Avalanches and Intermittency in Topological Defects Flows

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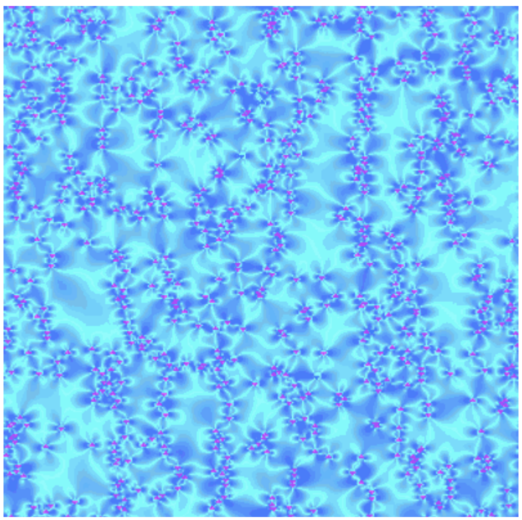
Motivation

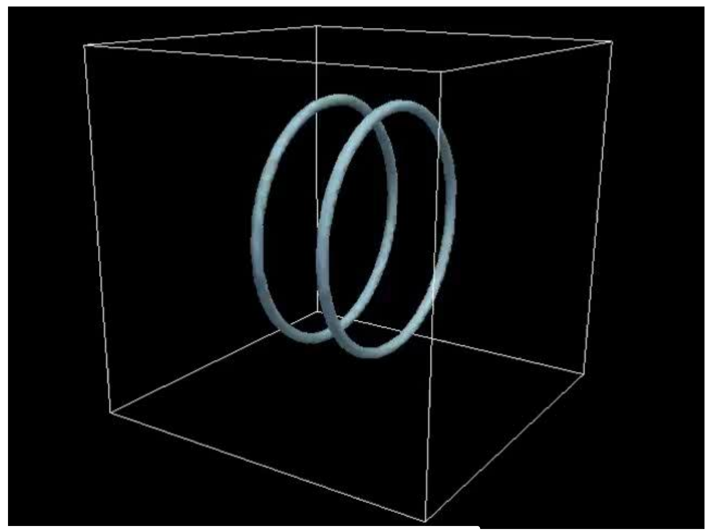
- **Quantum flows** are dominated by the discrete motion of vortex filaments in a similar way as **plastic deformations** are supported by the dynamics of dislocations
- **Avalanches** of plastic slips are due to the collective motion of dislocations interacting by long range elastic forces
- **Avalanches scenario** in quantum fluids analogous to dislocation slip avalanches in crystal plasticity

Dislocation dynamics



Miguel et al., Nature 2001

Vortex filaments dynamics



Simulation by P. Jeraldo

Outline

- **Plastic avalanches and non-Gaussian statistics in quantum fluids**
- **Extreme fluctuations from the long range interactions between topological defects:**
  - **dislocations for crystals**
  - **quantum vortices for superfluids**
- **Effect of correlations on the collective statistics**
- **Conclusions**

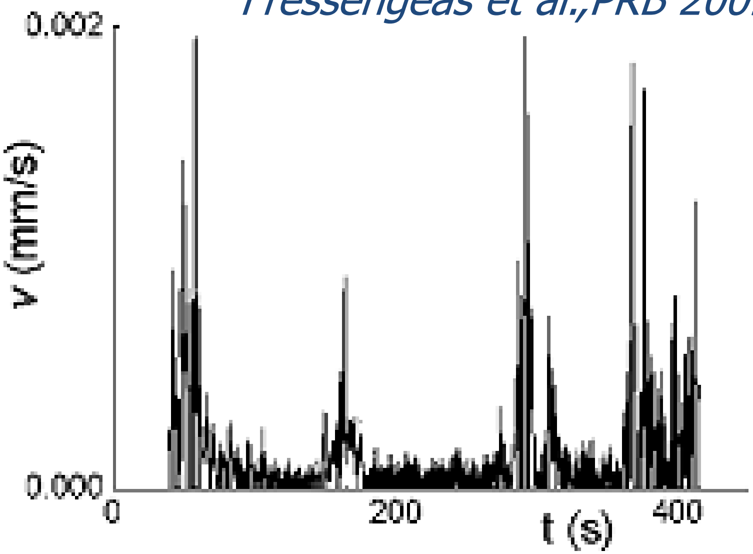
Plastic avalanches

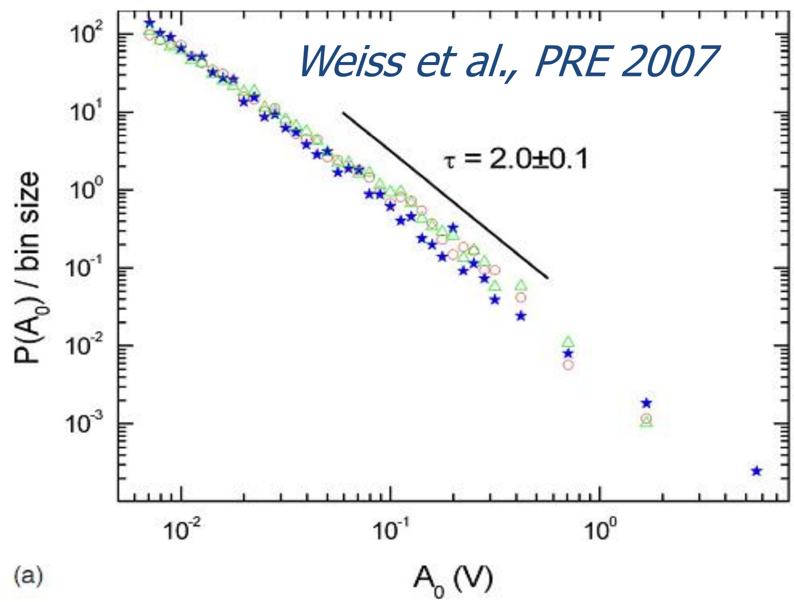
”seismology” of deformations at small scales and plastic creeps at large scales

"Seismological" activity in crystals

- **Experiments on single crystals under constant loads**
  - acoustic measurements, high-resolution extensometry
- **Robust power-law distribution of the acoustic events emitted during plastic deformations with avalanches**

Fressengeas et al.,PRB 2009

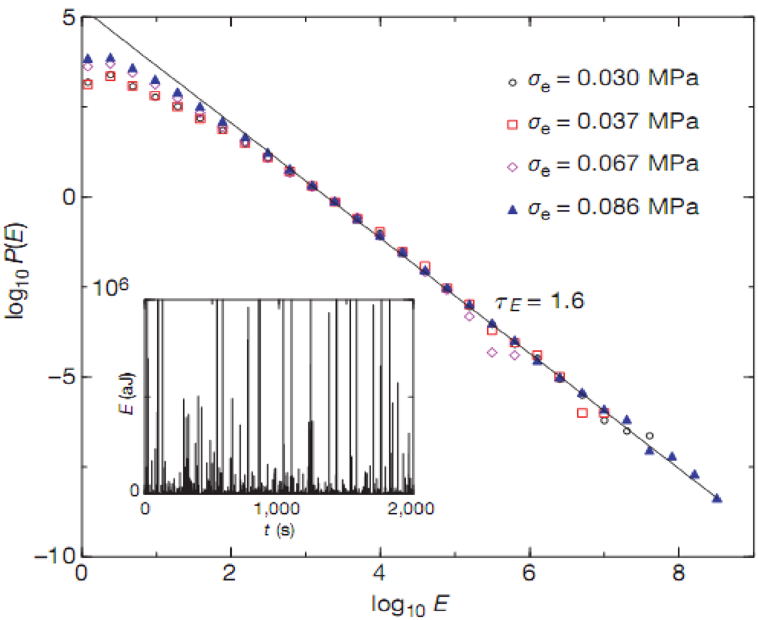


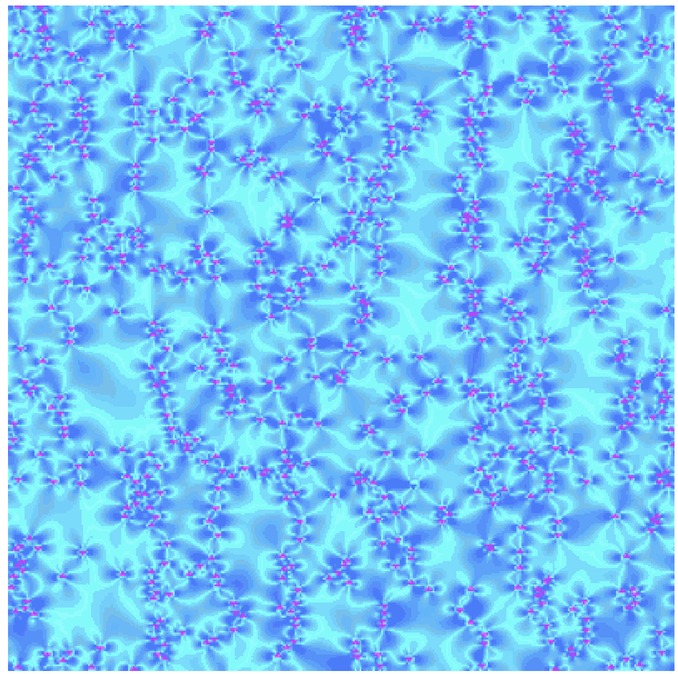


- **Different crystals: ice (single-slip), Zn or Cu (multislip)**
- **Different temperatures: near melting or far away**
- **Different loadings: compression, extension**

Intermittent plastic flow

- **Dislocation dynamics simulations**
- **Statistical comparison with plastic flow in ice crystals**
- **Dislocation avalanches and power-law distribution of plastic energy dissipation**

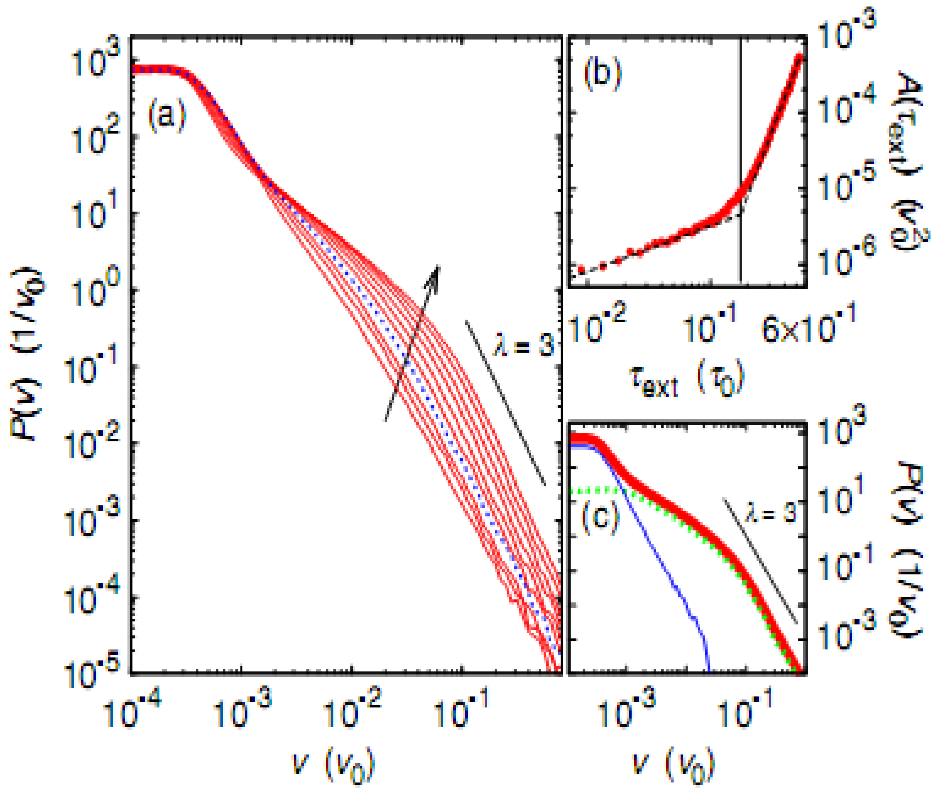




Miguel et al., Nature 2001

Cooperative dynamics in plasticity

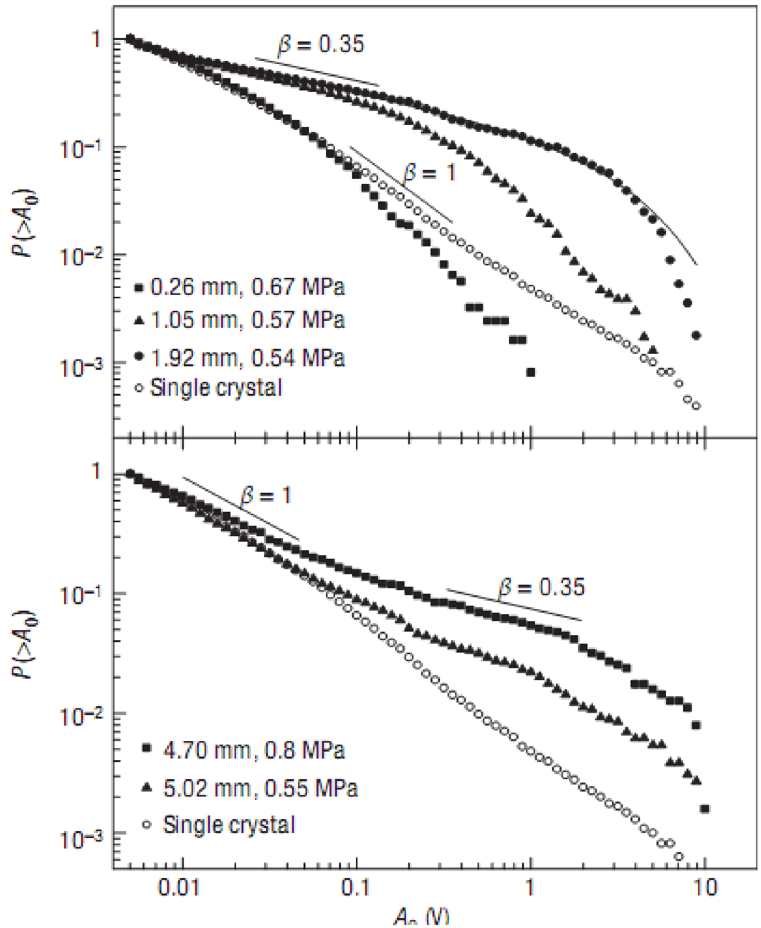
- **Discrete dislocation dynamics simulations**
- **Non-trivial yielding criteria at small scales**
- **Shoulder in the P(v) and avalanche regime**



Weygand et al., PRL 2010

Ways to suppress plastic avalanches

- **Polycrystalline materials**
  - **Grain size dependence of the avalanche cutoff and scaling exponents**
  - **Strain hardening**
  - **Smooth plastic flow at macroscopic scale**

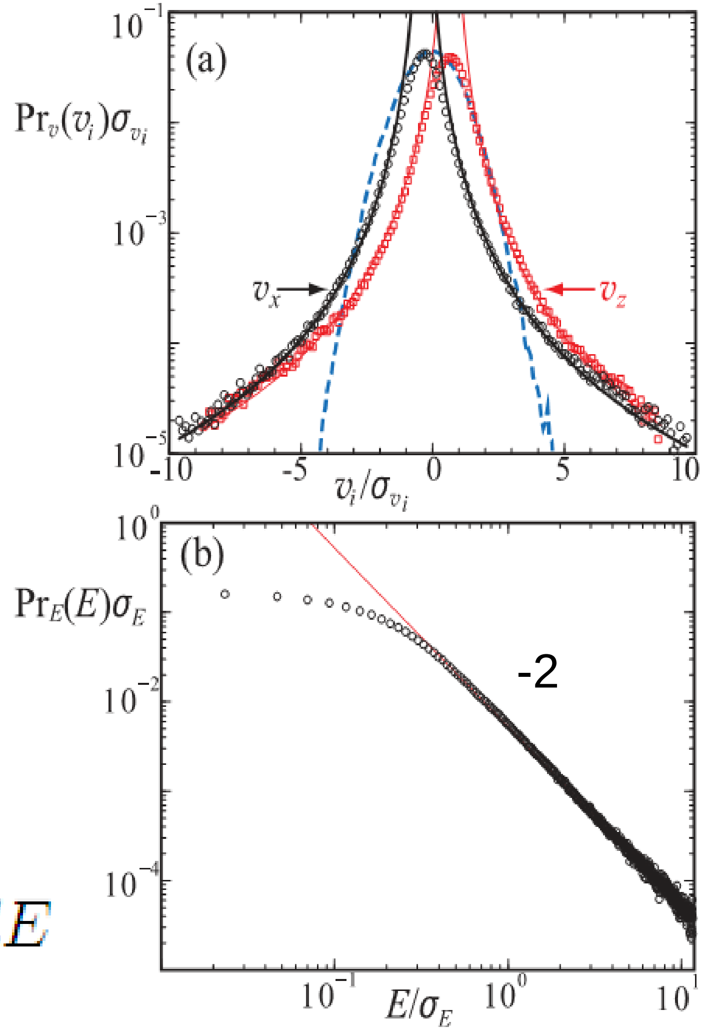


Richeton et al., Nature 2005

Non-Gaussian fluctuations of vortex velocity in quantum turbulence

Fluctuations in quantum turbulence

- **Decaying quantum turbulence with *^{4}He***
- **Velocity measured from the *H* passive tracers**
- **Large velocity is related to *H* tracers trapped near quantum vortices**
- **High vortex speeds near reconnection events**



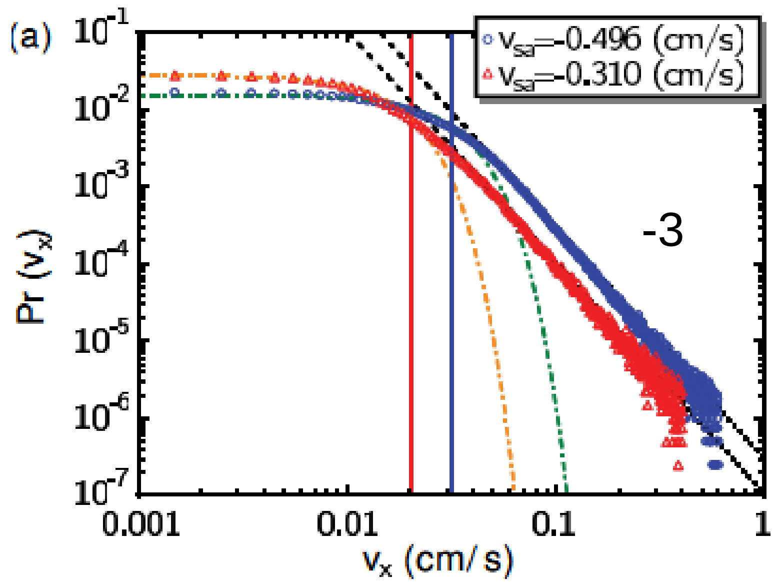
$$P(v)dv = P(E)dE$$
$$P(v) \sim v^{-3}$$

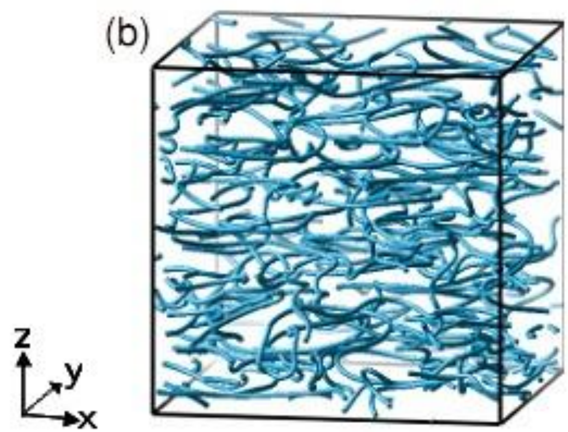
Paoletti et al., PRL 2008

Non-Gaussian statistics of Biot-Savart velocity field

- **Numerical study of thermal counterflow in a quantum fluid**
- **Biot-Savart: superflow velocity induced away from a vortex filament**
- **Cubic tail distribution**

$$P(v) \sim v^{-3}$$





Adachi and Tsubota, PRB 2011

Large fluctuation statistics in defects dominated flows

Dislocations, Topological defects, Long range interactions

Quantized defects in crystals: dislocations

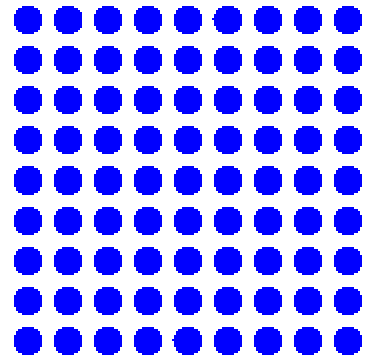
- **Screw, edge dislocations**
  - ***Burgers vector***

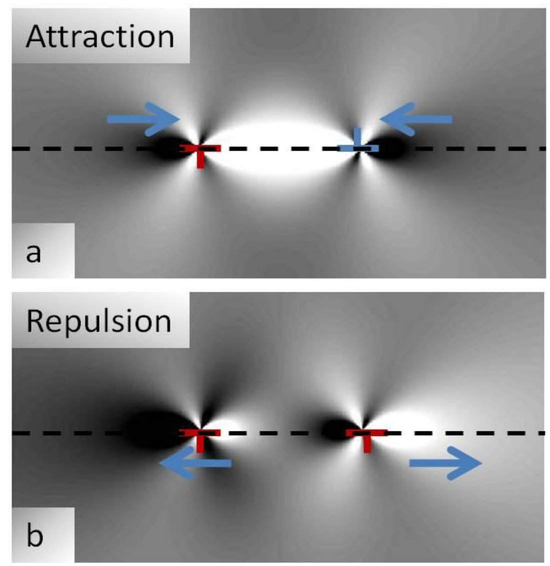
$$\oint u dl = b$$

- **Long range elastic fields near defects**

$$\tau \approx \frac{1}{r}$$

- **Motion in the elastic field induced by the other dislocations**





Other topological defects: quantum vortices

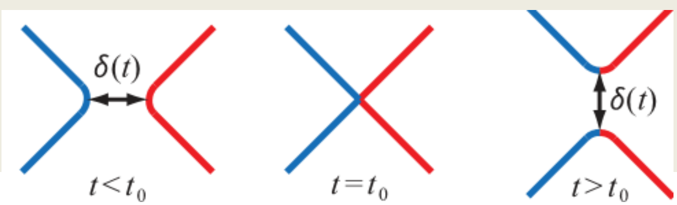
- **Quantized circulation**

$$\oint \boldsymbol{v} \cdot d\boldsymbol{r} = n\frac{h}{m}$$

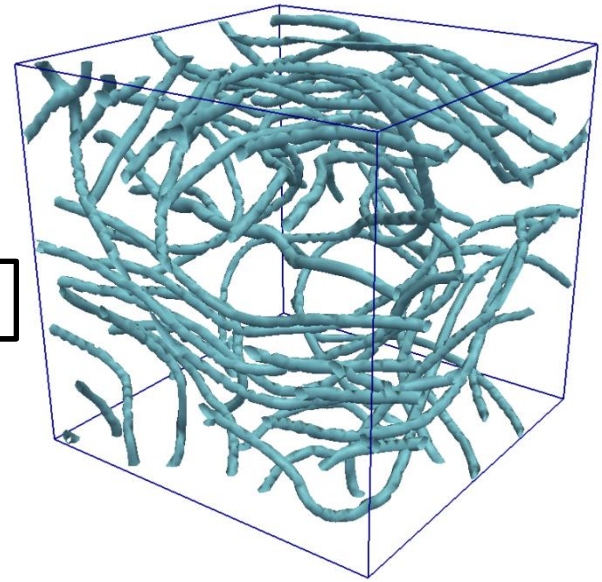
- **Velocity field near a vortex (unlike rigid body rotation)**

$$v \approx \frac{1}{r}$$

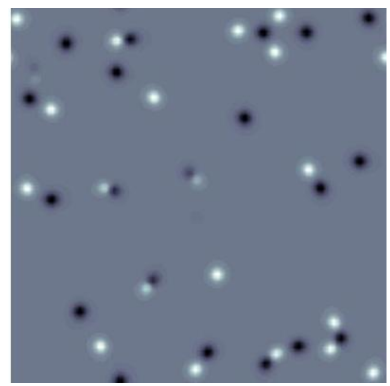
- **Motion due to interactions: annihilations, reconnections**



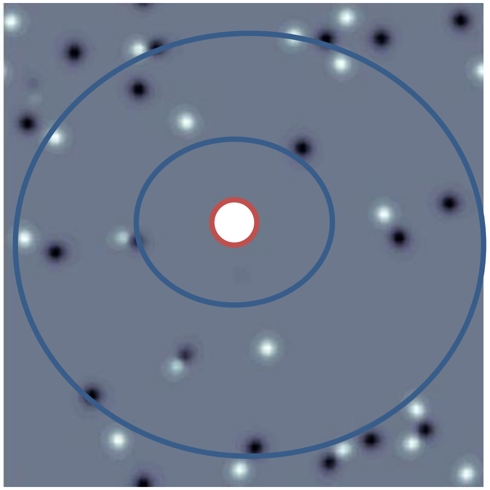
3D



2D



Non-Gaussian velocity distribution



Probability for a velocity between v and v+dv

$$P(v)dv = n(r)dr$$

Number of uncorrelated defects per unit volume with radius between r and r+dr

$$n(r)dr = L^2 f\left(L^{1/2} r\right) dr \sim L^{5/2} r dr$$

L **Filament density = Length/Volume**

Velocity at short distances

$$r << L^{-1/2}$$

$$v \approx \frac{1}{r} \Rightarrow P(v) \sim v^{-3}$$

Effective v at large distances

$$r >> L^{-1/2}$$

$$v \approx \frac{1}{r^2} \Rightarrow P(v) \sim v^{-2}$$

Chavanis, PRE 2002

Vortex dynamics in phase-ordering kinetics

O(2) model: complex order parameter in d-dimensions

$$\partial_t \psi = \nabla^2 \psi + (1 - |\psi|^2)\psi$$

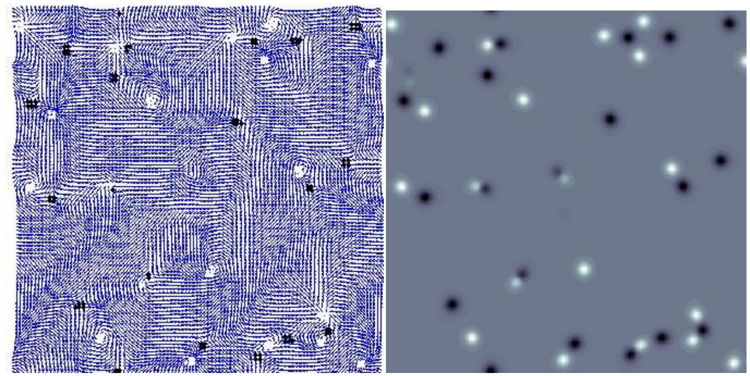
Location of defects

$$\mathcal{D} = \left|\left| \frac{\partial \psi_n}{\partial x_j} \right|\right|$$

Defect velocity (e.g. d=2)

$$\mathcal{D} v_x = -\frac{i}{2}\left(\partial_t \psi \partial_y \psi^* - \partial_t \psi^* \partial_y \psi\right)$$

$$\mathcal{D} v_y = \frac{i}{2}\left(\partial_t \psi \partial_x \psi^* - \partial_t \psi^* \partial_x \psi\right)$$



Topological defects = zeroes of ψ-field

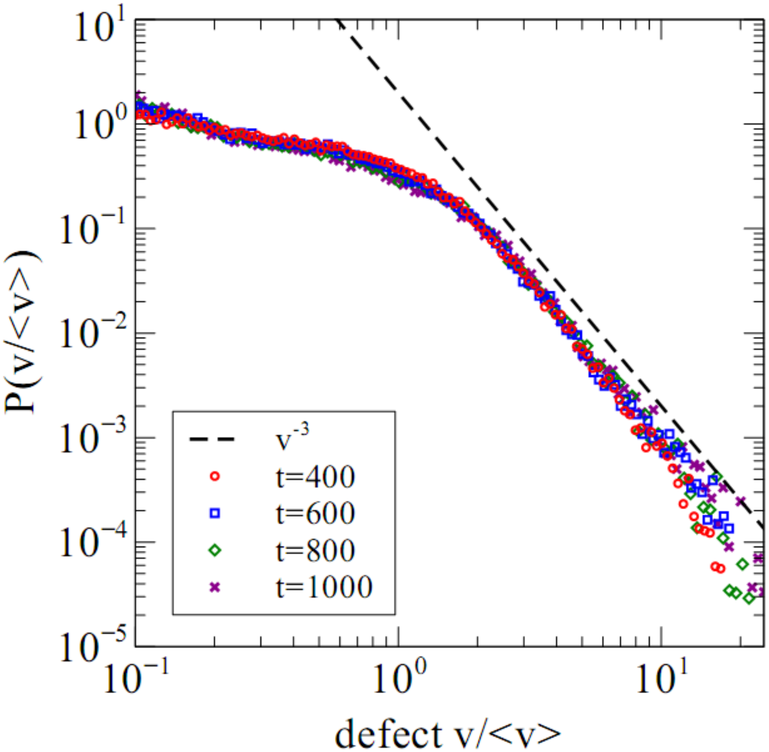
Topological charge at a given position

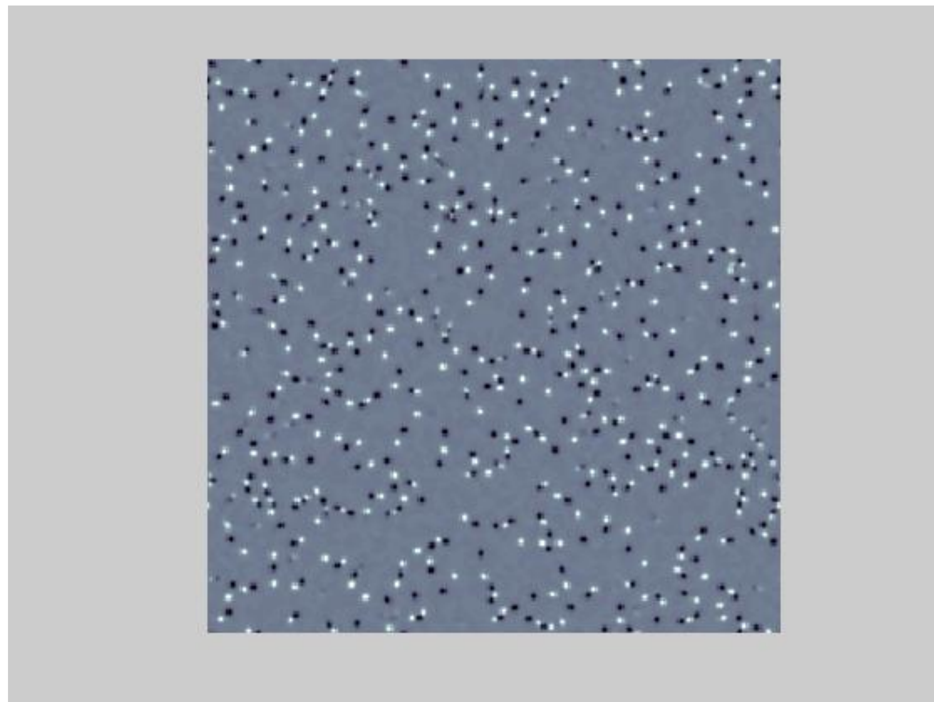
$$q_i = \frac{\mathcal{D}(\boldsymbol{r}_i)}{|\mathcal{D}(\boldsymbol{r}_i)|} = \pm 1$$

Mazenko, PRE 2002

Statistics of quenched vortices in 2D

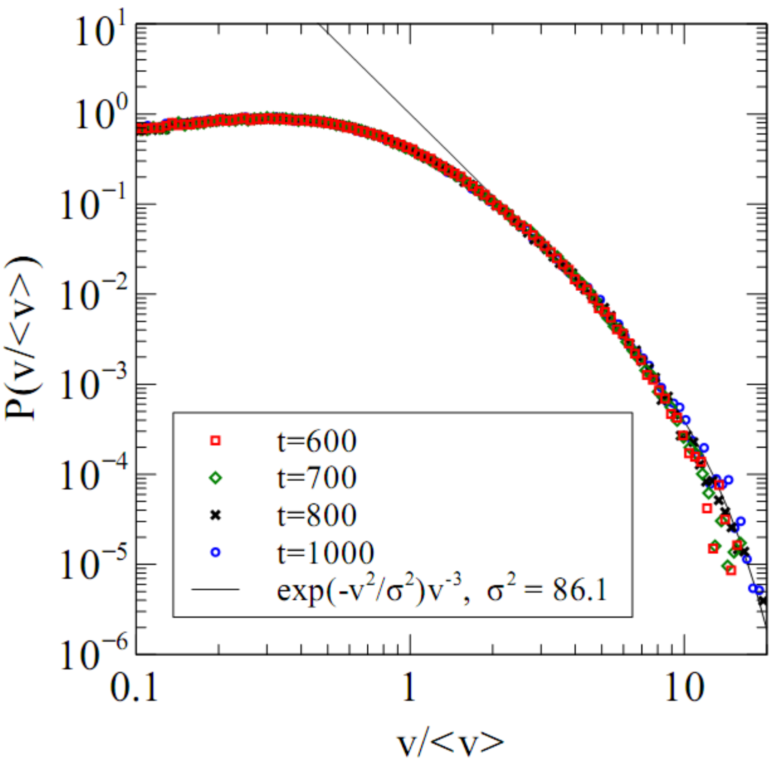
- **Coarsening dynamics due to annihilation of vortex pairs**
- **1/r interaction between vortices**

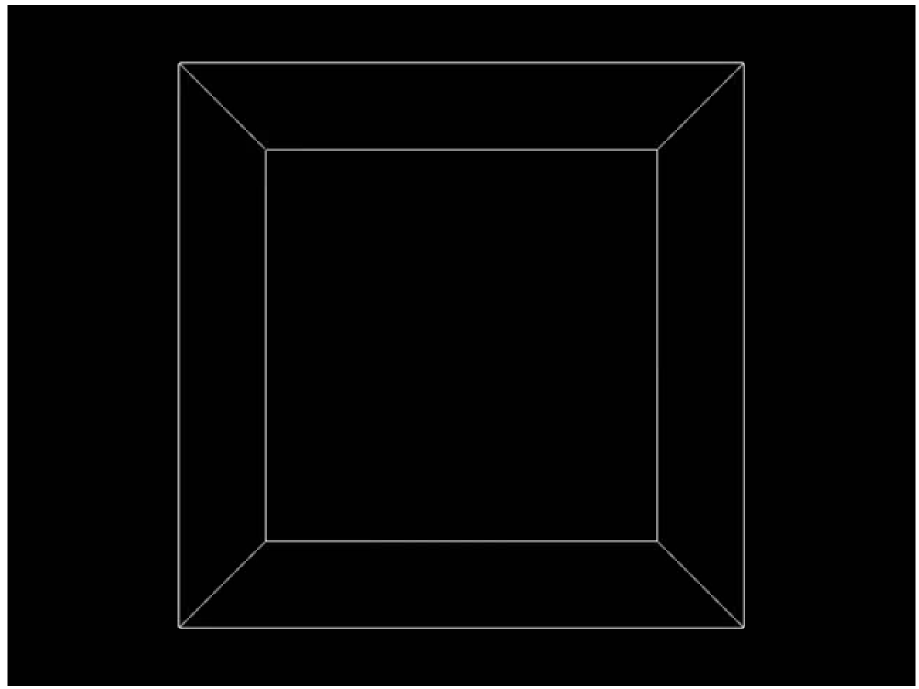




Vortex velocity in 3D quenches

- **Quenched dynamics of vortex filaments**
- **Gaussian cut-off due to vortex core size**



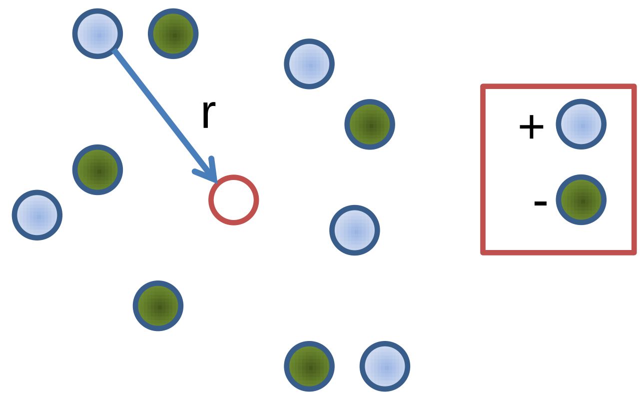


Correlated motion: Effect of cooperative interactions between defects

Distribution of collective stress

Stress induced by a single dislocation

$$\varphi^s(\mathbf{r}) = \frac{sK(\theta)}{r}$$



Collective stress at a point

$$\tau = \sum_{s=\pm1} \sum_{j=1}^{N^s} \varphi^s(\boldsymbol{r}_j)$$

Distribution of the collective stress

$$P_N(\tau) = \left\langle \delta\left(\tau - \sum_{s=\pm1} \sum_{j=1}^{N^s} \varphi_j^s\right)\right\rangle$$

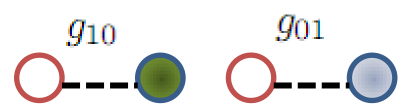
Cluster expansion

$$P_N(\tau) = \int dq e^{iq\tau} \left\langle \prod_{s=\pm 1} \prod_{j=1}^{N^s} e^{-iq\varphi_j^s} \right\rangle \sim \int dq e^{iq\tau} A(q)$$

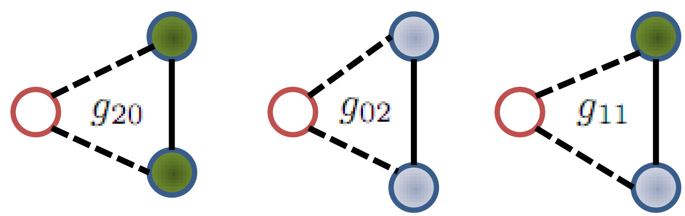
Linked cluster theorem:
A(q) = product of functions associated with irreducible k-clusters

$$A(q) = G_1(q) G_2(q) G_3(q) \ldots,$$

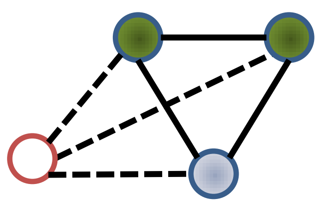
$G_1(q)$



$G_2(q)$



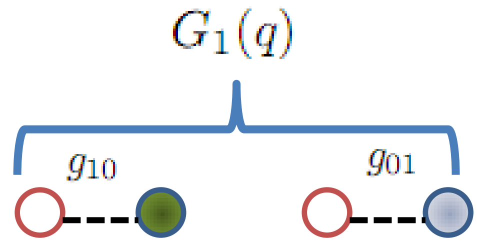
$G_3(q)$



Statistics of uncorrelated defects

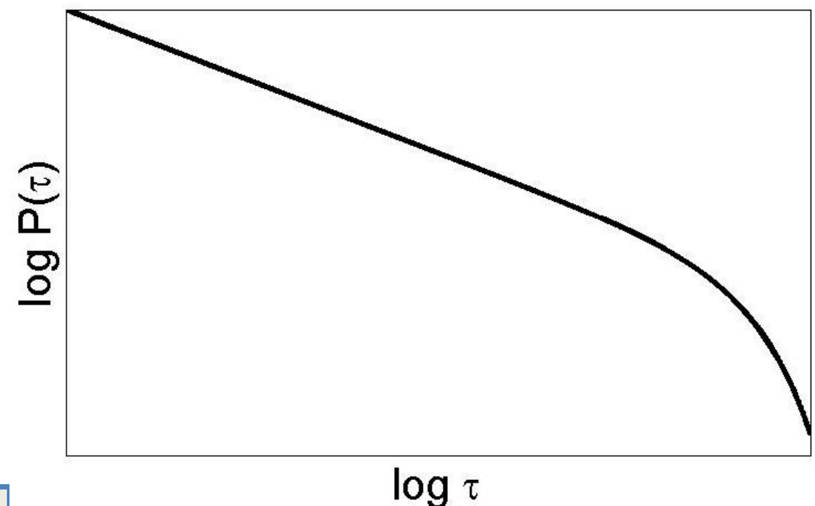
Characteristic function

$$\ln G_1(q) = -\frac{3-2\gamma}{4}\pi q^2 + \frac{\pi q^2}{2}\ln\frac{q}{N}$$



Tail distribution

$$P_N(\tau) \sim \int dq \cos(q\tau) G_1(q)$$
$$\sim \sqrt{2\pi}\tau^{-3}$$



For a finite core: Gaussian tail

Next order corrections

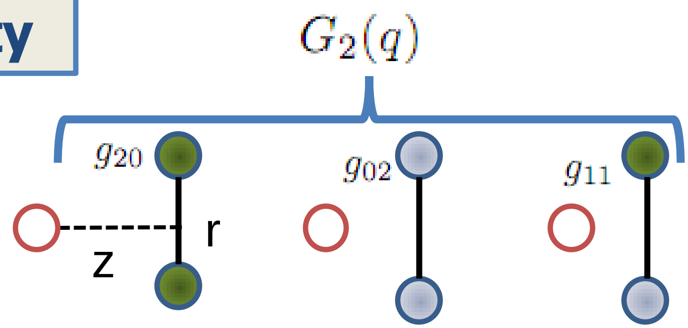
Slightly correlated defects, low density

$$\Gamma = n\left(\frac{Gb}{\tau_{ext}}\right)^2 \ll 1$$

$$G_2(q) = -\alpha\Gamma^{3/4}\frac{\pi q^2}{2}\ln\frac{q}{N} - \beta\Gamma^{3/4}q$$

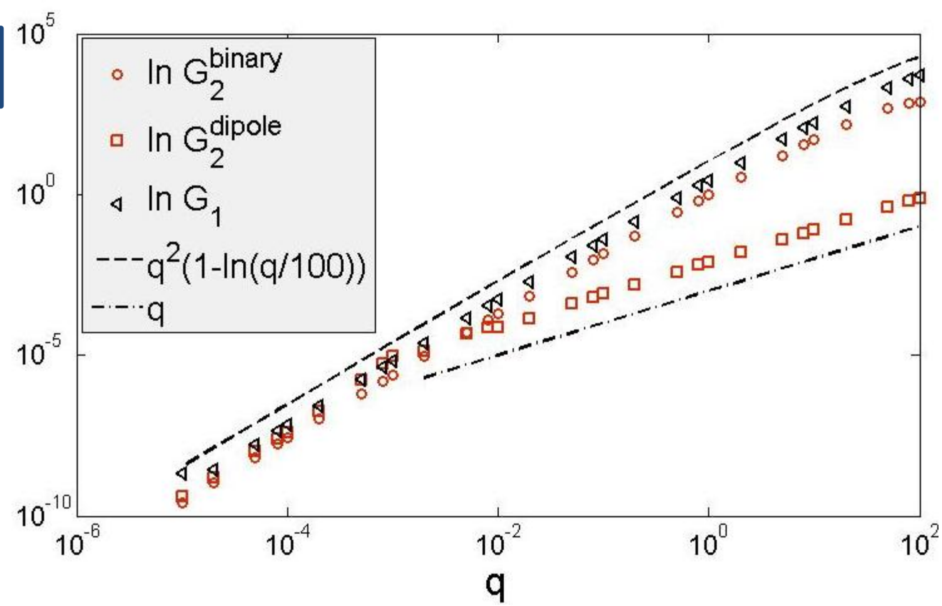
Binary

Dipole



Contribution from dipoles

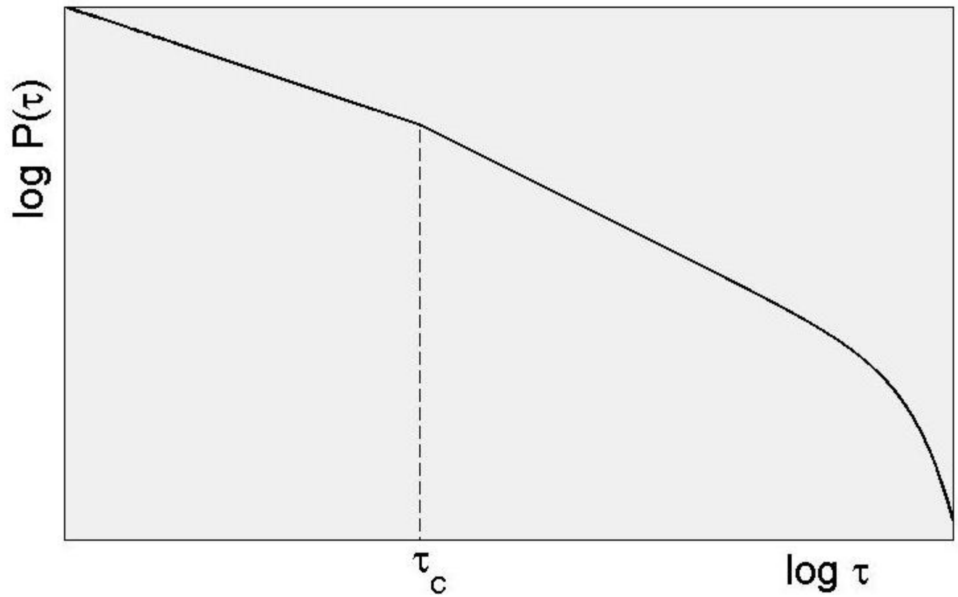
$$P(\tau) \sim \frac{\tau_0^2}{\tau_0^2 + \tau^2}$$



Statistics of correlated defects

Tail distribution

$$P(\tau) = \begin{cases} \tau^{-2}, \tau < \tau_c \\ \tau^{-3}, \tau > \tau_c \end{cases}$$



$\tau_0 = G\sqrt{n}$ Typical stress unit

$\tau_c \sim f(\tau_0, n)$ Cross-over stress

Conclusions

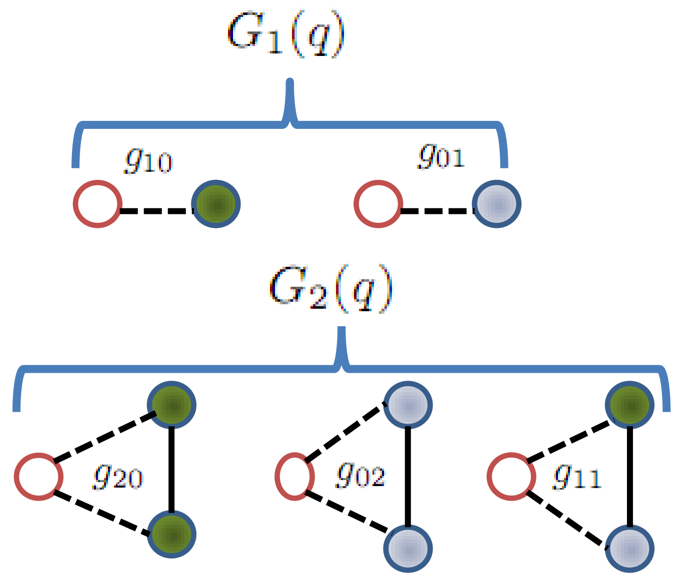
- **Intermittent flows induced by the collective motion of interacting topological defects**
- **Tail distribution of fluctuations determined by the long-range interactions between defects**
  - **Regime of -2 scaling: correlated motion**
  - **Regime of -3 scaling: local pairwise interactions**
- **Velocity statistics in the O(2) model is dominated by the local, uncorrelated interactions**
- **Avalanche statistics expected to be described by correlated interactions, thus associated with the -2 scaling regime**

Ongoing work on the velocity statistics in superfluid turbulence in the regime of correlated vortex interactions

Effect of pinning on defect avalanches

Cluster expansion of characteristic function

$$A(q) = G_1(q)G_2(q)G_3(q)\ldots,$$



With the cluster functions of kth-order

$$G_k(q) = \exp\left(\sum_{j+m=1}^{k} \frac{(n^+)^j}{j!}\frac{(n^-)^m}{m!}h_{jm}(q)\right)$$

Cluster integrals

$$h_{jm}(q) = \int \prod_{i=1}^{j} d^2\boldsymbol{r}_i \chi^+(\boldsymbol{r}_i) \prod_{n=1}^{m} d^2\boldsymbol{r}_n \chi^-(\boldsymbol{r}_n) g_{jm}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{j+m}).$$

$$\chi_j^s = e^{iq\varphi_j^s} - 1$$