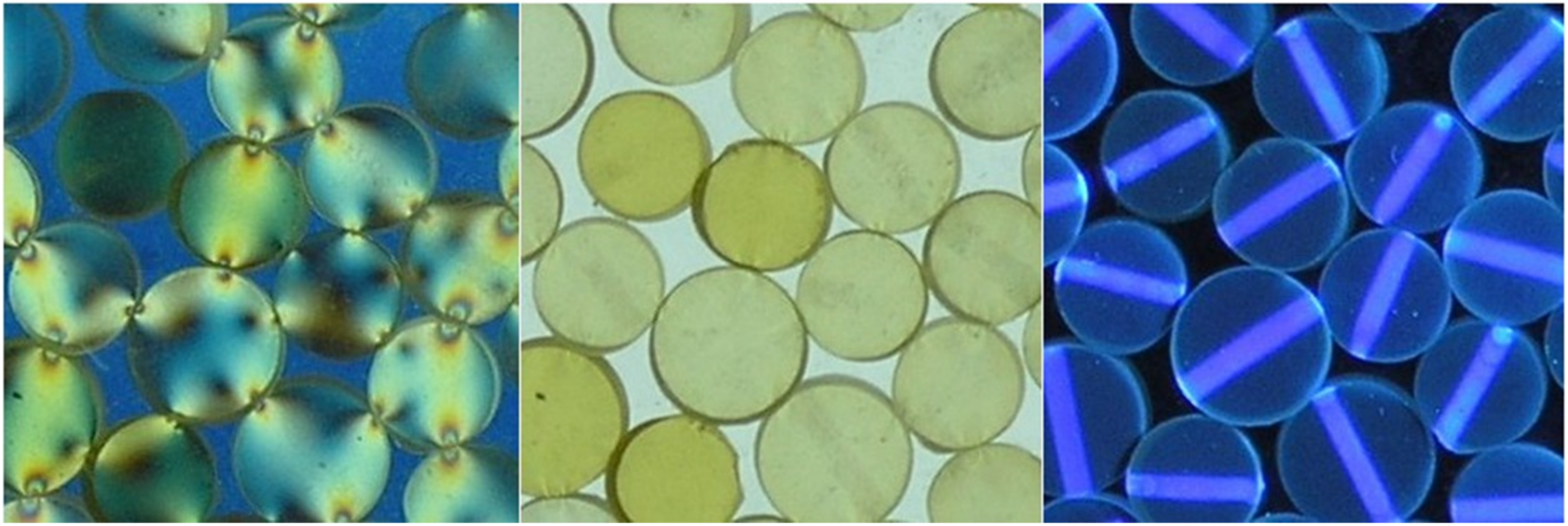




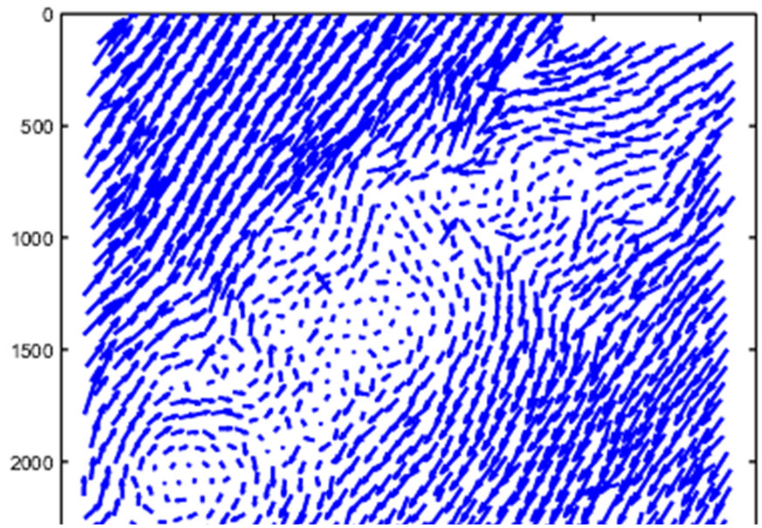
Shear

~2500 particles, bi-disperse, d_L*=0.9cm,* d_S*= 0.8cm,* $N_S/N_L = 4$

Track Particle Displacements/Rotations/Forces



Following a small strain step we track particle displacements



Under UV light—bars allow us to track particle rotations

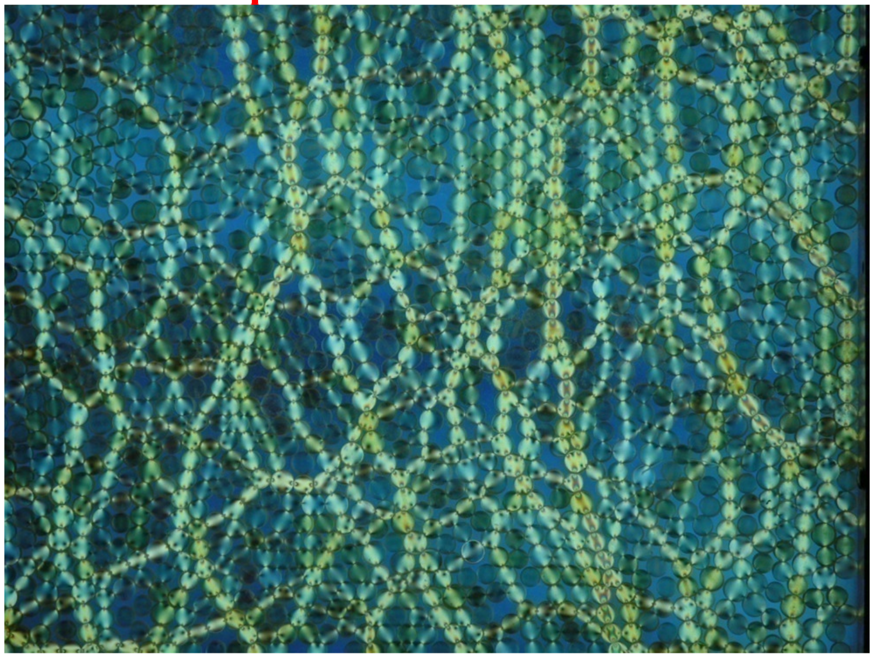
Basic principles of technique

Inverse problem:
photoelastic image of each disk → contact forces

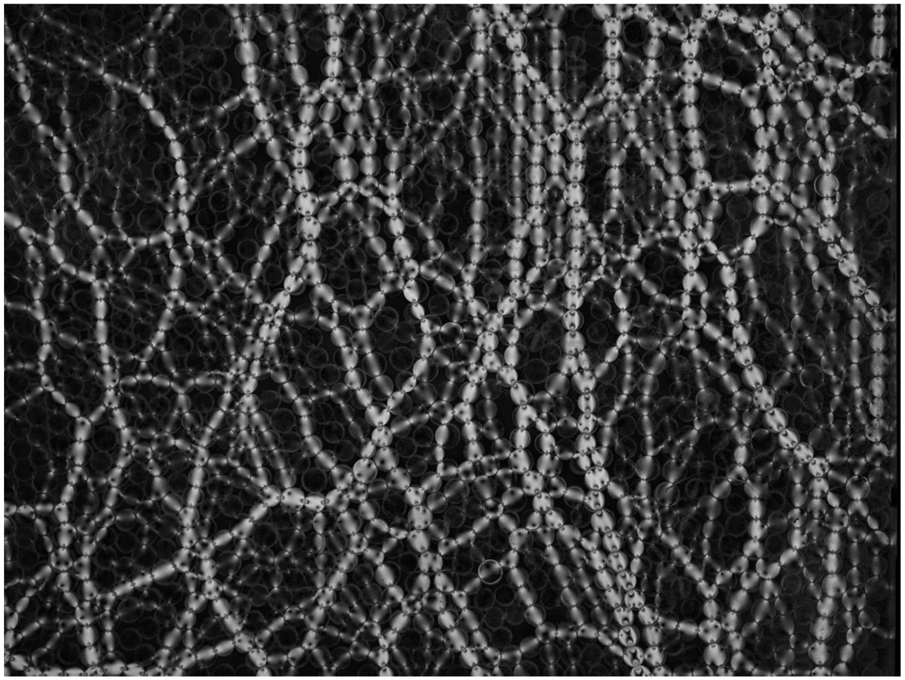
- Process images to obtain particle centers and contacts
- Invoke exact solution of stresses within a disk subject to localized forces at circumference
- Make a nonlinear fit to photoelastic pattern using contact forces as fit parameters
- $I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$
- In the previous step, invoke force and torque balance
- Newton’s 3d law provides error checking

Examples of Experimental and 'Fitted' Images

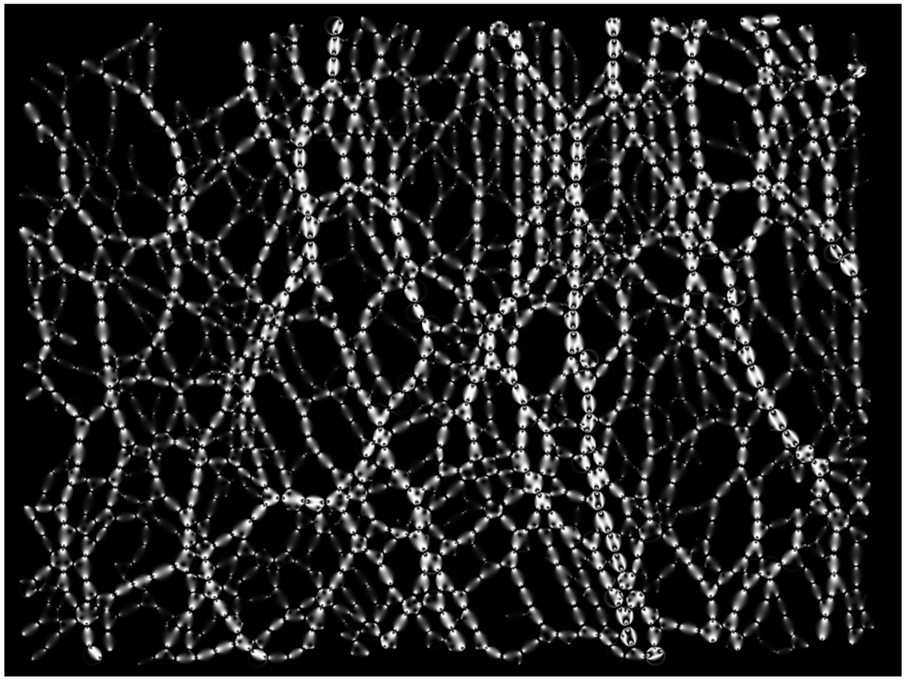
Experiment--raw



Experiment
Color filtered

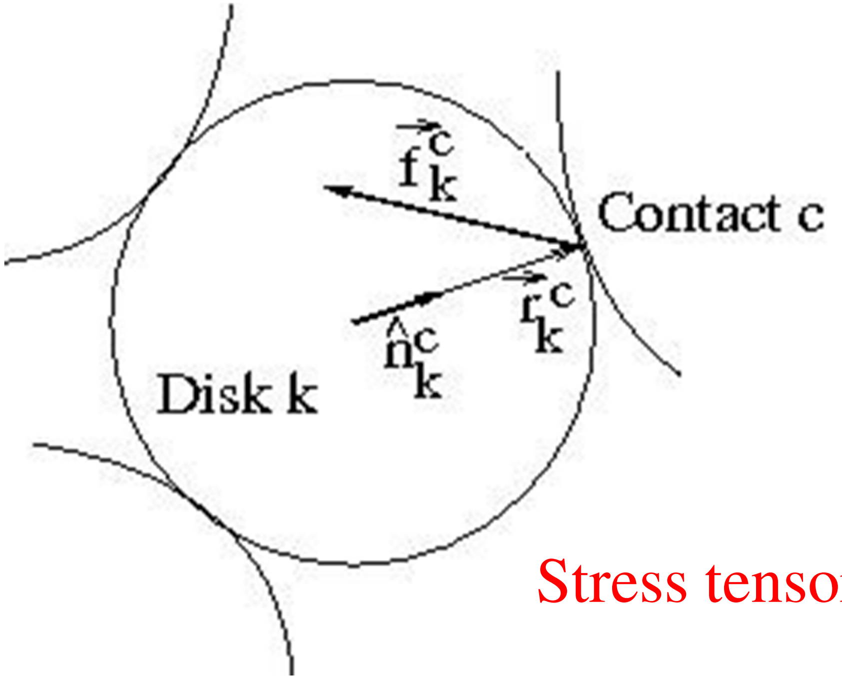


Reconstruction
From force
inverse algorithm



How do we obtain stresses and Z?

(Note: unique (?) for experiments to probe forces between particles inside a granular sample)



Fabric tensor

$R_{ij} = \Sigma_{k,c}\, n^c_{ik}\, n^c_{jk}$

$Z = \mathrm{trace}[R]$

Stress tensor (intensive)

-- $\sigma_{ij} = (1/A)\, \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk}$

and force moment tensor (extensive)

$\Sigma_{ij} = \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk} = A\, \sigma_{ij}$

A is system area

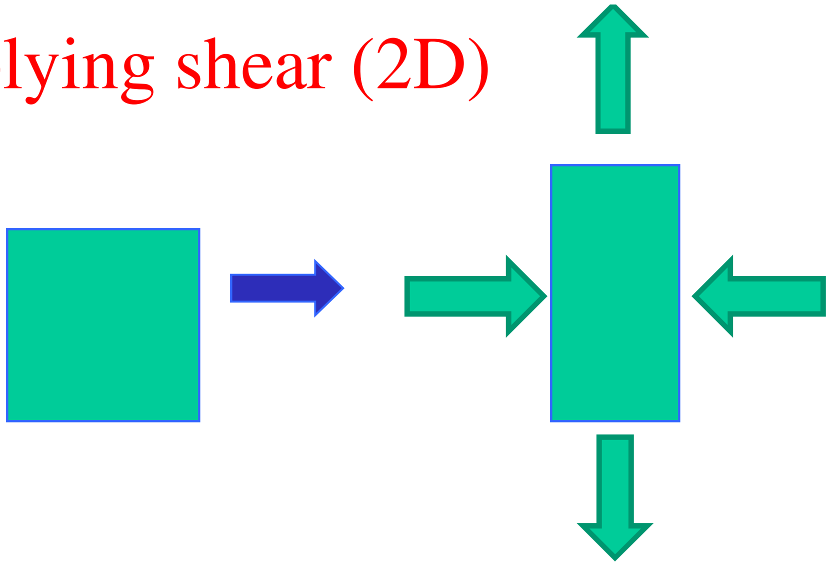
Pressure, P and Γ

$P = \mathrm{Tr}\,(\sigma)/2$

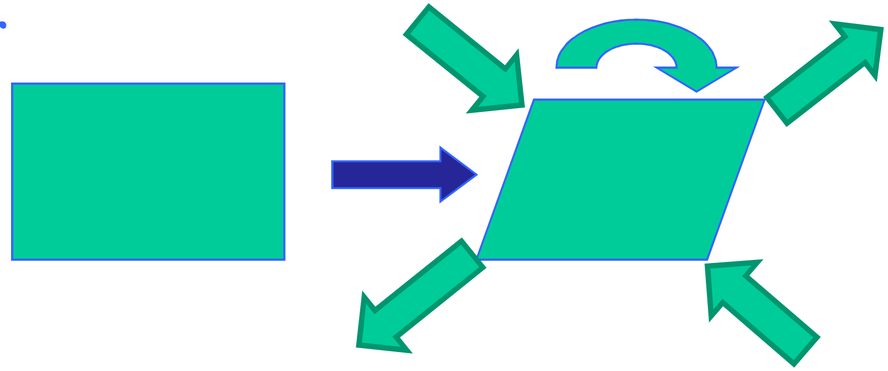
$\Gamma = \mathrm{Tr}\,(\Sigma)/2$

Different types methods of applying shear (2D)

- Example1: pure shear

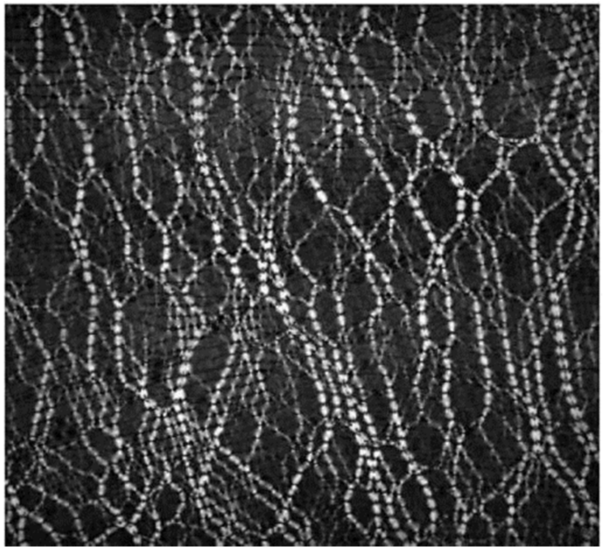


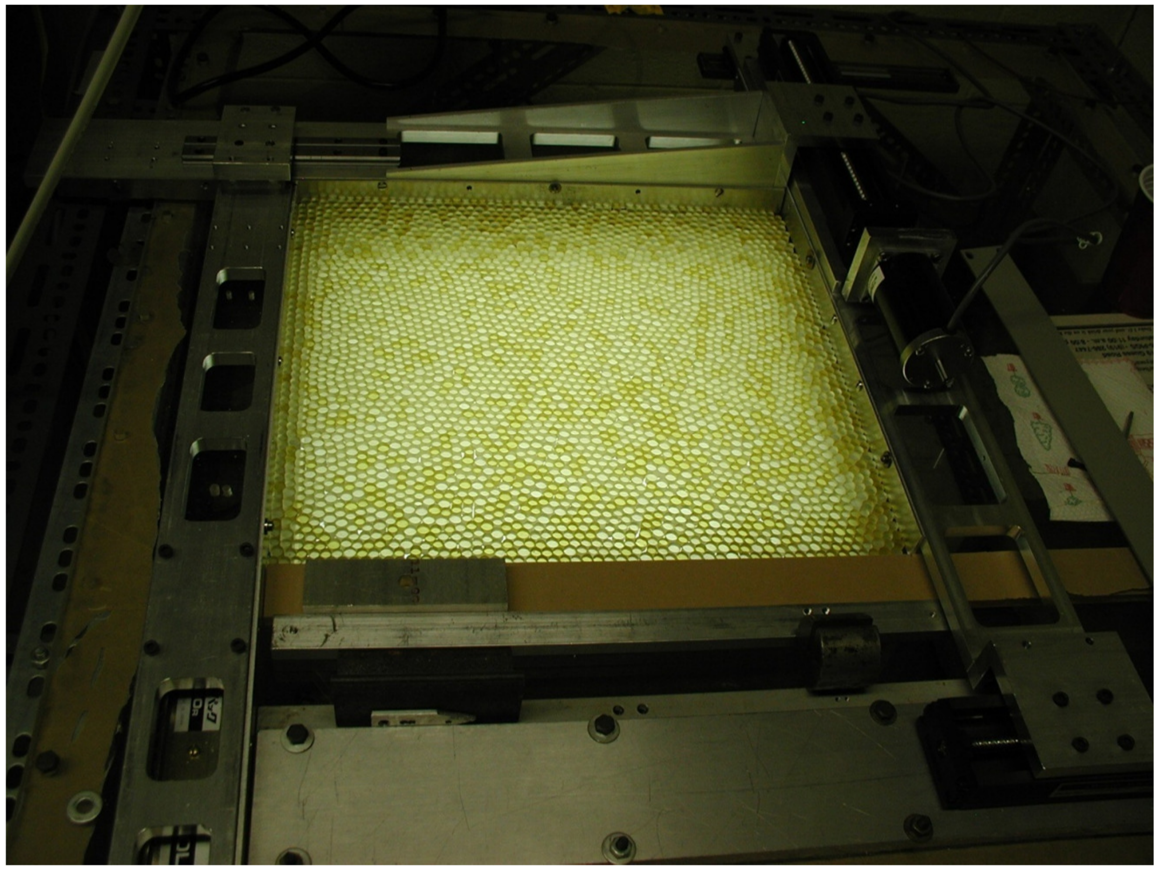
- Example 2: simple shear



- Example 3: steady shear

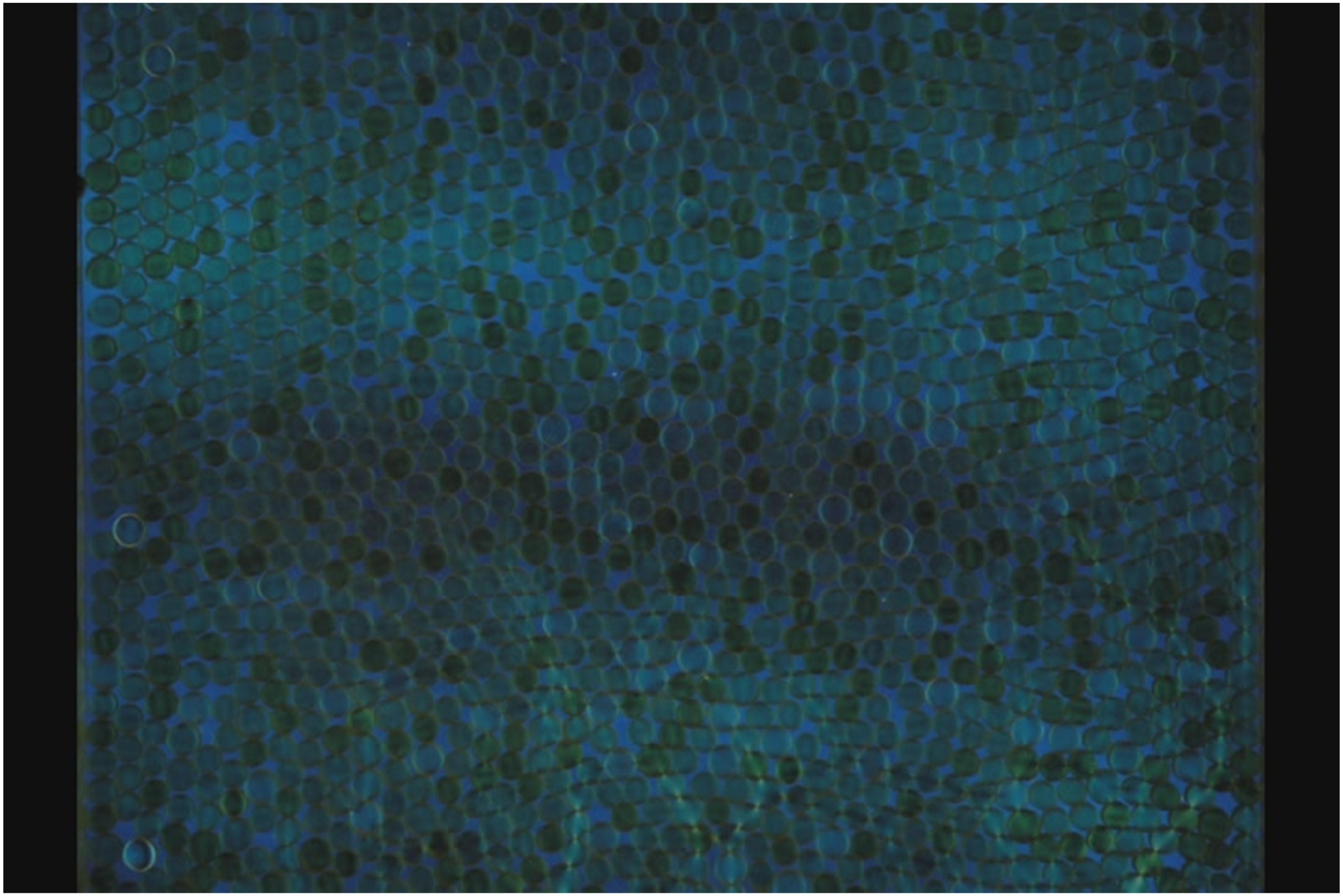
First: Pure Shear Experiment (both use photoelastic particles):

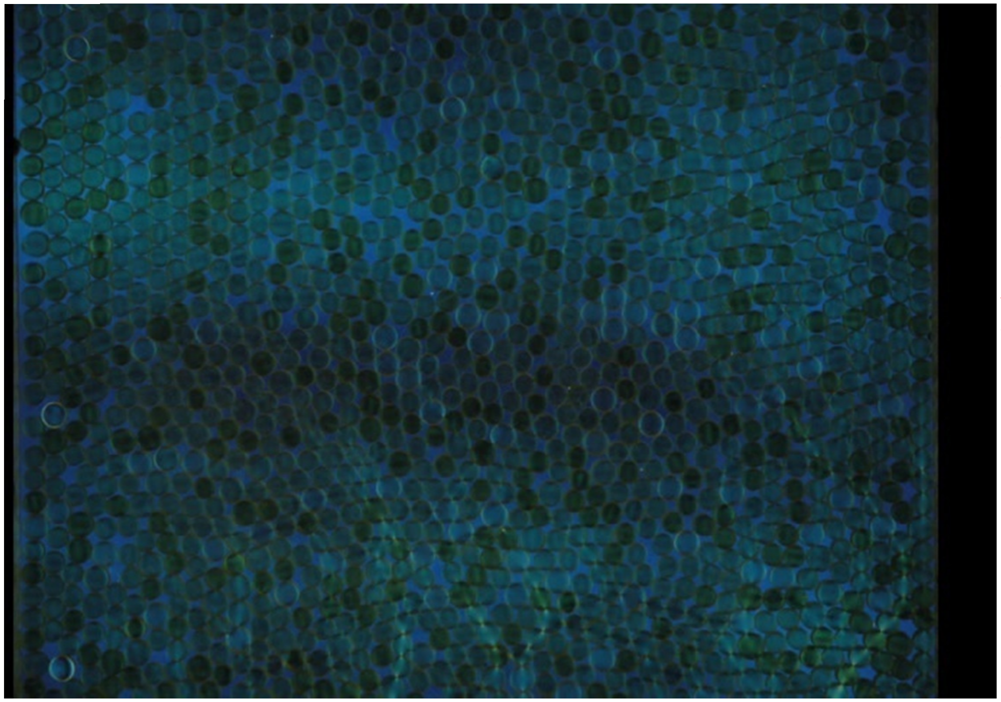




(Trush Majmudar and RPB,
Nature, June 23, 2005)
J. Zhang et al.
Granular Matter **12**, 159 (2010))

Time-lapse video (one pure shear cycle) shows force network evolution (J. Zhang et al. Granular Matter **12**, 159 (2010))

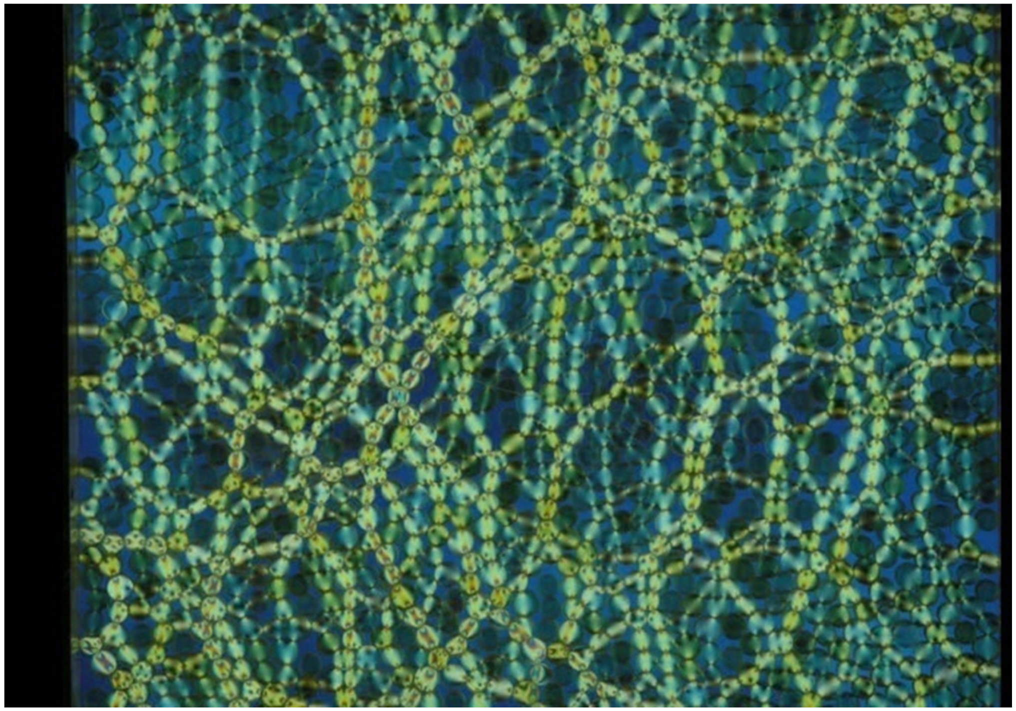




Initial and final states following a shear cycle—no change in area

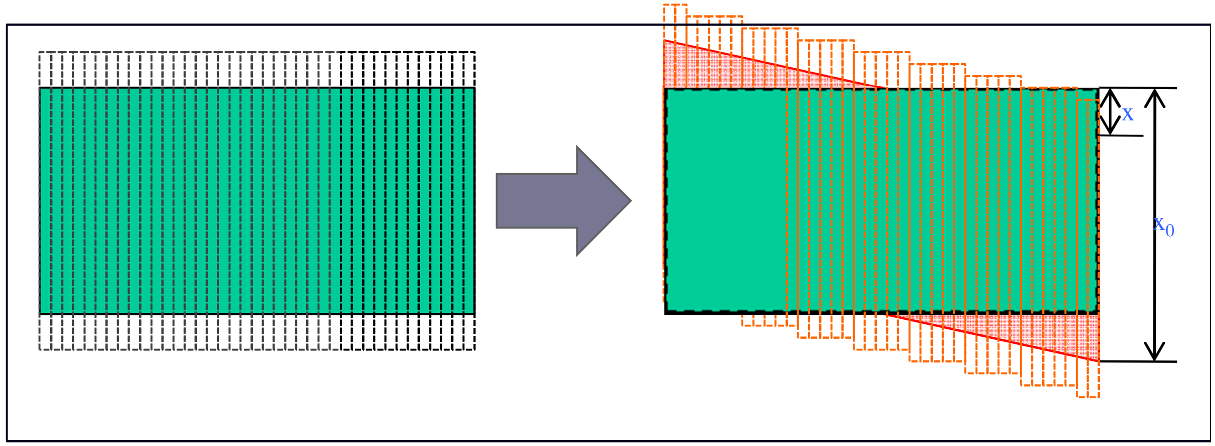
← Initial state, isotropic, no stress

Final state → large stresses



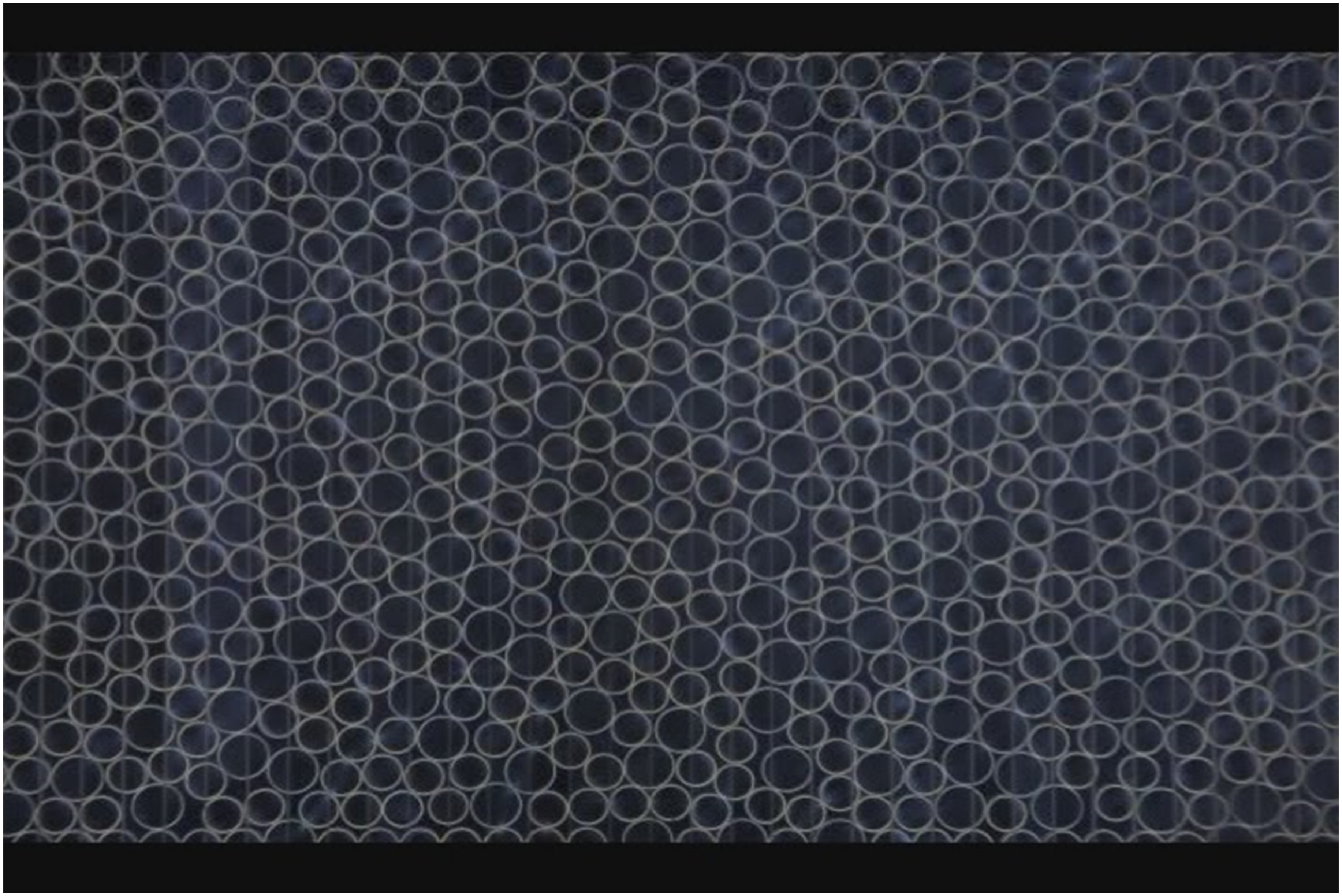
2nd apparatus: quasi-uniform simple shear

J. Ren et al. to be published

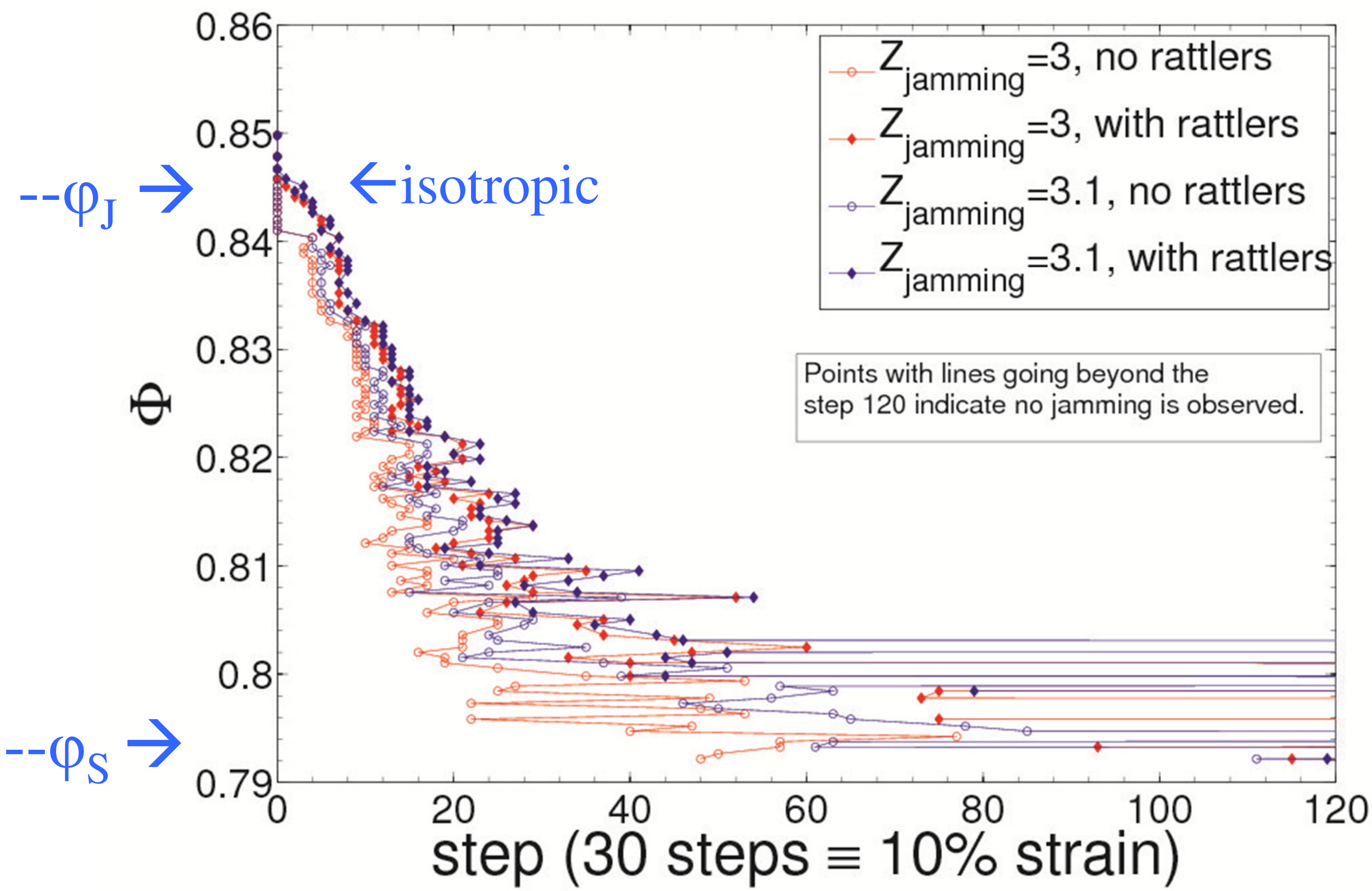


Goal of this experiment: Apply uniform shear everywhere, not just by deforming walls

Time-Lapse Video of Shear-Jamming

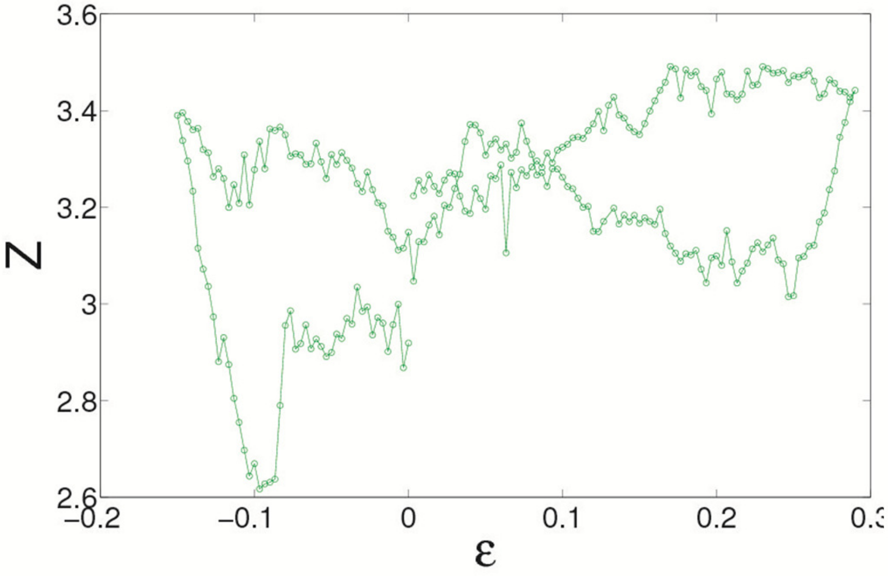


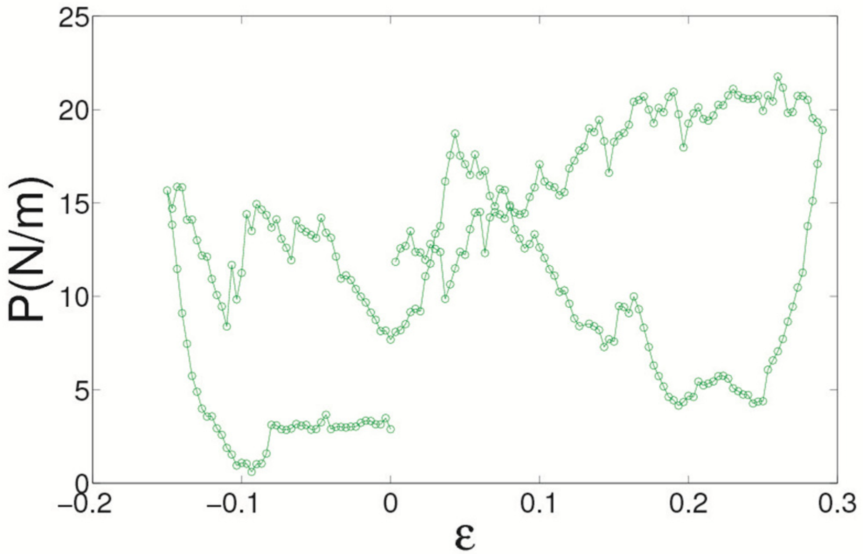
Return to biax--
Shear jamming for densities below φ_J
$\varphi_S < \varphi < \varphi_J$

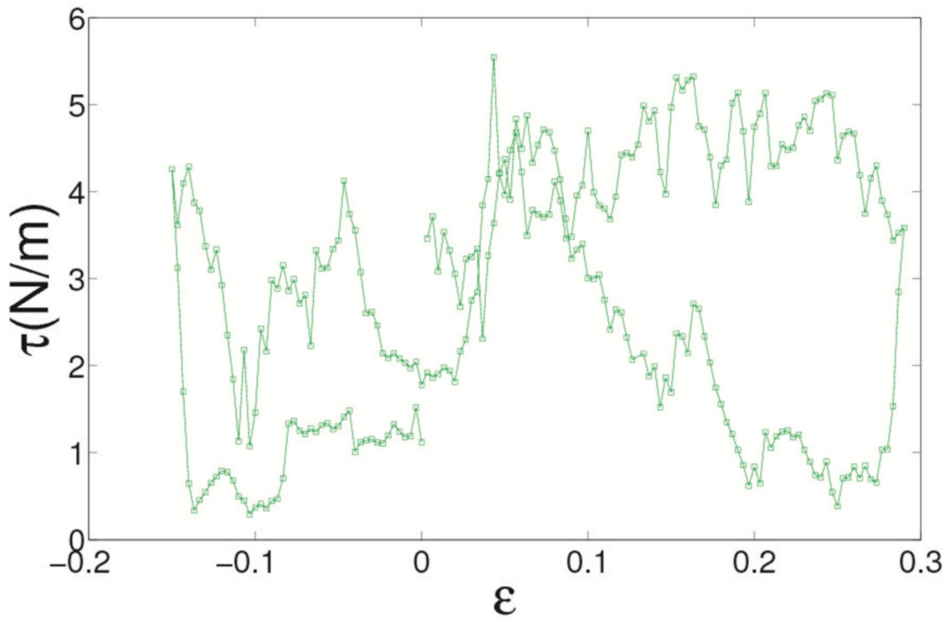


Note: $\pi/4 = 0.785\ldots$

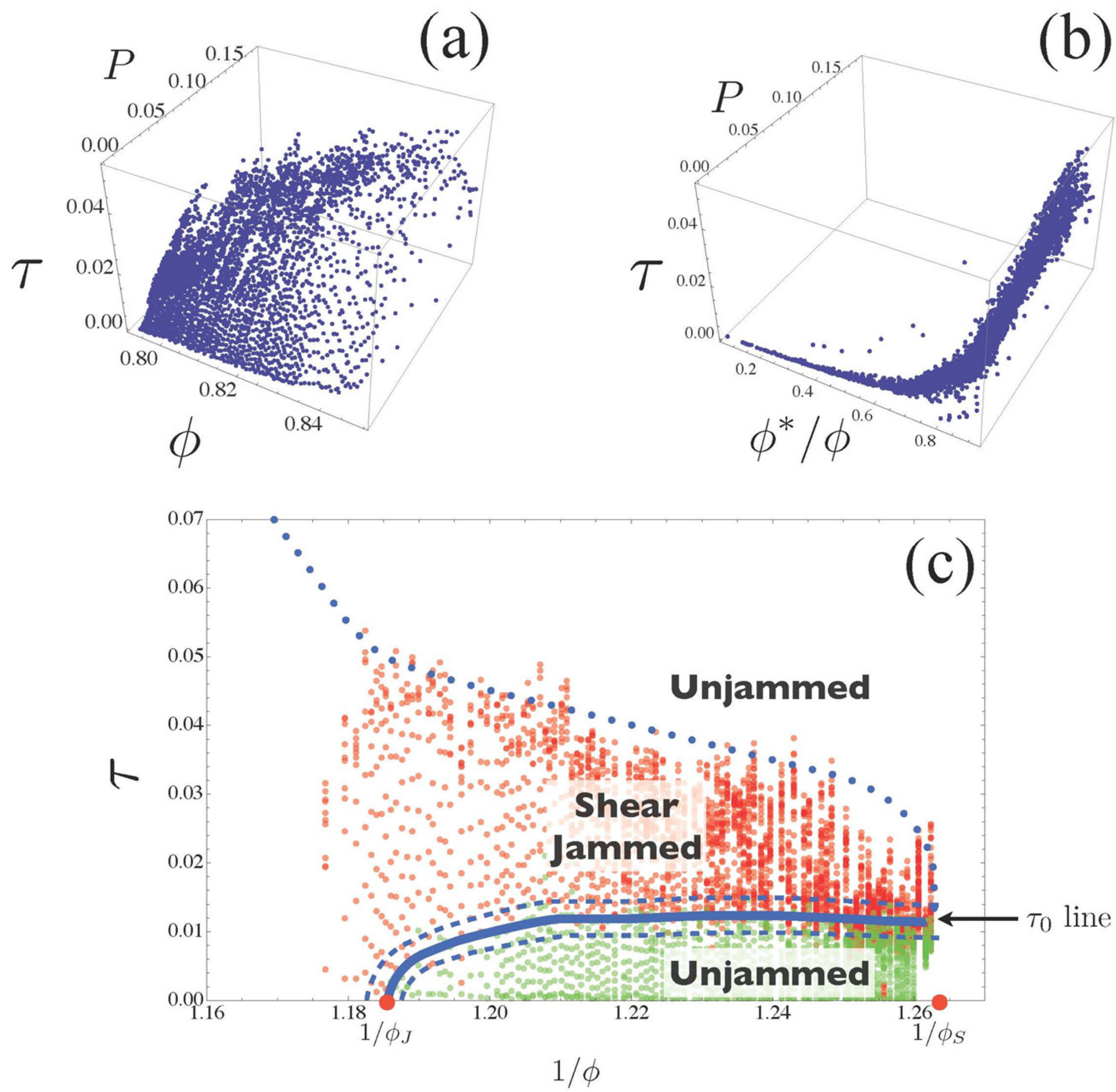
Hysteresis in stress-strain and Z-strain curves—one cycle





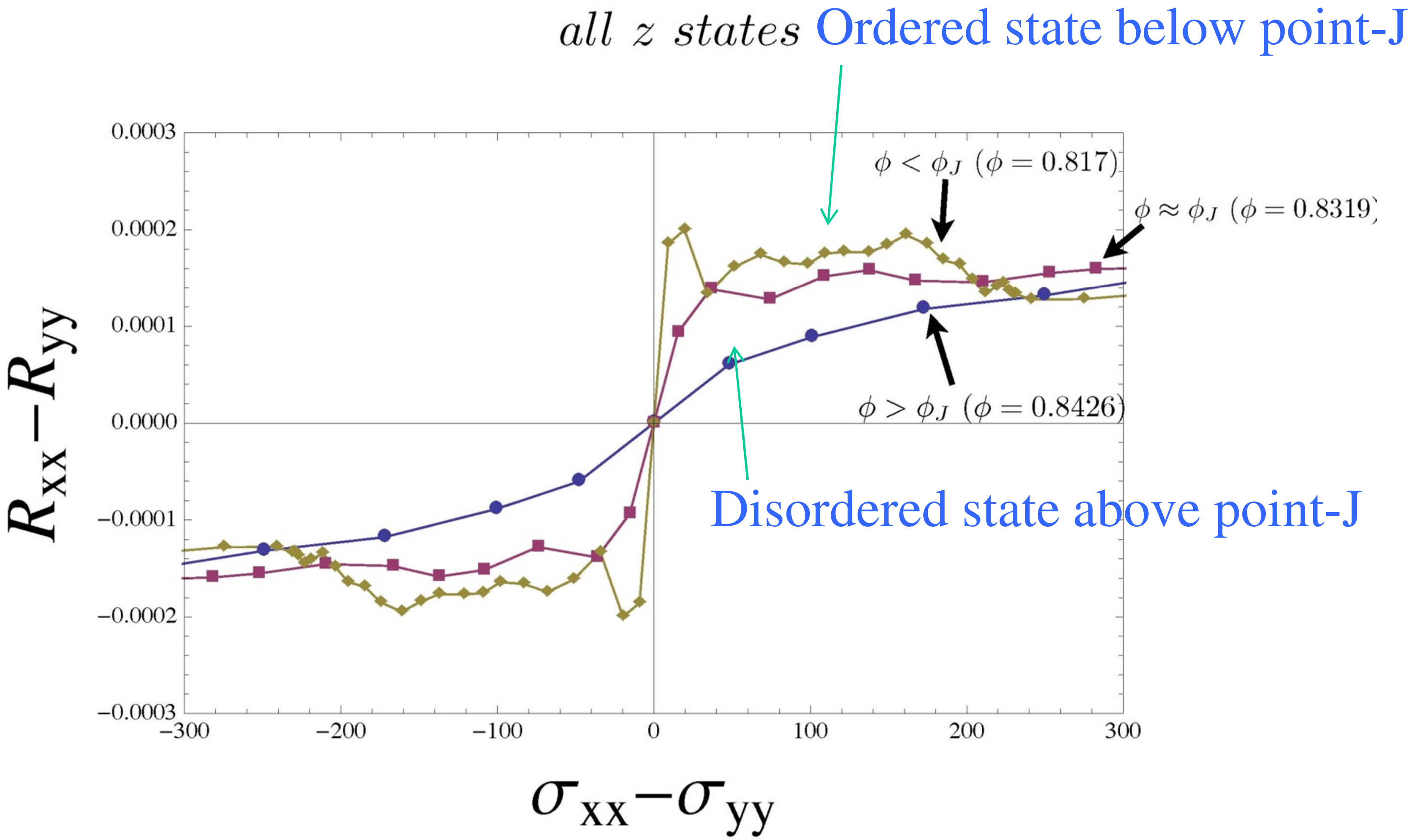


Despite hysteresis: striking collapse of data: Use φ*/φ = number of non-rattler particles



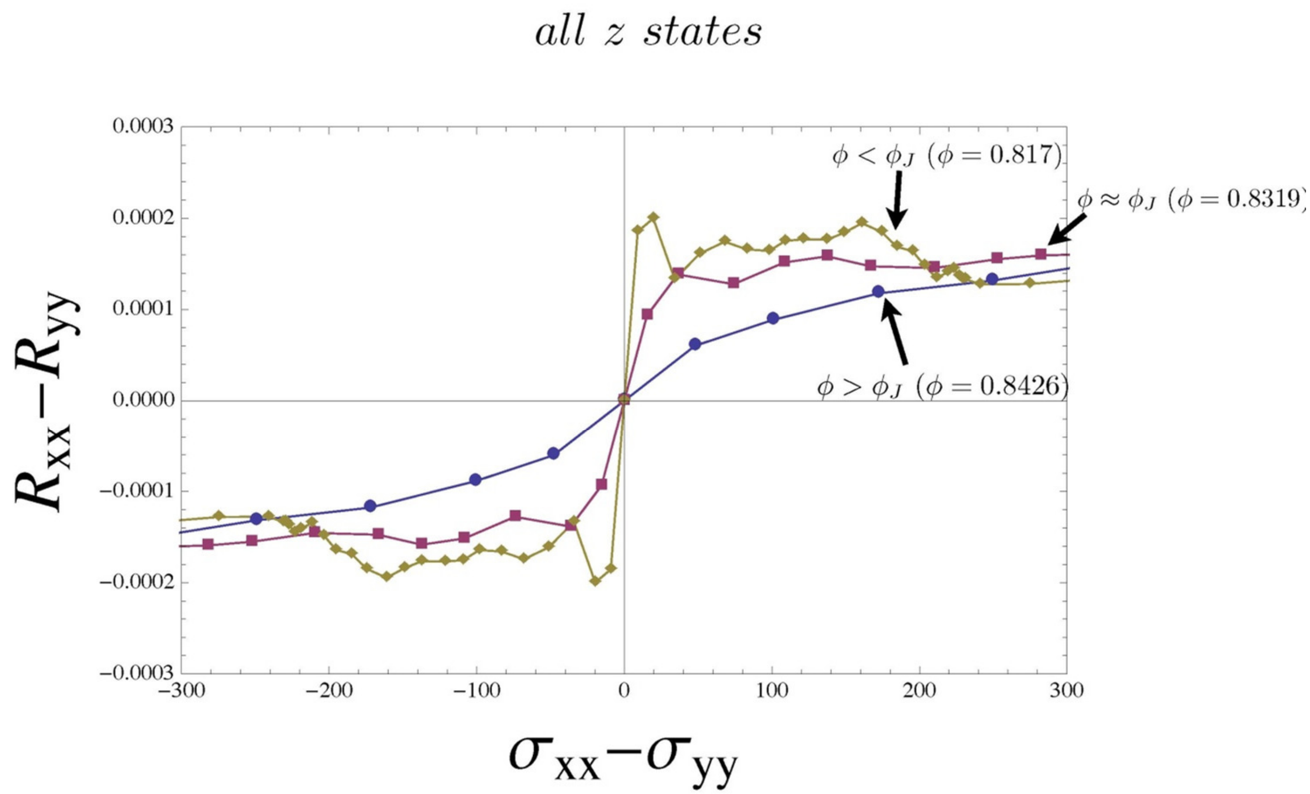
Fabric—Shear stress analogue to ferromagnetic critical point

Fabric tensor, R, gives
Geometric structure of network



Fabric—Shear stress analogue to ferromagnetic critical point

Key point: shear ordered states arise for $\varphi_S < \varphi < \varphi_J$. These anisotropic states appear to have a critical point at φ_J. Nature of φ_S to be determined.

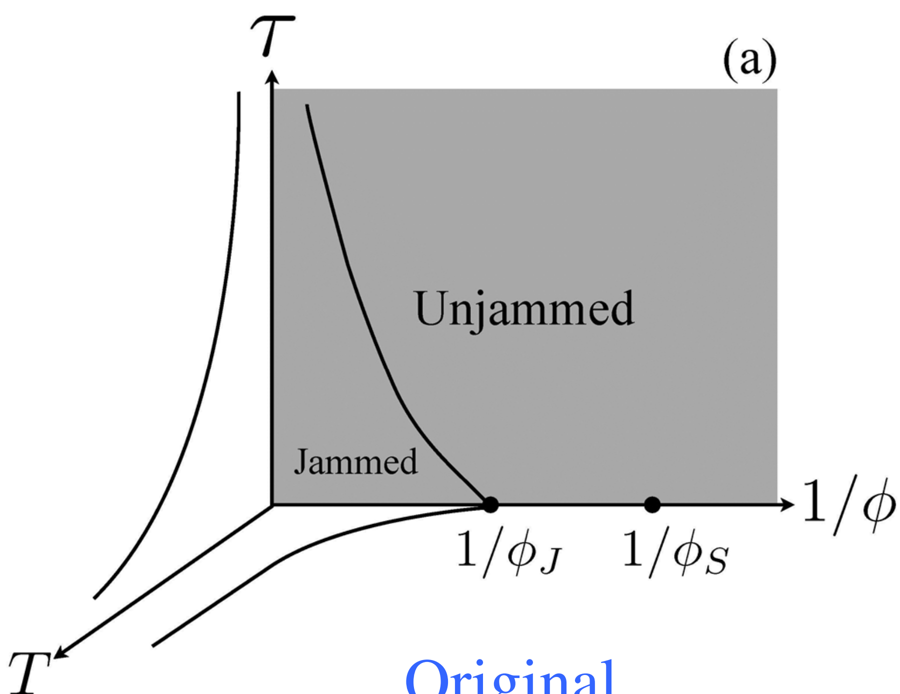


Jamming diagram for Frictional Particles

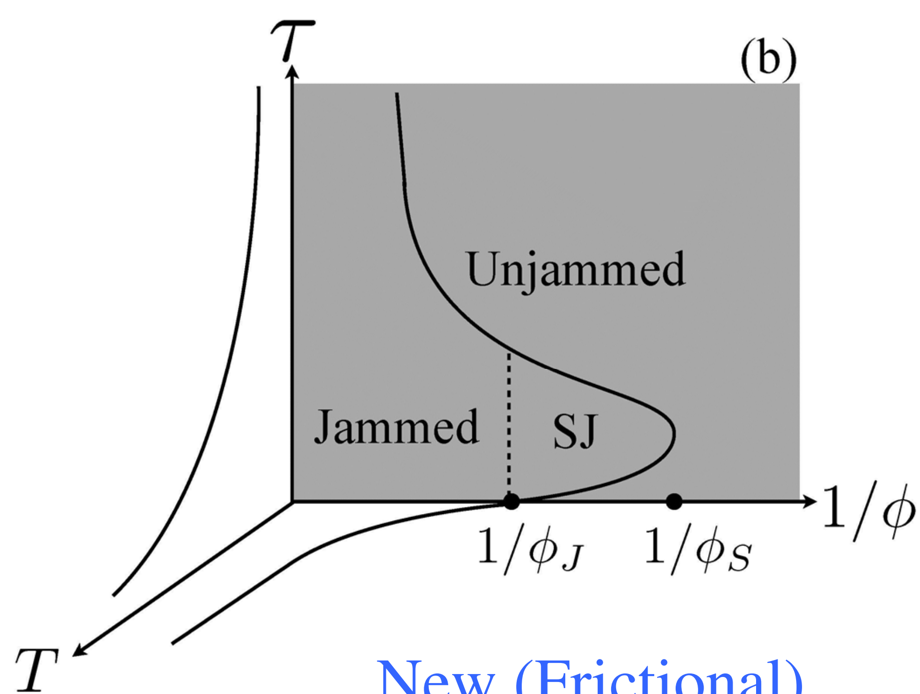
3D picture with axes P, τ and 1/φ

Two kinds of state, depending on φ

1) …$\varphi_S < \varphi < \varphi_J$—states arise under shear, $|\tau| > 0$
2) …$\varphi > \varphi_J$—jammed states occur at $\tau = 0$



Original



New (Frictional)

Conclusions--Questions

- MFT of KD and TE promising tool for characterizing fluctuations in granular shear flow
- …but rate dependence still an interesting and not completely resolved issue
- Shear strain applied to granular materials causes jamming for densities below φ_J
- What is nature of fluctuations associated with shear jamming?